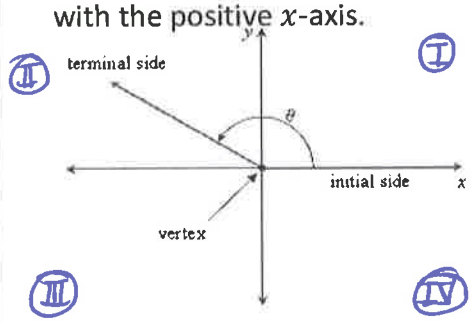
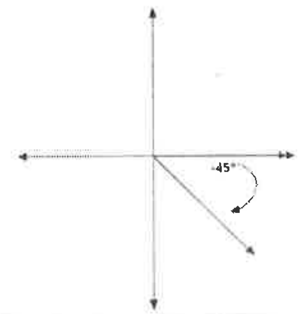
angles in standard position

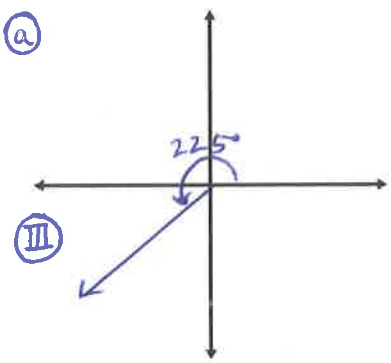
An angle that is drawn in **standard position** must have its vertex at the origin of the Cartesian plane, and its initial arm must coincide with the positive x-axis.

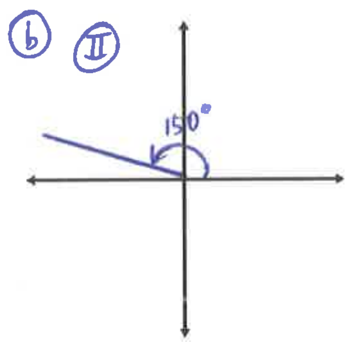


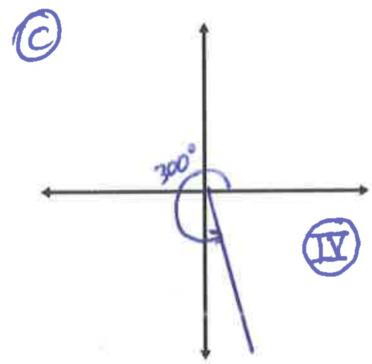
Clockwise angles have a negative measure:



Example 1– Draw each angle in standard position and identify the quadrant in which it lies: a) 225° b) 150° c) 300°

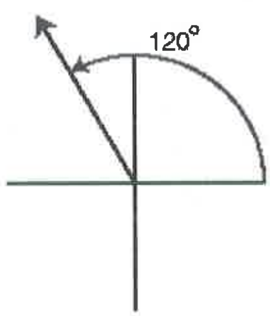


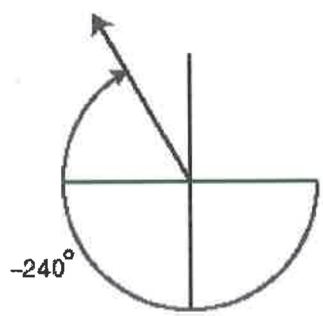


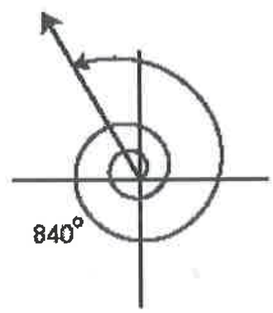


coterminal angles

Angles in standard position that have the same terminal side are **coterminal**.







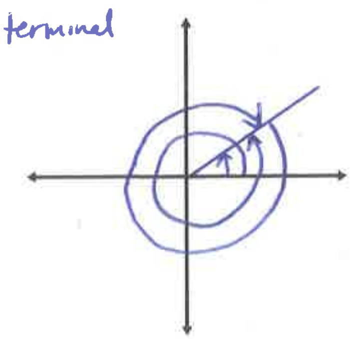
also:
$120+360 = 480°$

Example 2 – Find each to the nearest thousandth.

a) cos 30° = 0.866 b) cos 390° = 0.866 c) cos -330° = 0.866

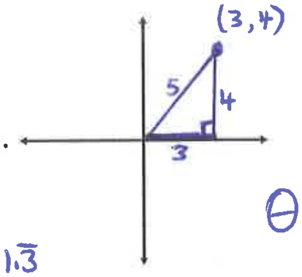
these angles are all coterminal so the terminal arm is in the same position

What do you notice?



