3.1 & 3.2 – Factors and Multiples

Name:

Date:

Goal: to determine prime factors, GCFs, and LCMs of whole numbers, and find roots using prime factors

Toolkit:	**Main Ideas:**
• Division • Multiplication • Writing repeated multiplication using powers, e.g. 2×2×2×2×2×2=	

Definitions

Factor – a term which divides evenly into another term

Prime number – when a number has only 2 distinct factors (1 and itself). **Examples:**

Composite number – when a number has more than 2 factors. **Examples:**

Prime factorization – a term written as a product of prime factors

every composite number can be expressed as a product of prime factors

Greatest common factor (GCF) – the largest term which will divide evenly into a series of separate terms

Least (or Lowest) common multiple (LCM) – the smallest multiple which is common to series of separate terms

Prime Factorization

Ex1) Write the prime factorization for each of the **composite** numbers:

a) 3 b) 6 c) 45 d) 47 e) 3300

Perfect Squares & Cubes

What is a perfect square?

What is a perfect cube?

Ex2) Using prime factors, is 1296 a perfect square? If so, what is the square root?

Can the prime factors be arranged into TWO identical groups?

What are the parts of a root called?

Is 1296 a perfect cube? ***Three identical groups?***

Ex3) Is 1728 a perfect square? Is it a perfect cube?

Ex4) Determine the edge length of a cube with volume $64x^6$.

Finding the GCF by listing all the factors of each number (the rainbow method)	Ex5) Determine the greatest common factor of 126 and 144 **Method 1** – list all the factors and find the largest one in common *(write small!)*
Finding the GCF by writing the prime factorization of each number	**Method 2** 1) write the prime factorization for each number 2) highlight the factors that they have in common 3) multiply all the common factors together go get the GCF
Finding the LCM by listing the first multiples of each number	Ex6) Find the least common multiple of 28, 42, and 63 **Method 1** – list the first few multiples of each number until you find (the first, lowest) one in common
Finding the LCM by writing the prime factorization of each number	**Method 2** 1) write the prime factors of each number 2) highlight the greatest power of each prime in ANY of the lists 3) multiply the greatest powers of each prime together to get the LCM
What types of real-world problems involve GCFs and LCMs?	Ex7) Beside each problem, write whether you would need the GCF or the LCM, then answer the question! a) A bathroom wall (the part above the bathtub) is a rectangle that measures 78" by 60". If you wanted to cover it exactly with square tiles, what is the largest possible square tile you could use? b) You have red bungee cords that are 18cm long and green bungee cords that are 14cm long. What is the shortest length of connected bungees you can make with each colour so that they make the same length?

3.7 – Multiplying Polynomials

Name:
Date:

Goal: to expand monomial and binomial products (multiply out!)

Toolkit:

- Adding, subtracting, multiplying polynomials
- Multiplying powers with the same base: add the exponents

  Ex: $(x^3)(x^4) =$
- Collecting like terms: same variable(s) with same exponents

  Ex: $2x^2 + 3x - x^2 + 2x + 1 =$

Main Ideas:

Definitions

Polynomial –

Monomial – Binomial – Trinomial –

F O I L

Ex1) Expand and simplify → translates to:

a) $3x^2(x+3)$

b) $(x+2)(x+3)$

c) $(2y+z)(3y-2z)$

d) $(2a-1)(2a+3)+(a-1)(a-2)$

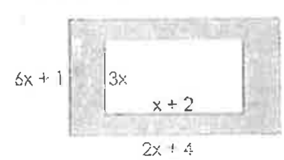
Ex2) Expand and simplify:

a) $(x + 3y)(x + y - 3)$

b) $(x + 2)^3$

c) $(r^2 + 3r - 1)(2r^2 - r + 2)$

Ex3) Find the area of the shaded region (simplified!):



3.3 – Common Factors of a Polynomial

Name:
Date:

Goal: to determine the factors of a polynomial by identifying the GCF

Toolkit:	**Main Ideas:**
• Finding the GCF • Distributive Property	

Factor a binomial using the GCF	Ex 1) Factor the binomial: $3g + 6$
	Ex 2) Factor the binomial: $-8y + 16y^2$
Factor a trinomial using the GCF	Ex 3) Factor the trinomial: $3x^2 + 12x - 6$
	Ex 4) Factor the trinomial: $6 - 12z + 18z^2$
Factor polynomials in more than one variable	Ex 5) Factor the trinomial: $-20c^4d - 30c^3d^2 - 25cd$

Reflection: How are the processes of factoring and expanding related?

