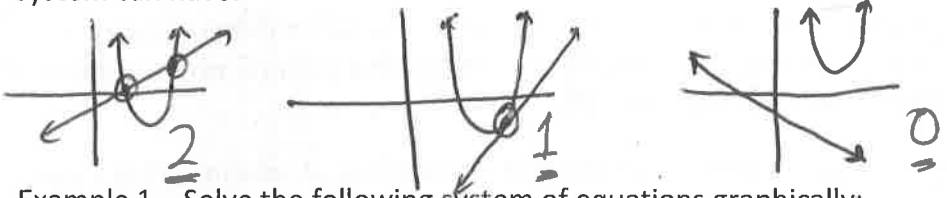
Math 10 Honours - PC 11 Systems of Equations Preview Key

Linear-Quadratic

A Linear-Quadratic System of Equations is a linear equation and a quadratic equation involving the same two variables. The solution(s) to this system are the point(s) where the line intersects the parabola (if it does at all).

Draw pictures to represent the possible number of solutions that a linear-quadratic system can have:



Example 1 – Solve the following system of equations graphically:

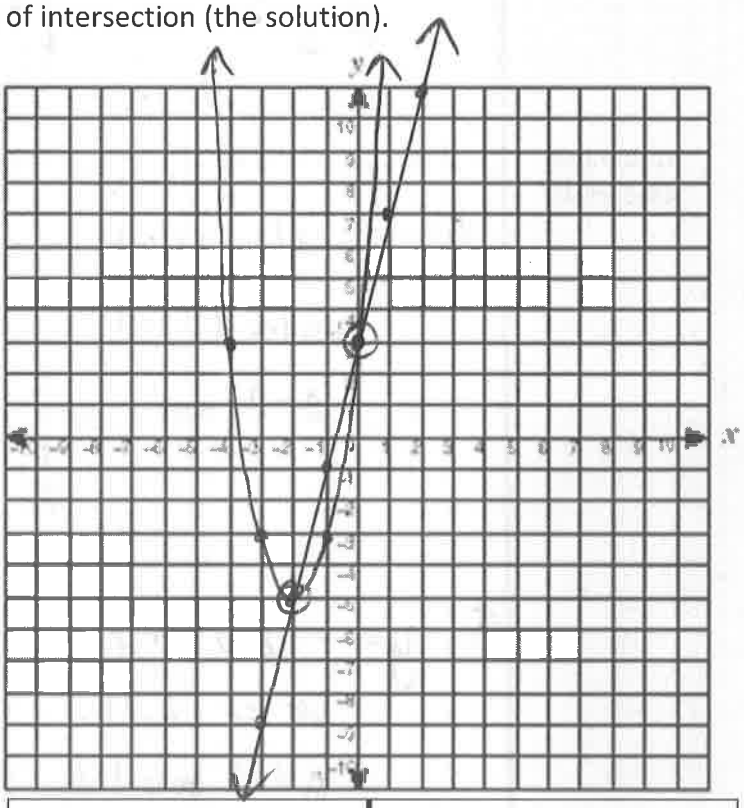
1) $4x - y + 3 = 0$
2) $y = 2(x + 2)^2 - 5$

a) Get the linear equation into $y = mx \pm b$ form and graph.
b) Graph the quadratic equation.
c) Identify and write down the points of intersection (the solution).
d) Verify the solution by checks.

a) $4x - y + 3 = 0$
$+y \quad +y$
$y = 4x + 3$
y-int = 3
$m = \frac{4 \leftarrow \text{up}}{1 \leftarrow \text{right}}$

b) $y = 2(x+2)^2 - 5$
vertex $(-2, -5)$
$a = 2$ over 1 up 2
2 8
3 18

c) $(-2, -5)$ & $(0, 3)$



d) $4(-2) - (-5) + 3 = 0$
$-8 + 5 + 3 = 0$
$-3 + 3 = 0$ ✓

$4(0) - (3) + 3 = 0$
$0 - 3 + 3 = 0$ ✓

$-5 = 2(-2+2)^2 - 5$
$-5 = 2(0)^2 - 5$
$-5 = 0 - 5$ ✓

$3 = 2(0+2)^2 - 5$
$3 = 2(2)^2 - 5$
$3 = 2(4) - 5$
$3 = 8 - 5$ ✓

Example 2 – Is (5, 7) a solution to the system 1) $3x^2 - 10y = 5$ and 2) $-y = x - 11$?

Do a check: ① $3(5)^2 - 10(7) = 5$

$3(25) - 10(7) = 5$

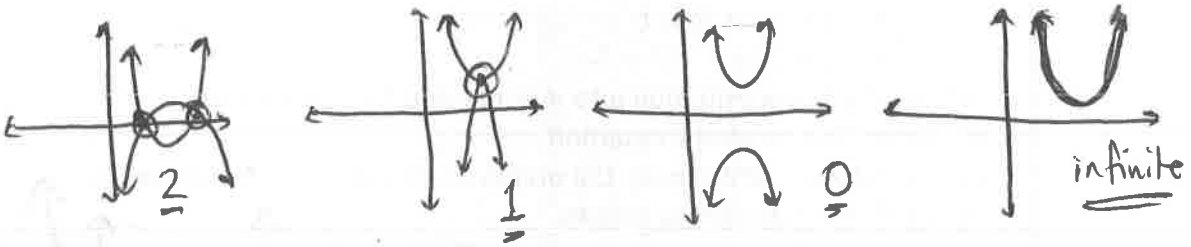
$75 - 70 = 5$

$5 = 5$ ✓

Quadratic-Quadratic

A Quadratic-Quadratic System of Equations is two quadratic equations involving the same variables. The solution(s) to this system are the point(s) where the parabola intersects the other parabola (if it does at all).