Let's draw a 3, 4, 5 triangle in Quadrant 1:

Write sin, cos, and tan ratios in fraction & decimal.

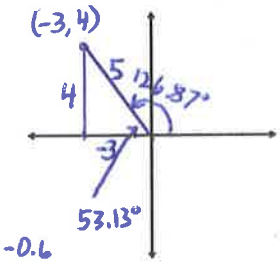


Find θ.

$\theta = 53.13°$

$\sin\theta = \frac{4}{5} = 0.8$; $\cos\theta = \frac{3}{5} = 0.6$; $\tan\theta = \frac{4}{3} = 1.\overline{3}$

Draw the angle 126.87° in standard position



Make a triangle with your terminal arm and the x-axis.

What is the angle inside the triangle? 53.13°

Label the three sides of the triangle. $\sin\theta = \frac{4}{5} = 0.8$; $\cos\theta = \frac{-3}{5} = -0.6$

Write sin, cos, and tan ratios for the triangle in fraction and decimal. $\tan\theta = \frac{4}{-3} = -1.\overline{3}$

Notice that the target angle is again 53.13°, but this time cosine and tangent are different ratios than you saw when the triangle was drawn in Quadrant I (cosine and tangent are negative).

Though the target angle **inside** the triangle is 53.13°, this triangle is in Quadrant II, so to represent that, ***θ* is actually the angle in standard position**, which in this case is 126.87°.

Compute sin 126.87, cos 126.87, and tan 126.87.

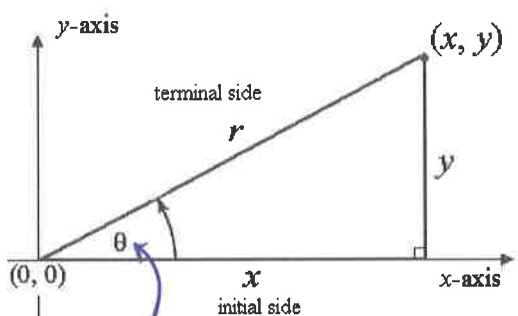
$\sin 126.87° = 0.8$

$\cos 126.87° = -0.6$

$\tan 126.87° = -1.\overline{3}$

In Quadrant II, the x-coordinate is negative (the adjacent side), so since cosine and tangent are built with adj involved, they are negative in Quadrant II.

Suppose θ is an angle in standard position. Suppose the point at the end of the terminal arm is labeled $P(x, y)$, at a distance r from the origin.



the angle inside the triangle is called the 'reference' angle

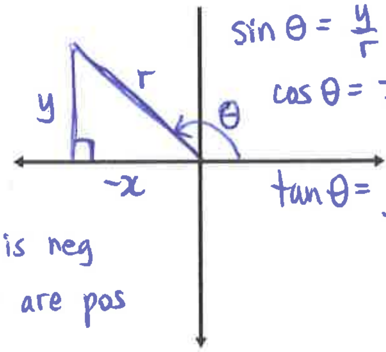
You can use a reference angle to determine the three trigonometric ratios in terms of x, y, and r.

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

r is the length of the terminal arm, and lengths are always positive.

Trigonometry ratios in the four quadrants:

Quadrant 2 $90° < \theta < 180°$



$\sin\theta = \frac{y}{r}$ (+)

$\cos\theta = \frac{-x}{r}$ (−)

$\tan\theta = \frac{y}{-x}$ (−)

x is neg

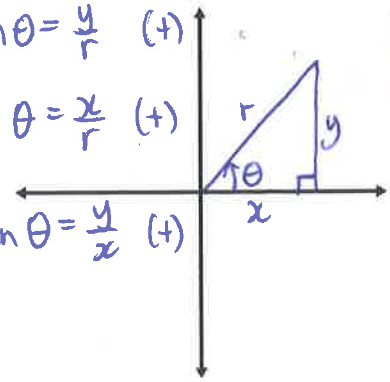
y, r are pos

Quadrant 1 $0° < \theta < 90°$

$\sin\theta = \frac{y}{r}$ (+)

