4.1 – Estimating Roots & 4.2 – Irrational Numbers

Name: Key
Date:

Goal: to explore decimal representations of different roots of numbers & to classify numbers and order irrational numbers

Toolkit:
- Finding a square root
- Finding a cube root
- Multiplication
- Estimating

Main Ideas:

Definitions:

Radical: an expression consisting of a radical sign, a radicand, and an index.

$\sqrt[n]{x}$ — index, radical sign, radicand

Perfect squares and cubes to memorize: $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{16} = 4$, $\sqrt{25} = 5$, $\sqrt{36} = 6$, $\sqrt{49} = 7$, $\sqrt{64} = 8$, $\sqrt{81} = 9$, $\sqrt[3]{8} = 2$, $\sqrt[3]{27} = 3$, $\sqrt[3]{64} = 4$, $\sqrt[3]{125} = 5$

Ex 1) Evaluate the following radicals, identify the radicand and index for each:

a) $\sqrt{16} = 4$

Radicand: 16
Index: 2

b) $\sqrt[3]{64} = 4$

Radicand: 64
Index: 3

Estimating square roots

Ex 2) Estimate the value of $\sqrt{20}$ to one decimal place.

$\sqrt{16}$ $\quad\sqrt{20}\quad$ $\sqrt{25}$

4 $\quad$ 4.4 or 4.5 $\quad$ 5

Estimating cube roots

Ex 3) Estimate the value of $\sqrt[3]{16}$

$\sqrt[3]{8}$ $\quad\sqrt[3]{16}\quad$ $\sqrt[3]{27}$

2 $\quad$ 2.4 $\quad$ 3

Why can you take the cube root of a negative number but not the square root of a negative number?

Ex 4) Estimate the value of $\sqrt[3]{-32}$

$\sqrt[3]{-27}$ $\quad\sqrt[3]{-32}\quad$ $\sqrt[3]{-64}$

-3 $\quad$ -3.2 $\quad$ -4

Ex 5) Evaluate $\sqrt{0.64}$
$= 0.8$

b/c
$0.8 \times 0.8 = 0.64$

Ex 6) Evaluate $\sqrt{0.0144}$
$= 0.12$

b/c
$0.12 \times 0.12 = 0.0144$

4.2 – Irrational Numbers

Number Systems:

$\mathbb{N}$ **Natural Numbers:** Counting Numbers 1, 2, 3, 4, etc…

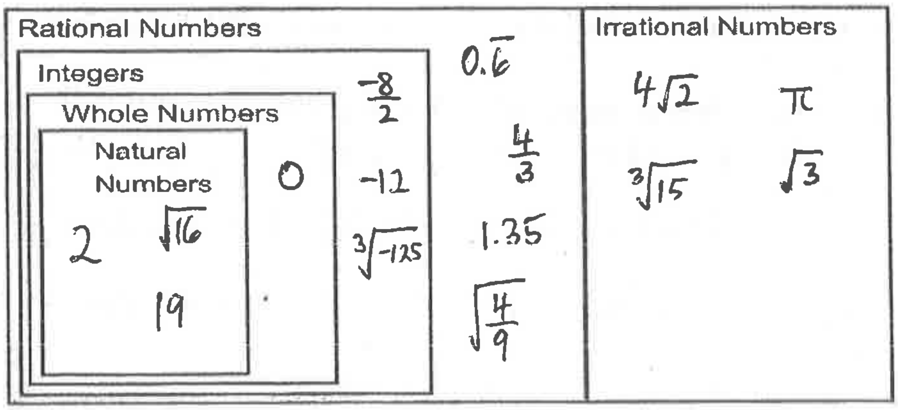
$\mathbb{W}$ **Whole Numbers:** Counting Numbers AND zero

$\mathbb{Z}$ **Integers:** Positive and negative non-decimals AND zero

$\mathbb{Q}$ **Rational Numbers:** Any number that can be written as a fraction (any number that terminates or repeats)

$\overline{\mathbb{Q}}$ **Irrational Numbers:** Any number that cannot be written as a fraction (any number that neither terminates or repeats)

Real Numbers:



Ex 7) Where do these number belong in the diagram of Real Numbers?