Draw pictures to represent the possible number of solutions that a quadratic-quadratic system can have:



Quadratic-Quadratic

Example 3 – Solve 1) $y = 2(x-2)^2 - 1$ and 2) $y = -(x-2)^2 + 2$

① $y = 2(x-2)^2 - 1$

vertex (2, -1)

$a = 2$ over 1 up 2

2 8

3 18

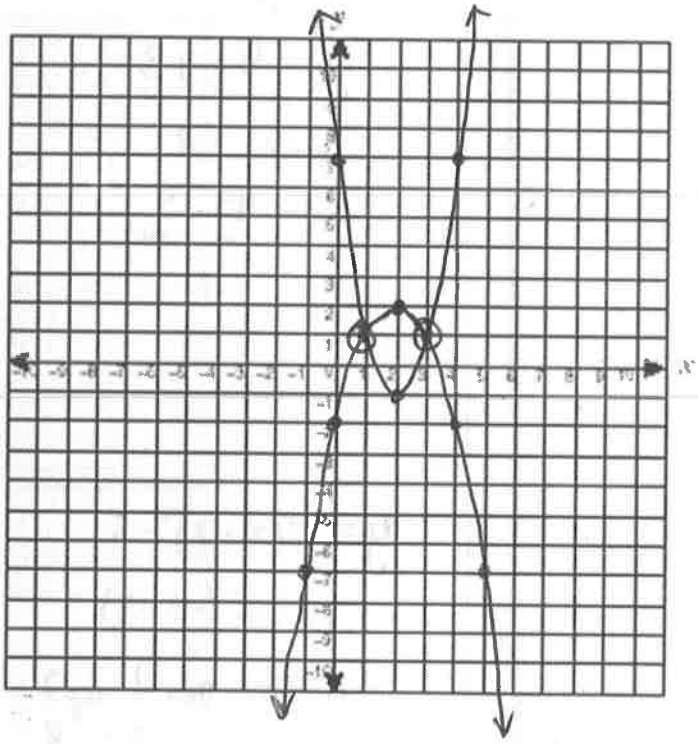
② $y = -(x-2)^2 + 2$

vertex (2, 2)

$a = -1$ over 1 down 1

2 4

3 9



Solutions: (1, 1) & (3, 1)

Linear-Quadratic

Example 4 – Solve the following linear-quadratic system using **substitution**:

1) $5x - y = 10$ and $x^2 + x - 2y = 0$

a) Solve the linear equation for y.
b) Substitute the linear equation for y in the quadratic equation.
c) Solve the quadratic equation by factoring.
d) Substitute the resulting x value(s) into the original linear equation to determine the corresponding y values.

a) $5x - y = 10$

$y = 5x - 10$

b &c) $x^2 + x - 2y = 0$

$x^2 + x - 2(5x - 10) = 0$

$x^2 + x - 10x + 20 = 0$

$x^2 - 9x + 20 = 0$ $\quad \times 20$, $+ -9$, $-5, -4$

$(x-5)(x-4) = 0$

$x = 5, 4$

d) ① $5x - y = 10$

$5(5) - y = 10$

$25 - y = 10$

$y = 15$ $\quad (5, 15)$

$5(4) - y = 10$

$20 - y = 10$

$y = 10$ $\quad (4, 10)$

Example 5 – Solve by elimination: 1) $5x - y = 10$ and 2) $x^2 + x - 2y = 0$

**Align the terms with the same degree. Since the squared term is the variable x, eliminate the y-term.*

② $x^2 + x - 2y = 0$

① $(5x - y = 10) \times 2$

② $x^2 + x - 2y = 0$

$-\quad (10x - 2y = 20)$

$x^2 - 9x \quad = -20$

$x^2 - 9x + 20 = 0$

⋮

rest is same as last example

Quadratic-Quadratic

For a Quadratic-Quadratic Systems of Equations, what are all the possible # of solutions?

0, 1, 2, infinite

Example 6 – Solve the following system first by substitution, then by elimination.

1) $6x^2 - x - y = -1$
2) $4x^2 - 4x - y = -6$

Substitution:

① $6x^2 - x - y = -1$

$y = 6x^2 - x + 1$

② $4x^2 - 4x - y = -6$

$4x^2 - 4x - (6x^2 - x + 1) = -6$

$4x^2 - 4x - 6x^2 + x - 1 = -6$

$-2x^2 - 3x + 5 = 0$ move it all to other side to get x^2 term positive

$2x^2 + 3x - 5 = 0$

x	ac	x
+	b	+
		5, -2

$2x^2 - 2x + 5x - 5 = 0$

$2x(x-1) + 5(x-1) = 0$

$(x-1)(2x+5) = 0$

$x = 1, -\frac{5}{2}$

Elimination:

$6x^2 - x - y = -1$

$-(4x^2 - 4x - y = -6)$

$2x^2 + 3x = 5$

$2x^2 + 3x - 5 = 0$

continue ↓

① $6(1)^2 - (1) - y = -1$

$6 - 1 - y = -1$

$5 - y = -1$

$y = 6$ $(1, 6)$

① $6\left(\frac{-5}{2}\right)^2 - \left(\frac{-5}{2}\right) - y = -1$

$6\left(\frac{25}{4}\right) + \frac{5}{2} - y = -1$

$\frac{150}{4} + \frac{5}{2} - y = -1$

$\frac{75}{2} + \frac{5}{2} - y = -1$

$\frac{80}{2} - y = -1$

$40 - y = -1$

$y = 41$ $\left(\frac{-5}{2}, 41\right)$