$\cos\theta = \frac{x}{r}$ (+)

$\tan\theta = \frac{y}{x}$ (+)

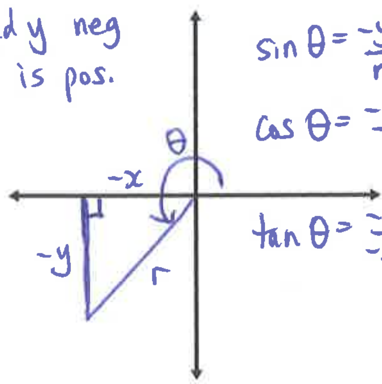


x, y, r all positive

Quadrant 3 $180° < \theta < 270°$

x and y neg

r is pos.



$\sin\theta = \frac{-y}{r}$ (−)

$\cos\theta = \frac{-x}{r}$ (−)

$\tan\theta = \frac{-y}{-x} = \frac{y}{x}$ (+)

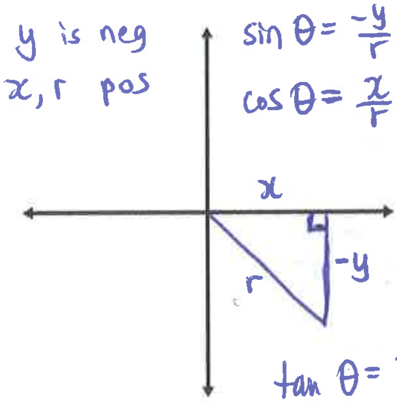
Quadrant 4 $270° < \theta < 360°$

y is neg

x, r pos

$\sin\theta = \frac{-y}{r}$ (−)

$\cos\theta = \frac{x}{r}$ (+)



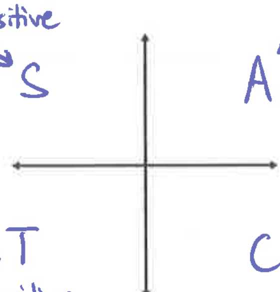
$\tan\theta = \frac{-y}{x}$ (−)

CAST

Here is a way to remember the sign of the trigonometric ratios in each quadrant:

"S"in positive → S

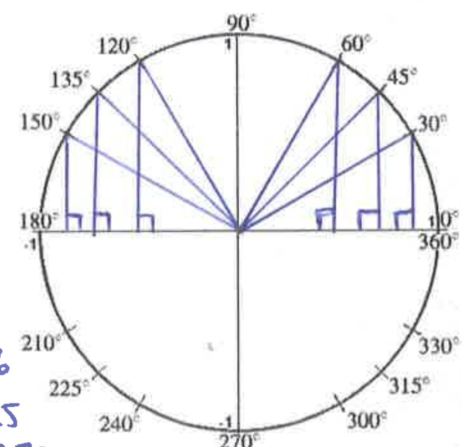
A ← "A"ll ratios positive



→ T

"T"an positive

C ← "C"os positive



With calc, find sin, cos, tan for 30°, 45° 60°

$\sin 30° = 0.5$ $\sin 45° = 0.707$ $\sin 60° = 0.866$

$\cos 30° = 0.866$ $\cos 45° = 0.707$ $\cos 60° = 0.5$

$\tan 30° = 0.577$ $\tan 45° = 1$ $\tan 60° = 1.73$

Without calc, give sin, cos, tan for: 120, 135, 150,

$\sin 150° = 0.5$ $\sin 135° = 0.707$ $\sin 120° = 0.866$

$\cos 150° = -0.866$ $\cos 135° = -0.707$ $\cos 120° = -0.5$

$\tan 150° = -0.577$ $\tan 135° = -1$ $\tan 120° = -1.73$

210, 225, 240

$\sin 210° = -0.5$ $\sin 225° = -0.707$ $\sin 240° = -0.866$

$\cos 210° = -0.866$ $\cos 225° = -0.707$ $\cos 240° = -0.5$

$\tan 210° = 0.577$ $\tan 225° = 1$ $\tan 240° = 1.73$

300, 315, 330

$\sin 330° = -0.5$ $\sin 315° = -0.707$ $\sin 300° = -0.866$

$\cos 330° = 0.866$ $\cos 315° = 0.707$ $\cos 300° = 0.5$

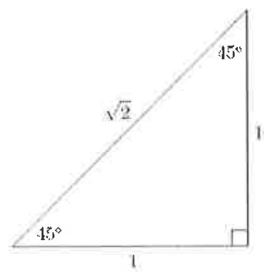
$\tan 330° = -0.577$ $\tan 315° = -1$ $\tan 300° = -1.73$

Day 2 of Notes: **Watch unit circle animation:**

https://www.desmos.com/calculator/n8kwtwfrce - toggle 'a' value on left

Special Right Triangles

A 45°-45°-90° triangle with legs of each 1 unit has a hypotenuse of $\sqrt{2}$.



$$sin\theta = \frac{opposite}{hypotenuse} \quad cos\theta = \frac{adjacent}{hypotenuse} \quad tan\theta = \frac{opposite}{adjacent}$$

S O H C A H T O A

For the 45°-45°-90° triangle,

$$sin45° = \frac{1}{\sqrt{2}} \quad cos45° = \frac{1}{\sqrt{2}} \quad tan45° = \frac{1}{1} = 1$$

The trigonometric ratios are given as **exact values** (in fraction/radical form as opposed to an approximated decimal).

Make triangles for 45, 135, 225, 315:

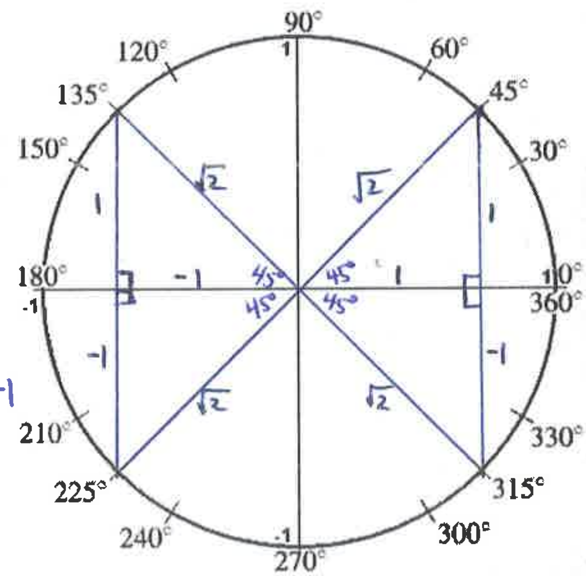
Build sin, cos, tan ratios for each:

$\sin 45° = \frac{1}{\sqrt{2}}$ $\cos 45° = \frac{1}{\sqrt{2}}$ $\tan 45° = 1$

$\sin 135° = \frac{1}{\sqrt{2}}$ $\cos 135° = \frac{-1}{\sqrt{2}}$ $\tan 135° = -1$

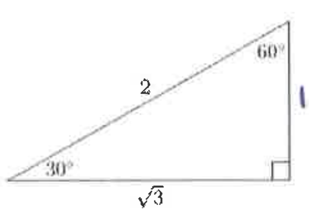
$\sin 225° = \frac{-1}{\sqrt{2}}$ $\cos 225° = \frac{-1}{\sqrt{2}}$ $\tan 225° = 1$

$\sin 315° = \frac{-1}{\sqrt{2}}$ $\cos 315° = \frac{1}{\sqrt{2}}$ $\tan 315° = -1$



Verify with a calculator.

A 30°-60°-90° triangle has legs of 1 unit and $\sqrt{3}$ units, with a hypotenuse of 2 units.



For the 30°-60°-90° triangle,

$$sin30° = \frac{1}{2} \quad cos30° = \frac{\sqrt{3}}{2} \quad tan30° = \frac{1}{\sqrt{3}}$$

$$sin60° = \frac{\sqrt{3}}{2} \quad cos60° = \frac{1}{2} \quad tan60° = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Make triangles for 30, 150, 210, 330:

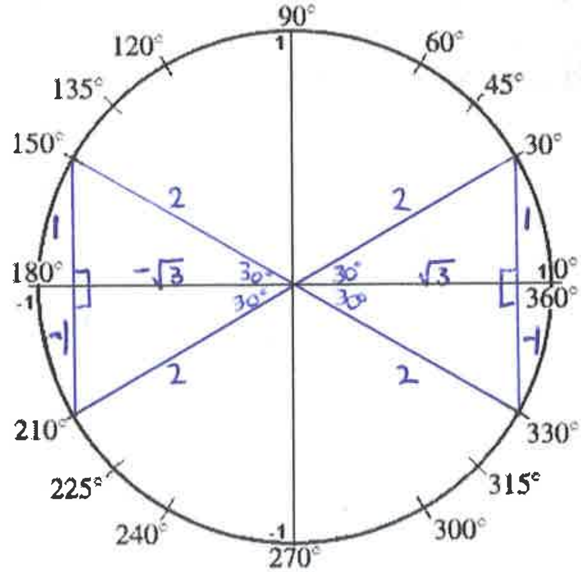
Build sin, cos, tan ratios for each:

