

radical review

Identify and define all parts of the radical, then simplify:

$5\sqrt[3]{8}$
 Coefficient \rightarrow 5 Index \leftarrow 3 radical sign \leftarrow $\sqrt{\quad}$
 radicand \leftarrow 8

$5\sqrt[3]{8}$
 $5(2)$
 $\underline{10}$

Ex1 - Simplify

a) $\sqrt{8}$ b) $\sqrt{27}$ c) $\sqrt{52}$ d) $\sqrt[3]{24}$ e) $\sqrt[3]{81}$ f) $\sqrt[4]{32}$
 $\frac{\sqrt{4 \cdot 2}}{\sqrt{4} \cdot \sqrt{2}}$ $\frac{\sqrt{9 \cdot 3}}{\sqrt{9} \cdot \sqrt{3}}$ $\frac{\sqrt{4 \cdot 13}}{\sqrt{4} \cdot \sqrt{13}}$ $\frac{\sqrt[3]{8 \cdot 3}}{\sqrt[3]{8} \cdot \sqrt[3]{3}}$ $\frac{\sqrt[3]{27 \cdot 3}}{\sqrt[3]{27} \cdot \sqrt[3]{3}}$ $\frac{\sqrt[4]{16 \cdot 2}}{\sqrt[4]{16} \cdot \sqrt[4]{2}}$
 $2\sqrt{2}$ $3\sqrt{3}$ $2\sqrt{13}$ $2\sqrt[3]{3}$ $3\sqrt[3]{3}$ $2\sqrt[4]{2}$

| radicand index / | x | x ² | x ³ | x ⁴ | x ⁵ | x ⁶ | x ⁷ | x ⁸ | x ⁹ |
|------------------|---------------|------------------|--------------------------|--------------------------------|----------------------------------|------------------|------------------|--------------------|------------------|
| 2 | \sqrt{x} | $\sqrt{x^2} = x$ | $\sqrt{x^3} = x\sqrt{x}$ | $\sqrt{x^4} = x^2$ | $\sqrt{x^5} = x^2\sqrt{x}$ | x^3 | $x^3\sqrt{x}$ | x^4 | $x^4\sqrt{x}$ |
| 3 | $\sqrt[3]{x}$ | $\sqrt[3]{x^2}$ | $\sqrt[3]{x^3} = x$ | $\sqrt[3]{x^4} = x\sqrt[3]{x}$ | $\sqrt[3]{x^5} = x\sqrt[3]{x^2}$ | x^2 | $x^2\sqrt[3]{x}$ | $x^2\sqrt[3]{x^2}$ | x^3 |
| 4 | $\sqrt[4]{x}$ | $\sqrt[4]{x^2}$ | $\sqrt[4]{x^3}$ | $\sqrt[4]{x^4} = x$ | $\sqrt[4]{x^5} = x\sqrt[4]{x}$ | $x\sqrt[4]{x^2}$ | $x\sqrt[4]{x^3}$ | x^2 | $x^2\sqrt[4]{x}$ |
| 5 | $\sqrt[5]{x}$ | $\sqrt[5]{x^2}$ | $\sqrt[5]{x^3}$ | $\sqrt[5]{x^4}$ | $\sqrt[5]{x^5} = x$ | $x\sqrt[5]{x}$ | $x\sqrt[5]{x^2}$ | $x\sqrt[5]{x^3}$ | $x\sqrt[5]{x^4}$ |

Ex2 - Simplify

a) $\sqrt{18x^3y^6}$ b) $\sqrt{63n^7p^4}$ c) $\sqrt{32x^8y^{11}}$ d) $\sqrt[3]{40a^4b^8c^{15}}$
 $\frac{\sqrt{9 \cdot 2 \cdot x^2 \cdot x \cdot y^6}}{3xy^3\sqrt{2x}}$ $\frac{\sqrt{9 \cdot 7 \cdot n^6 \cdot n \cdot p^4}}{3n^3p^2\sqrt{7n}}$ $\frac{\sqrt{16 \cdot 2 \cdot x^8 \cdot y^{10} \cdot y}}{4x^4y^5\sqrt{2y}}$ $\frac{\sqrt[3]{8 \cdot 5 \cdot a^3 \cdot a \cdot b^6 \cdot b^2 \cdot c^{15}}}{2ab^2c^5\sqrt[3]{5ab^2}}$

e) $\sqrt[3]{54a^5b^{10}}$ f) $\sqrt[4]{m^7} = \sqrt[4]{m^4 \cdot m^3}$ g) $\sqrt[4]{162x^3y^{11}z^5}$
 $\frac{\sqrt[3]{27 \cdot 2 \cdot a^3 \cdot a^2 \cdot b^9 \cdot b}}{3ab^3\sqrt[3]{2a^2b}}$ $m\sqrt[4]{m^3}$ $\frac{\sqrt[4]{81 \cdot 2 \cdot x^3 \cdot y^8 \cdot y^3 \cdot z^4 \cdot z}}{3y^2z\sqrt[4]{2x^3y^3z}}$

