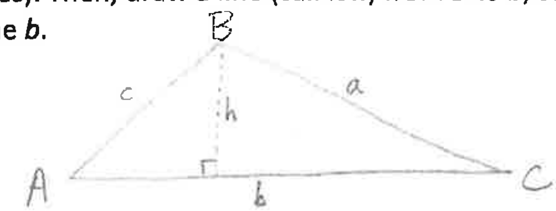
The Sine Law

So far, you have learned how to use trigonometry when working with right triangles. Now, you will learn how to use trigonometry for **oblique triangles** (non-right triangles).

developing the sine law

Draw an oblique triangle *ABC* and label the sides *a*, *b*, & *c* (opposite the respective corresponding angles). Then, draw a line (call it *h*) from *B* to *b*, so that it is perpendicular to line *b*.



Write a ratio for sin*A*, and then for sin*C*. Then, solve each for *h*.

$\sin A = \frac{h}{c}$ $\quad$ $\sin C = \frac{h}{a}$

$h = c \sin A$ $\quad$ $h = a \sin C$

Since each ratio is equal to *h*, they must also equal one another.

$c \sin A = a \sin C$ $\quad$ $\frac{\sin A}{a} = \frac{\cancel{a} \sin C}{\cancel{a}\, c}$

$\frac{c \sin A}{c} = \frac{a \sin C}{c}$ $\quad$ $\frac{\sin A}{a} = \frac{\sin C}{c}$

By using similar steps, you can also show the same for *b* and sin*B*.

sine law

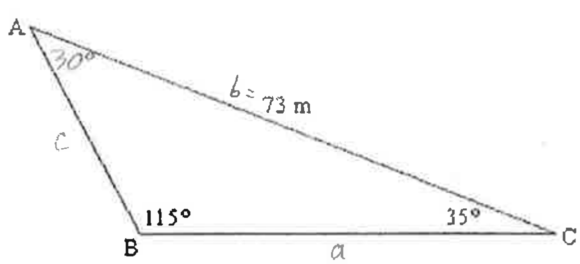
For any triangle, the sine law states that the sides of a triangle are proportional to the sines of the opposite angles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad OR \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1 – Solve for side AB and side BC to the nearest tenth.

c $\quad$ a $\quad$ side c

to find side AB:



$\frac{\sin B}{b} = \frac{\sin C}{c}$; $\frac{\sin 115°}{73} = \frac{\sin 35°}{c}$

$c \sin 115° = 73 \sin 35°$

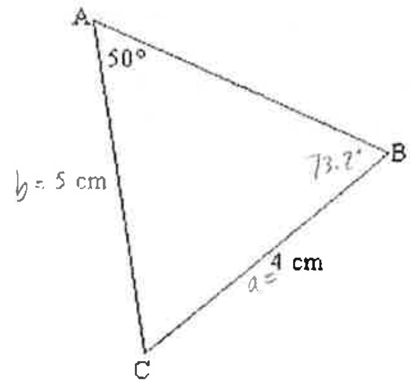
$c = \frac{73 \sin 35°}{\sin 115°} = 46.2\text{m}$

$\angle A = 180° - 115° - 35° = 30°$

To find side BC (side a): $\frac{\sin B}{b} = \frac{\sin A}{a}$; $\frac{\sin 115°}{73} = \frac{\sin 30°}{a}$

$a = \frac{73 \sin 30°}{\sin 115°} = 40.3\text{m}$

Example 2 – Solve for angle B to the nearest degree. Then find angle C to the nearest degree and side AB to the nearest tenth.



$\angle A = 50°$ $\quad a = 4cm$

$\angle B =$ ___ $\quad b = 5cm$

$\angle C =$ ___ $\quad c =$ ___

to find $\angle B$:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 50°}{4} = \frac{\sin B}{5}$$

$$\sin B = \frac{5 \sin 50°}{4}$$

$$\sin B = 0.95756$$

$$\angle B = \sin^{-1} 0.95756$$

$$\angle B = 73.2°$$

$$\angle C = 180 - 50° - 73.2$$

$$\angle C = 56.8° = 57°$$

side AB = side c

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 50°}{4} = \frac{\sin 56.8°}{c}$$

$$c = \frac{4 \sin 56.8°}{\sin 50°}$$

$$c = 4.4cm$$

information necessary to use the sine law

For oblique triangles, what is the minimum information needed in order to use the sine law to find new information?

Suppose we think of $\angle A$ and side a as 'partners'. Same for the other sets.

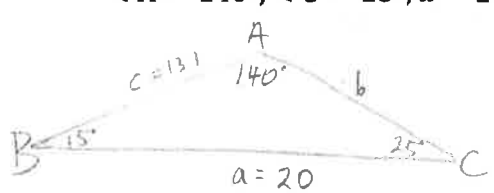
To use sine law, you must know info on 1 full set of 'partners' and half of another 'partnership'.

solving a triangle

When **solving a triangle**, you must find all of the unknown angles and sides.

Example – Sketch and solve the triangle (each answer to the nearest tenth).

$$\angle A = 140°, \angle C = 25°, a = 20$$



$\angle A = 140°$ $\quad a = 20$

$\angle B =$ ___ $\quad b =$ ___

$\angle C = 25°$ $\quad c =$ ___

$$\angle B = 180 - 140 - 25 = 15°$$

$$\angle B = 15°$$

To find side c: $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\frac{\sin 140°}{20} = \frac{\sin 25°}{c}; \quad c = \frac{20 \sin 25°}{\sin 140°}$$

$$c = 13.1$$

To find side b: $\frac{\sin A}{a} = \frac{\sin B}{b}$

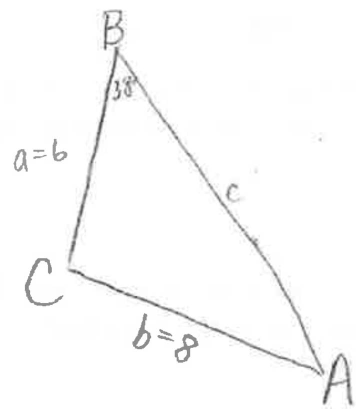
*can't use Pythag as not a right tri

$$\frac{\sin 140°}{20} = \frac{\sin 15°}{b}$$

$$b = \frac{20 \sin 15°}{\sin 140°}; \quad b = 8.1$$

Example – Solve the triangle (round to the nearest whole number).

$< B = 38°, b = 8, a = 6$



$\angle A =$ ___
$\angle B = 38°$
$\angle C =$ ___

$a = 6$
$b = 8$
$c =$ ___

To find $\angle A$: $\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin 38°}{8} = \frac{\sin A}{6}$

$\sin A = \frac{6 \sin 38°}{8}$

$\sin A = 0.4617461$

$\angle A = \sin^{-1} 0.4617461$

$\angle A = 27.4998 = 27°$

$\angle A = 27°$

To find $\angle C$:

$180 - 38 - 27.4998 = 114.5002$

$\angle C = 115°$

To find side c:

*cannot use Pythag!

$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 38°}{8} = \frac{\sin 114.5}{c}$

$c = \frac{8 \sin 114.5°}{\sin 38°}$

$c = 11.8 = 12$

$c = 12$

The Cosine Law

For right triangles, the trigonometric ratios sine, cosine, and tangent can be used to find unknown sides and angles. For oblique triangles, **sine law** and **cosine law** must be used.

An effective way to work with oblique triangles is to imagine the angle and its opposite side as 'partners'. Thus, angle A and side a are partners, $<B$ and b are partners, and $<C$ and c are partners.

In order to use the sine law, you must know one full set of partners and half of another set. If you know only half of each set of the three partners, at least two of which are sides, you must use **cosine law**.

Example – For each oblique triangle, state which law you would use.

a) $x = 30$cm, $y = 28$cm, $z = 32$cm (b) $<C = 27°$, $a = 17$m, $c = 13$m (c) $<J = 41°$, $k = 16$cm, $p = 14$cm

a)
- 3 half partners
- all are sides

COSINE LAW

(b) full set of partners & half of another

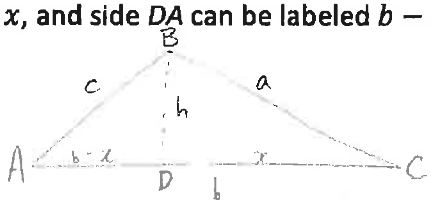
SINE LAW

(c) 3 half partners, 2 of which are sides

COSINE LAW

deriving cosine law

1. The **cosine law** can be developed by starting with oblique ΔABC and drawing vertical line h from $<B$ to side b. Where h meets side b, call that vertex D. Side CD can then be labeled x, and side DA can be labeled $b - x$.



2. For ΔBCD, find $\cos C$ and rearrange the equation to isolate x. Then write a Pythagorean equation for ΔBCD.

$\cos C = \frac{x}{a}$; $x = a\cos C$

$x^2 + h^2 = a^2$

3. Next, for ΔABD, write a Pythagorean equation. Then FOIL $(b - x)^2$. Can you see where a^2 can now replace a part of the equation? What can you replace for x?