$\sin 30° = \frac{1}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\tan 30° = \frac{1}{\sqrt{3}}$

$\sin 150° = \frac{1}{2}$ $\cos 150° = \frac{-\sqrt{3}}{2}$ $\tan 150° = -\frac{1}{\sqrt{3}}$

$\sin 210° = \frac{-1}{2}$ $\cos 210° = \frac{-\sqrt{3}}{2}$ $\tan 210° = \frac{1}{\sqrt{3}}$

$\sin 330° = \frac{-1}{2}$ $\cos 330° = \frac{\sqrt{3}}{2}$ $\tan 330° = -\frac{1}{\sqrt{3}}$



Verify with a calculator.

Make triangles for 60, 120, 240, 300.

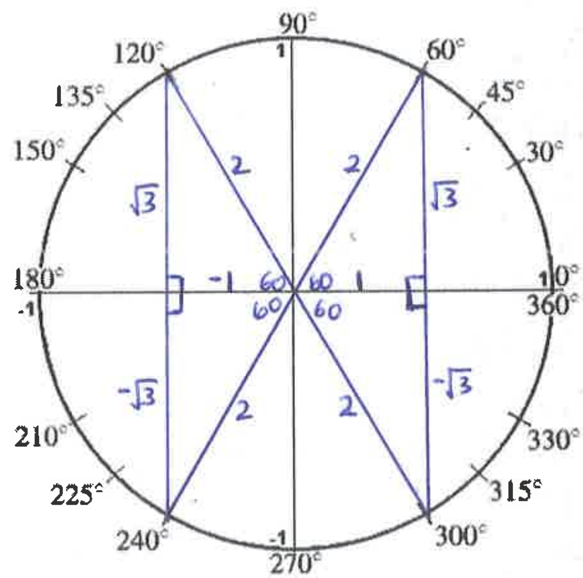
Build sin, cos, tan ratios for each:

$\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$ $\tan 60° = \sqrt{3}$

$\sin 120° = \frac{\sqrt{3}}{2}$ $\cos 120° = \frac{-1}{2}$ $\tan 120° = -\sqrt{3}$

$\sin 240° = \frac{-\sqrt{3}}{2}$ $\cos 240° = \frac{-1}{2}$ $\tan 240° = \sqrt{3}$

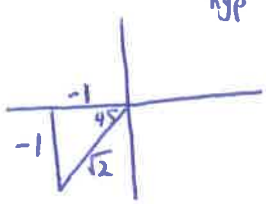
$\sin 300° = -\frac{\sqrt{3}}{2}$ $\cos 300° = \frac{1}{2}$ $\tan 300 = -\sqrt{3}$

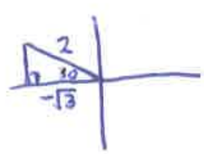


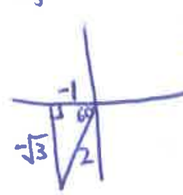
Verify with a calculator

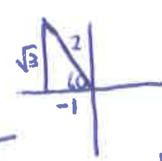
Example 1) Find the exact ratio of each:

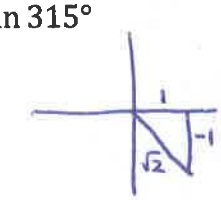
a) sin 225° $\frac{opp}{hyp}$ b) cos 150° $\frac{adj}{hyp}$ c) tan 240° $\frac{opp}{adj}$ d) sin 120° e) tan 315°











$\sin 120° = \frac{\sqrt{3}}{2}$ $\tan 315° = \frac{-1}{1} = -1$

$\sin 225° = \frac{-1}{\sqrt{2}}$ $\cos 150° = -\frac{\sqrt{3}}{2}$ $\tan 240° = \sqrt{3}$