3.5 – Factoring Trinomials of the form $x^2 + bx + c$, where a=1

Name:
Date:

Goal: to use models and algebraic strategies to multiply binomials and to factor trinomials.

Toolkit:	**Main Ideas:**
• Factoring	

Definitions:
Descending order: the terms are written in order from the term with the greatest exponent to the term with the least exponent
Ascending order: the terms are written in order from the term with the least exponent to the term with the greatest exponent

Steps for Factoring a Trinomial in the form: x^2+bx+c, where a=1

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable *(biggest to smallest)*
Step 2: Find two numbers that multiply to equal the *c* term and add to equal the *b* term (add to the middle, multiply to the end)
Step 3: Factor into two binomials using the numbers from step 2, with the variable from the question placed first in each bracket

Multiplying two binomials

Ex 1) Expand and Simplify: $(x-1)(x-7)$ use FOIL

Remember: expanding and factoring are opposite operations....they UNDO each other!

Factoring a trinomial in the form x^2+bx+c

Ex 2) Factor the trinomial: $x^2 - 8x + 7$ ....we should end up with $(x-1)(x-7)$!

Notice that a (the number in front of the x^2) will always end up being 1 in these questions!

Ex 3) Factor: $a^2 - 2a - 8$

Factoring a trinomial written in ascending order

Ex 4a) Factor: $-30 + 7m + m^2$

b) $x^2 - 4xy + 21y^2$

Ex 5) Factor: $-5h^2 - 20h + 60$

Always check to see if there is a GCF you can factor out first! IF there is a negative number in front of the x^2, factor out the negative as well.

Ex 6) Factor: $-12 - 9g + 3g^2$

Ex 7) Factor: $2x^2 - 6x - 80$

Ex 8) Factor: $x^2 + x - 2$

Reflection: Does the order in which the binomial factors are written affect the solution? Explain.

3.6 – Polynomials of the Form $ax^2 + bx + c$, $a \neq 1$

Name:
Date:

Goal: to extend the strategies for multiplying binomials and factoring trinomials

Toolkit:	**Main Ideas:**
• Multiplying binomials • Factoring	

Factoring by Decomposition: (needed when the $a \neq 1$ in $ax^2 + bx + c$)

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable *(biggest to smallest)*

Step 2: Find two numbers that multiply to equal *ac* and add to equal *b* *(add to the middle, multiply to product of first and last)*

Step 3: Re-write the expression but split or *decompose* the *b* term using the two numbers from step 2.

Step 4: Now the expression has FOUR terms, so we can factor by grouping the first two terms and the last two terms.

Step 5: When fully factored, the remaining two brackets need to be identical! These are now a common factor, and can be factored out, and what is left becomes the components of the second bracket.

Factoring a trinomial of the form $ax^2 + bx + c$

notice that a (the number in front of x^2) is not = 1 in any of these questions!

Ex 1) Factor the trinomial: $4g^2 + 11g + 6$ by decomposition

Ex 2) Factor the trinomial: $-7m - 10 + 6m^2$

Ex 3) Factor: $8p^2 - 18pq - 5q^2$

Ex 4) Factor: $6x^2 + 14x - 12$

If you can make a trinomial have a=1 by removing a G.C.F., then you can use "the simple way"!

Ex 5) Factor: $3x^2 + 6x - 9$

Ex 6) Find an integer to replace □ so that the trinomial can be factored. How many integers can you find?

$4x^2 + \square x + 9$

Reflection: Will decomposition work if the *a* value of a trinomial is 1? Do an example to prove this.

3.8 – Factoring Special Polynomials

Name:
Date:

Goal: to investigate perfect square trinomials and difference of squares

Toolkit:	**Main Ideas:**
• Finding a square root • Finding GCF • Multiplying Polynomials	

Definitions:

Perfect Square Trinomial: a trinomial of the form $m^2 + 2mn + n^2$; it can be factored as $(m + n)^2$
or of the form $m^2 - 2mn + n^2$; it can be factored as $(m - n)^2$

Difference of Squares: a binomial of the form $m^2 - n^2$; it can factored as $(m - n)(m + n)$

Warmup: Factor the trinomial $4x^2 - 4x + 1$ using decomposition.

Factoring a perfect square trinomial

Decomposition works, but it is time consuming. Test to see if the trinomial is a perfect square! If so, it will be quicker to factor. $4x^2 - 4x + 1$

Step 1: Is the trinomial in order? Can you factor out a GCF?

Step 2: Are the first and last terms perfect squares?