Ex3 - Change
to entire

$$\begin{aligned} \text{a) } 4\sqrt{3} &= \frac{\sqrt{16} \cdot \sqrt{3}}{\sqrt{16 \cdot 3}} \\ &= \frac{\sqrt{48}}{\sqrt{48}} \end{aligned}$$

$$\begin{aligned} \text{b) } 3\sqrt{5} &= \frac{\sqrt{9} \sqrt{5}}{\sqrt{9 \cdot 5}} \\ &= \frac{\sqrt{45}}{\sqrt{45}} \end{aligned}$$

$$\begin{aligned} \text{c) } 2\sqrt[3]{7} &= \frac{\sqrt[3]{8} \sqrt[3]{7}}{\sqrt[3]{8 \cdot 7}} \\ &= \frac{\sqrt[3]{56}}{\sqrt[3]{56}} \end{aligned}$$

$$\begin{aligned} \text{d) } 2x\sqrt{6x} &= \frac{\sqrt{4x^2} \sqrt{6x}}{\sqrt{4x^2 \cdot 6x}} \\ &= \frac{\sqrt{24x^3}}{\sqrt{24x^3}} \end{aligned}$$

$$\begin{aligned} \text{e) } x^3\sqrt{x} &= \frac{\sqrt{x^6} \sqrt{x}}{\sqrt{x^6 \cdot x}} \\ &= \frac{\sqrt{x^7}}{\sqrt{x^7}} \end{aligned}$$

$$\begin{aligned} \text{f) } 2k^2(\sqrt[3]{4k}) &= \frac{\sqrt[3]{8k^6} \sqrt[3]{4k}}{\sqrt[3]{8k^6 \cdot 4k}} \\ &= \frac{\sqrt[3]{32k^7}}{\sqrt[3]{32k^7}} \end{aligned}$$

$$\begin{aligned} \text{g) } 3a^2b\sqrt[3]{b^2c} &= \frac{\sqrt[3]{27a^6b^3} \sqrt[3]{b^2c}}{\sqrt[3]{27a^6b^3 \cdot b^2c}} \\ &= \frac{\sqrt[3]{27a^6b^5c}}{\sqrt[3]{27a^6b^5c}} \end{aligned}$$

Like Radicals

'Like Radicals' are radicals with... *the same index and same radicand*

Verify:

$$\text{Simplify } 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$$

$$\begin{aligned} 3\sqrt{2} + 2\sqrt{2} &= 5\sqrt{2} \\ 4.24 + 2.83 &= 7.07 \\ &= 7.07 \end{aligned}$$

Steps for adding & subtracting like radicals:

- ① Simplify to try to get like radicals
- ② Add/subtract coefficients.
- ③ Leave radicands the same.

Ex1 - Simplify

$$\begin{aligned} \text{a) } 7\sqrt{3} - 2\sqrt{3} &= 5\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b) } -5\sqrt[3]{10} - 6\sqrt[3]{10} &= -11\sqrt[3]{10} \end{aligned}$$

$$\begin{aligned} \text{c) } 2\sqrt{75} + 3\sqrt{3} &= 2\sqrt{25 \cdot 3} + 3\sqrt{3} \\ &= 2 \cdot 5\sqrt{3} + 3\sqrt{3} \\ &= 10\sqrt{3} + 3\sqrt{3} \\ &= 13\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{e) } -\sqrt{27} + 3\sqrt{5} - \sqrt{80} - 2\sqrt{12} &= -3\sqrt{3} + 3\sqrt{5} - 4\sqrt{5} - 4\sqrt{3} \\ &= -7\sqrt{3} - \sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{f) } \sqrt{9b} - 3\sqrt{16b}, b \geq 0 &= 3\sqrt{b} - 12\sqrt{b} \\ &= -9\sqrt{b} \end{aligned}$$