$h^2 + (b-x)^2 = c^2$

$h^2 + (b-x)(b-x) = c^2$

$h^2 + b^2 - 2bx + x^2 = c^2$

$\underbrace{x^2 + h^2}_{} + b^2 - 2bx = c^2$

$a^2 + b^2 - 2b\underbrace{x}_{a\cos C} = c^2$

$c^2 = a^2 + b^2 - 2ba\cos C$

$c^2 = a^2 + b^2 - 2ab\cos C$ Cosine law!

cosine law

The **cosine law** describes the relationship between the cosine of an angle and the lengths of the three sides of any triangle.

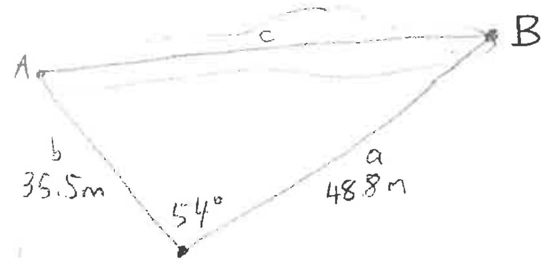
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Cosine law can also be written as $a^2 = b^2 + c^2 - 2bc\cos A$ OR

$$b^2 = a^2 + c^2 - 2ac\cos B$$

Example 1 – Kohl wants to find the distance between two points, *A* and *B*, on opposite sides of a pond. She locates a point *C* that is 35.5m from *A* and 48.8m from *B*. If the angle at *C* is 54°, determine the distance *AB*, to the nearest tenth of a metre.

3 half partners, 2 of which are sides
COSINE LAW



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (48.8)^2 + (35.5)^2 - 2(48.8)(35.5)\cos 54°$

$c^2 = 2381.44 + 1260.25 - 2036.56$

$c^2 = 1605.13$

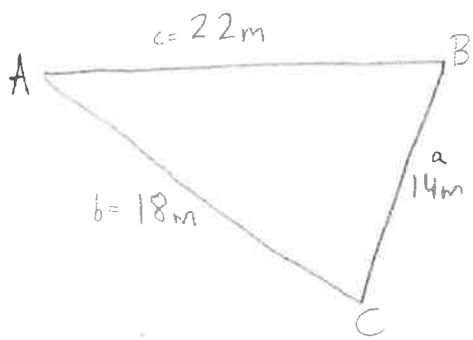
$c = \sqrt{1605.13}$

$c = 40.1\text{m}$

<A = ___ a = 48.8
<B = ___ b = 35.5
<C = 54° c = ___

The pond is 40.1m long

Example 2 – A triangular brace has side lengths 14m, 18m, and 22m. Determine the measure of the angle opposite the 18m side, to the nearest degree.



Must find <B

$b^2 = a^2 + c^2 - 2ac\cos B$

$18^2 = 14^2 + 22^2 - 2(14)(22)\cos B$

$324 = 196 + 484 - 616\cos B$

$324 = 680 - 616\cos B$
$-680 \quad \cancel{-680}$

$\frac{-356}{-616} = \frac{-\cancel{616}\cos B}{\cancel{-616}}$

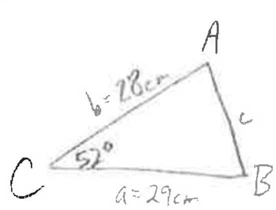
$0.57792 = \cos B$

$<B = \cos^{-1} 0.57792$

$<B = 54.7° \approx 55°$

using cosine law & sine law

Example 3 – In Δ*ABC*, *a* = 29cm, *b* = 28cm, and <*C* = 52°. Sketch a diagram and determine the length of the unknown side and the measures of the unknown angles, to the nearest tenth.



must start with cos law
(no full set of partners)

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (29)^2 (28)^2 - 2(29)(28)\cos 52°$

$c^2 = 841 + 784 - 999.834$

$c^2 = 625.166$

$c = \sqrt{625.166}$

$c = 25\text{ cm}$

<A = ___ a = 29
<B = ___ b = 28
<C = 52° c = ___

To find <A:

$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin 52°}{25} = \frac{\sin A}{29}$

$\sin A = \frac{29 \sin 52°}{25}$

$\sin A = 0.914$

$<A = \sin^{-1} 0.914$

$<A = 66.1°$

$<C = 180 - 52 - 66.1$

$<C = 61.9°$