Step 3: Make two brackets, and write the square roots into each. Then, figure out if the brackets should have a '+' or '-' in between the terms.

Step 4: Now test that the middle terms (the 'O' and 'I' of FOIL) add to the middle term of the original polynomial. If so, the trinomial is a perfect square.

Ex 1) Factor the trinomial: $36x^2 + 12x + 1$

Ex 2) Factor the trinomial: $18x^2 - 48xy + 32y^2$

Ex 3) Factor the trinomial: $25c^2 - 29cd + 4d^2$

Factoring a Difference of Squares	Difference of Squares is only possible if you have a binomial. The binomial must have a SUBTRACT (difference) in between two PERFECT SQUARES (of squares). Ex 4) Factor the binomial: $81m^2 - 49$ Step 1: Is there a subtract in the middle? Step 2: Is each term a perfect square? Step 3: If not, is there a GCF to factor out? Step 4: Make two brackets, one with a '+' and one with a '-'. Step 5 Square root each term and put into the appropriate position in each bracket. CHECK: Ex 5) Factor: $m^2 - 36$ Why is one bracket '+' and one '-' ? Ex 6) Factor: $32v^2 - 2w^2$ Ex 7) Factor: $\frac{x^2}{25} - \frac{y^2}{4}$ Ex 8) Factor: $x^2 + 9$ Ex 9) Factor: $2x^4 - 162$ *If you have a 4^{th} power variable, there is a good chance there will be TWO LAYERS of factoring to complete.

Reflection: Does a sum of squares factor? Explain.

3.9 – Factoring Synthesis

Name:
Date:

FACTORING FLOW CHART

STEP 1 Take out COMMON FACTORS (**GCF**)

STEP 2 Ask: How many terms are there? Are they in order?

TWO

Test for **difference of squares:**

*You need **subtraction** ("difference") and each term must be a **perfect square**

If you don't have perfect squares, check to see if you can factor out a GCF.

$a^2 - b^2 = (a + b)(a - b)$

Example:
$4x^2 - 9$
$(2x + 3)(2x - 3)$

Example:
$2m^2 - 32n^2$
$2(m^2 - 16n^2)$
$2(m + 4n)(m - 4n)$

Example:
$4w^2 + 9y^2$
*cannot factor
As it is a SUM
of squares*

THREE

Factoring **trinomials:** $ax^2 + bx + c$
Is the trinomial in order?
Can you factor out a GCF?

Type 1: $a = 1$
Example: $x^2 - 3x + 2$
Ask: what ADDS to "b" (here –3)
& MULTIPLIES to "c" (here +2)
Answer: –1, –2
Write factors: $(x - 1)(x - 2)$

Type 2: $a \neq 1$
Is it a perfect square trinomial?
Are first and last terms perfect squares?
Is the middle term correct?
Example: $4x^2 - 12x + 9$
Factor using square roots:
$(2x - 3)(2x - 3)$
Middle term: $-6x - 6x = -12x$

If it isn't a perfect square trinomial, factor using DECOMPOSITION.
Example: $2x^2 - x - 1$
Ask: what ADDS to "b" (here –1)
& MULTIPLIES to "ac" (here $2(-1) = -2$)
Answer: –2, 1
Use these to split (decompose) the middle term into two separate terms:

$$2x^2 - \mathbf{x} - 1$$
$$\underline{2x^2 - \mathbf{2x}} + \underline{\mathbf{1x} - 1}$$

Factor using grouping:

$$2x(x - 1) + 1(x - 1)$$

See if two brackets are the same.
Factor the bracket out front as a GCF, & the 'leftovers' make up the 2nd bracket.

$$(x - 1)(2x + 1)$$

STEP 3 Ask: **FF?** Look inside each factor (bracket) and see if you can **FACTOR FURTHER.** *If the original question has an x^4 term, there is a good chance there will be 2 layers of factoring!

Practice factoring expressions using the flowchart for assistance.	Ex 1) Factor: $2x^2 - 22x + 60$ Ex 2) Factor: $p^2 - 25q^2$ Ex 3) Factor: $3y^2 - 7y - 6$ Ex 4) Factor: $4m^2 + 12m - 56$ Ex 5) Factor: $9x^2 - 42xy + 49y^2$ Ex 6) Factor: $8b^2 + 2c^2$ Ex 7) Factor: $8x^2 + 40x + 18$ Ex 8) Factor: $32x^2 - 50y^2$ Ex 9) Factor: $3n^4 - 48$

Reflection: