

Math 10 Honours – PC Math 11 Preview: Simplifying Rational Expressions

how zero affects division

A rational expression is: an algebraic fraction with a numerator and/or denominator that are monomials or polynomials

$$\text{Evaluate } \frac{0}{3} = 0$$

When zero is divided by any non-zero real number, ... the result is 0.

$$\text{Evaluate } \frac{7}{0} = \text{undefined}$$

Division by zero is undefined because... you can't put a finite number of items into zero groups.

undefined values

For the expression $\frac{3}{x-2}$, what value for x is undefined? $x=2$

What is an **undefined value**? Any value for a variable that makes a denominator equal to zero

Write a rule that explains how to determine undefined values:

For Any denominator expression that includes a variable:

- set the expression to zero
- solve for the variable

Example 1 - Determine the undefined values for each rational expression

a) $\frac{4a}{3b}$

$$b \neq 0$$

b) $\frac{x-1}{(x+2)(x-3)}$

$$x \neq -2, 3$$

c) $\frac{2y^2}{y^2-4}$

$$\frac{2y^2}{(y+2)(y-2)}$$

$$y \neq \pm 2$$

simplifying rational expressions

When simplifying rational expressions:

1) Factor as much as possible

2) Reduce/Cancel common factors

Example 2 - Simplify the rational expressions. Keep a running list of undefined values.

a) $\frac{3x-3}{6x-6}$ $x \neq 1$

$$\frac{3(x-1)}{6(x-1)}$$

$$= \left(\frac{1}{2}\right)$$

b) $\frac{x-2}{x^2-4}$ $x \neq \pm 2$

$$\frac{\cancel{x-2}}{(x+2)(\cancel{x-2})}$$

$$= \left(\frac{1}{x+2}\right)$$

c) $\frac{3x-6}{2x^2+x-10}$ $2x^2+x-10$
 $2x^2-4x+5x-10$
 $2x(x-2)+5(x-2)$
 $(x-2)(2x+5)$

$$\frac{3(x-2)}{(x-2)(2x+5)}$$

$$= \left(\frac{3}{2x+5}\right)$$

$x \neq 2, -\frac{5}{2}$

d) $\frac{2y^2+y-10}{y^2+3y-10}$ $y \neq -5, 2$

$$\frac{(y-2)(2y+5)}{(y+5)(y-2)}$$

$$= \left(\frac{2y+5}{y+5}\right)$$

e) $\frac{6-2m}{m^2-9}$ $m \neq \pm 3$

$$\frac{2(3-m)}{(m+3)(m-3)}$$

$$\frac{-2(-3+m)}{(m+3)(m-3)}$$

$$\frac{-2(\cancel{m-3})}{(m+3)(\cancel{m-3})} = \left(\frac{-2}{m+3}\right)$$

f) $\frac{x^2y+xy^2}{xy+y^2}$ $y \neq 0$
 $x \neq -y$

$$\frac{xy(x+y)}{y(x+y)}$$

$$= (x)$$

Example 3 - Simplify

a) $\left(\frac{4x^2}{3xy}\right)\left(\frac{y^2}{8x}\right)$ $x \neq 0$
 $y \neq 0$

$$\frac{4x^2y^2}{24x^2y}$$

$$= \left(\frac{y}{6}\right)$$

b) $\left(\frac{d}{2\pi r}\right)\left(\frac{2\pi rh}{d-2}\right)$ $r \neq 0$
 $d \neq 2$

$$\frac{2\pi rhd}{2\pi r(d-2)}$$

$$= \left(\frac{hd}{d-2}\right)$$

c) $\frac{y^2-9}{r^3-r} \times \frac{r^2-r}{y+3}$ $r \neq \pm 1$
 $y \neq -3$

$$\frac{(y+3)(y-3)r(r-1)}{r(r^2-1)(y+3)}$$

$$\frac{r(y+3)(y-3)(\cancel{r-1})}{r(r+1)(\cancel{r-1})(y+3)}$$

$$= \left(\frac{y-3}{r+1}\right)$$

d) $\left(\frac{x^2-x-12}{x^2-9}\right)\left(\frac{x^2-4x+3}{x^2-4x}\right)$ $x \neq -3, 0, 3, 4$

$$\frac{(x-4)(x+3)(x-3)(x-1)}{x(x+3)(x-3)(x-4)}$$

$$= \left(\frac{x-1}{x}\right)$$