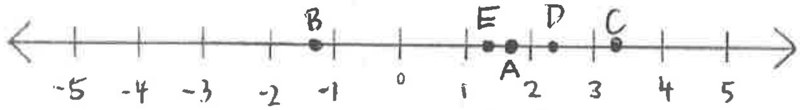
$2 \qquad 0.\overline{6} \qquad 4\sqrt{2} \qquad \frac{4}{3} \qquad \frac{-8}{2} \qquad -12 \qquad \pi \qquad 0 \qquad \sqrt{16}$

$1.35 \qquad \sqrt[3]{-125} \qquad \sqrt{3} \qquad \sqrt[3]{15} \qquad 19 \qquad \sqrt{\frac{4}{9}}$

Ex 8) Use a number line to order these irrational numbers from least to greatest:

A	B	C	D	E
$\sqrt[3]{6}$	$\sqrt[3]{-2}$	$\sqrt{11}$	$\sqrt[4]{30}$	$\sqrt{2}$
$\cong 1.82$	$\cong -1.26$	$\cong 3.32$	$\cong 2.34$	$\cong 1.41$

B, E, A, D, C



4.3 – From Entire to Mixed (Simplifying) Radicals, and Back

Name:
Date:

Goal: to express an entire radical as a mixed radical, and *visa vera*

Toolkit:	**Main Ideas:**
• Understanding Radicals • Identifying Factors of a Number	

Perfect Squares - 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ...

Perfect Cubes - 1, 8, 27, 64, 125, 216, ...

What is an entire radical?

A radical sign with a number (radicand) under it ex $\sqrt{28}$, $\sqrt[3]{16}$

What is a mixed radical?

A radical with a coefficient other than 1. ex $2\sqrt{3}$

Equivalent Forms:

Ex 1)

a) $\sqrt{16 \cdot 9}$ is equivalent to $\sqrt{16} \cdot \sqrt{9}$ because:

$\sqrt{144}$
$= 12$

$4 \cdot 3$
$= 12$

b) $\sqrt[3]{8 \cdot 27}$ is equivalent to $\sqrt[3]{8} \cdot \sqrt[3]{27}$ because:

$\sqrt[3]{216}$
$= 6$

$= 2 \cdot 3$
$= 6$

What is the Multiplication Property of Radicals?

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, where n is a natural number, and a and b are real numbers

***We can use this property to simplify square roots and cube roots that are *not* perfect squares or perfect cubes, but have *factors* that are perfect squares or perfect cubes.**

Simplifying Square Roots

We can simplify $\sqrt{24}$ because 24 has a perfect square factor of 4.
(hint: look at list of perfect squares!)

- Re-write $\sqrt{24}$ as a product of two factors, with the first one being the perfect square:

$= \sqrt{4 \cdot 6}$
$= \sqrt{4} \cdot \sqrt{6}$
$= 2 \cdot \sqrt{6}$
$= 2\sqrt{6}$

ex $\sqrt{45}$
$= \sqrt{9 \cdot 5}$
$= \sqrt{9} \cdot \sqrt{5}$
$= 3 \cdot \sqrt{5}$
$= 3\sqrt{5}$

Simplifying Cube Roots

We can also simplify $\sqrt[3]{24}$ because 24 has a perfect cube factor of 8.
(hint: look at list of perfect cubes!)

- Re-write $\sqrt[3]{24}$ as a product of two factors, with the first one being the perfect cube:

$= \sqrt[3]{8 \cdot 3}$
$= \sqrt[3]{8} \cdot \sqrt[3]{3}$
$= 2 \cdot \sqrt[3]{3}$
$= 2\sqrt[3]{3}$

Tip: If there is MORE than one perfect square or perfect cube factor, choose the LARGEST one!

Ex 2) Simplify each radical: (remember your list of perfect squares and perfect cubes!)

a) $\sqrt{80}$
$= \sqrt{16 \cdot 5}$
$= \sqrt{16} \cdot \sqrt{5}$
$= 4\sqrt{5}$

b) $\sqrt{32}$
$= \sqrt{16 \cdot 2}$
$= \sqrt{16} \cdot \sqrt{2}$
$= 4\sqrt{2}$

c) $\sqrt{98}$
$= \sqrt{49 \cdot 2}$
$= \sqrt{49} \cdot \sqrt{2}$
$= 7\sqrt{2}$

d) $\sqrt[3]{162}$
$= \sqrt[3]{27 \cdot 6}$
$= \sqrt[3]{27} \cdot \sqrt[3]{6}$
$= 3\sqrt[3]{6}$

e) $\sqrt[3]{108}$
$= \sqrt[3]{27 \cdot 4}$
$= \sqrt[3]{27} \cdot \sqrt[3]{4}$
$= 3\sqrt[3]{4}$

How do you simplify something with an index of 4? (a fourth root?)

Ex 3) Simplify $\sqrt[4]{162}$

Perfect Fourths : 16, 81, 256

$= \sqrt[4]{81 \cdot 2}$

$= \sqrt[4]{81} \cdot \sqrt[4]{2}$

$= 3\sqrt[4]{2}$

-Rewrite radical with the prime factorization of 162

-Since $\sqrt[4]{162}$ is a fourth root, look for a factor that appears 4 times!

Ex 4) Simplify $\sqrt[4]{48}$

$= \sqrt[4]{16 \cdot 3}$

$= \sqrt[4]{16} \cdot \sqrt[4]{3}$

$= 2\sqrt[4]{3}$

Word Problem

Ex 5) A cube has a volume of $128cm^3$. Write the edge length of the cube in simplest radical form.

$V = e^3$

$e^3 = 128$

$e = \sqrt[3]{128}$

$e = \sqrt[3]{64 \cdot 2}$

$e = \sqrt[3]{64} \cdot \sqrt[3]{2}$

$e = 4\sqrt[3]{2}$ cm

How do you write a mixed radical as an entire radical?

Write the mixed radical $4\sqrt{3}$ as an entire radical:

$4\sqrt{3}$

$= 4 \cdot \sqrt{3}$

$= \sqrt{16} \cdot \sqrt{3}$

$= \sqrt{16 \cdot 3}$

$= \sqrt{48}$

- Use the Multiplication Property of Radicals (re-write 4 as a radical.....think .....$4 = \sqrt{?}$.....$\sqrt{16}$!)
- Combine these under the same radical sign and multiply

(***NOTICE...these are the *opposite* steps to writing an entire radical as a mixed radical)

Ex. 6) Write each as an entire radical:

a) $5\sqrt{2}$

$5 \cdot \sqrt{2}$

$\sqrt{25} \cdot \sqrt{2}$

$\sqrt{25 \cdot 2}$

$\sqrt{50}$

b) $3\sqrt{3}$

$3 \cdot \sqrt{3}$

$\sqrt{9} \cdot \sqrt{3}$

$\sqrt{9 \cdot 3}$

$\sqrt{27}$

c) $3\sqrt[3]{2}$

$3 \cdot \sqrt[3]{2}$

$\sqrt[3]{27} \cdot \sqrt[3]{2}$

$\sqrt[3]{27 \cdot 2}$

$\sqrt[3]{54}$

d) $2\sqrt[3]{6}$

$2 \cdot \sqrt[3]{6}$

$\sqrt[3]{8} \cdot \sqrt[3]{6}$

$\sqrt[3]{8 \cdot 6}$

$\sqrt[3]{48}$

What do you do if the index is 4 or 5 (or higher?)

Write $3\sqrt[5]{2}$ as an entire radical:

First, re-write 3 as $\sqrt[5]{?}$ .... $3 = \sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \sqrt[5]{243}$

Perfect Fifths : 32, 243, 1024

So now, $3\sqrt[5]{2}$

$= 3 \cdot \sqrt[5]{2}$

$= \sqrt[5]{243} \cdot \sqrt[5]{2}$ now, using the Multiplication Property of Radicals...

$= \sqrt[5]{243 \cdot 2}$

$= \sqrt[5]{486}$

Ex. 7) Write each as an entire radical:

a) $2\sqrt[4]{5}$

$2 \cdot \sqrt[4]{5}$

$\sqrt[4]{16} \cdot \sqrt[4]{5}$

$\sqrt[4]{16 \cdot 5}$

$\sqrt[4]{80}$

b) $4\sqrt[5]{2}$

$4 \cdot \sqrt[5]{2}$

$\sqrt[5]{1024} \cdot \sqrt[5]{2}$

$\sqrt[5]{1024 \cdot 2}$

$\sqrt[5]{2048}$

How can entire radicals be used to help you order a set of mixed radicals with the same index?

Ex. 8) Arrange the following in order from greatest to least: $3\sqrt{5}, 2\sqrt{13}, 4\sqrt{3}, 2, 9\sqrt{2}$

$3\sqrt{5}$	$2\sqrt{13}$	$4\sqrt{3}$	2	$9\sqrt{2}$
$3 \cdot \sqrt{5}$	$\sqrt{4} \cdot \sqrt{13}$	$\sqrt{16}\sqrt{3}$	$\sqrt{4}$	$\sqrt{81}\sqrt{2}$
$\sqrt{9} \cdot \sqrt{5}$	$\sqrt{52}$	$\sqrt{48}$		$\sqrt{162}$
$\sqrt{45}$				

$9\sqrt{2}, 2\sqrt{13}, 4\sqrt{3}, 3\sqrt{5}, 2$

4.4 Fractional Exponents and Radicals

Name: Notes
Date: Key

Goal: to relate rational exponents and radicals

oolkit:

- Exponent Laws
- Taking square and cube roots
- Converting decimals to fractions
- Order of operations

Main Ideas:

Evaluating powers of the form $a^{\frac{1}{n}}$

Powers with Rational Exponents with Numerator 1

When n is as natural number and x is a rational number,

$x^{\frac{1}{n}} = \sqrt[n]{x}$ ... for example... $16^{\frac{1}{2}} = \sqrt[2]{16} = 4$

Ex 1) Write each power as a radical then evaluate without using a calculator.

a) $1000^{\frac{1}{3}}$

$\sqrt[3]{1000}$

$= \boxed{10}$

b) $0.25^{0.5}$ Must change to a fraction first!

$0.25^{\frac{1}{2}}$

$\sqrt{0.25}$

$= \boxed{0.5}$

c) $(-8)^{\frac{1}{3}}$

$\sqrt[3]{-8}$

$= \boxed{-2}$

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{1/4}}{81^{1/4}}$

$\frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \boxed{\frac{2}{3}}$

Rewriting powers in radical and exponent form

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number,

$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^{m} = \left(\sqrt[n]{x}\right)^{m}$ ... ex) $25^{\frac{3}{2}} = \left(25^{\frac{1}{2}}\right)^{3} = \left(\sqrt[2]{25}\right)^{3} = (5)^3 = 125$

m → power, n → root

or

$x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$ ... ex) $25^{\frac{3}{2}} = (25^3)^{\frac{1}{2}} = \sqrt[2]{25^3} = \sqrt{15625} = 125$

Ex 2) Write $26^{\frac{2}{5}}$ in radical form in two different ways.

#1) $26^{2/5} = \left(26^{1/5}\right)^2 = \left(\sqrt[5]{26}\right)^2$

#2) $26^{2/5} = \left(26^2\right)^{1/5} = \sqrt[5]{26^2}$

Ex 3) Write the following in exponent form.

a) $\sqrt{6^5}$ root, → power

$= 6^{5/2}$

b) $\left(\sqrt[4]{19}\right)^3$ root, → power

$= 19^{3/4}$

*Think "root" underneath like a tree root. "Power" → being on top.

Evaluating powers with rational exponents and rational bases

Ex 4) Evaluate the following:

a) $0.01^{\frac{3}{2}}$

$\left(\sqrt[2]{0.01}\right)^3$

$(0.1)^3$

0.001

b) $(-27)^{\frac{4}{3}}$

$\left(\sqrt[3]{-27}\right)^4$

$(-3)^4$

81

c) $32^{0.4}$

$32^{\frac{4}{10}}$

$32^{\frac{2}{5}}$

$\left(\sqrt[5]{32}\right)^2$

$(2)^2$

4

d) $16^{0.75} = 16^{\frac{75}{100}}$

$16^{\frac{3}{4}}$

$\left(\sqrt[4]{16}\right)^3$

$(2)^3$

8

Applying rational exponents

Ex 5) Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass, m kilograms. Use the formula to estimate the brain mass of each animal.

a) A moose with a body mass of 512kg

$b = 0.01m^{\frac{2}{3}}$

$b = 0.01(512)^{\frac{2}{3}}$

$= 0.01\left(\sqrt[3]{512}\right)^2$

$= 0.01(8)^2$

$= 0.01(64)$

$= 0.64$ kg

The brain mass is 0.64 kg

b) A cat with a body mass of 5kg

$b = 0.01\ m^{\frac{2}{3}}$

$b = 0.01(5)^{\frac{2}{3}}$

$b = 0.01\left(\sqrt[3]{5}\right)^2$

$b = 0.01(1.71)^2$

$b = 0.01(2.9241)$

$b = 0.03$ kg

The brain mass is 0.03 kg

Reflection: In the power $x^{\frac{m}{n}}$, m and n are natural numbers and x is a rational number. What does the numerator m represent? What does the denominator n represent? Use an example to explain your answer.

n = index or 'root'

m = exponent or power

$x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$ or $\sqrt[n]{x^m}$

4.5 – Negative Exponents and Reciprocals

Name: Key
Date:

Goal: To relate negative exponents to reciprocals

Toolkit:
- Simplifying and evaluating with rational exponents
- Multiplying fractions

Main Ideas:

What is a reciprocal?

Two numbers with a product of 1 are reciprocals.

Ex1). Since $4 \cdot \frac{1}{4} = 1$, the numbers 4 and $\frac{1}{4}$ are <u>reciprocals</u>

Ex2). Since $\frac{2}{3} \cdot \frac{3}{2} = 1$, the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are <u>reciprocals</u>

Powers with Negative Exponents

When x is any non-zero number and n is a rational number, x^{-n} is the reciprocal of x^n.

That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, $x \neq 0$

Evaluate a power with a negative exponent

<u>Evaluate each power:</u>

Ex. 3) a) 3^{-2}

$\frac{1^2}{3^2} = \frac{1}{3^2}$

$\boxed{= \frac{1}{9}}$

b) $(-5)^{-3}$

$= \frac{1}{(-5)^3}$

$= \frac{1}{-125}$

$\boxed{= -\frac{1}{125}}$

c) $(-\frac{3}{4})^{-3}$

$= \left(-\frac{4}{3}\right)^3$

$= \frac{(-4)^3}{3^3}$

$\boxed{= \frac{-64}{27}}$

d) $(\frac{10}{3})^{-2}$

$= \left(\frac{3}{10}\right)^2$

$= \frac{3^2}{10^2}$

$\boxed{= \frac{9}{100}}$

Evaluate a power with a negative *rational* exponent

<u>To evaluate a power with a negative *rational* (fraction) exponent:</u>

Ex. 4) Evaluate $8^{-\frac{2}{3}}$

$= \frac{1}{8^{\frac{2}{3}}}$ power, root *write with a positive exponent*

$= \frac{1}{(\sqrt[3]{8})^2}$ root, power *re-write into radical form, then work from inside out*

$= \frac{1}{(2)^2}$ *evaluate (write answer with NO exponents)*

$= \frac{1}{4}$

Ex. 5) Evaluate:

a) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

$= \left(\frac{16}{9}\right)^{\frac{3}{2}}$ reciprocate the base, positive exponent

$= \frac{16^{\frac{3}{2}}}{9^{\frac{3}{2}}}$

$= \frac{(\sqrt{16})^3}{(\sqrt{9})^3} = \frac{4^3}{3^3}$

$\boxed{= \frac{64}{27}}$

b) $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$

$= \left(\frac{36}{25}\right)^{\frac{1}{2}}$

$= \frac{36^{\frac{1}{2}}}{25^{\frac{1}{2}}}$

$= \frac{\sqrt{36}}{\sqrt{25}}$

$\boxed{= \frac{6}{5}}$

c) $16^{-\frac{5}{4}}$

$= \left(\frac{1}{16}\right)^{\frac{5}{4}}$

$= \frac{1^{\frac{5}{4}}}{16^{\frac{5}{4}}}$

$= \frac{1}{(\sqrt[4]{16})^5}$

$= \frac{1}{(2)^5}$ $\boxed{= \frac{1}{32}}$

d) $-25^{-1.5}$

(hint: change 1.5 to a fraction in lowest terms!)

$= -25^{-\frac{3}{2}}$

$= -\left(\frac{1}{25}\right)^{\frac{3}{2}}$

$= -\left(\frac{1}{\sqrt[2]{25}}\right)^3$

$= -\left(\frac{1}{(5)^3}\right)$

$\boxed{= -\frac{1}{125}}$

Applying Negative Exponents (word problems)

Ex. 6) Use the formula $v = 0.155s^{\frac{5}{3}}f^{-\frac{7}{6}}$ to estimate the speed of a dinosaur when $s = 1.5$ and $f = 0.3$ (answer is a speed in m/s)

Substitute values into the proper places in the formula

$$V = 0.155(1.5)^{\frac{5}{3}}(0.3)^{-\frac{7}{6}}$$

brackets

1.5 y^x (5 ÷ 3) =

0.3 y^x (6 − 7 ÷ 6) =

Evaluate, using your calculator

$$V = 0.155 \cdot (1.5)^{\frac{5}{3}} \cdot (0.3)^{-\frac{7}{6}}$$

$$V = 0.155 \cdot (1.9656) \cdot (4.0740)$$

$$\boxed{V = 1.24 \text{ m/s}}$$

The speed of the dinosaur is 1.24 m/s

Reflection:

Check Out the Pattern!

$2^4 = 16$

$2^3 = 8$ ÷2

$2^2 = 4$ ÷2

$2^1 = 2$ ÷2

$2^0 = 1$ ÷2

$2^{-1} = \frac{1}{2}$ ÷2

$2^{-2} = \frac{1}{4} = \frac{1}{2^2}$

$2^{-3} = \frac{1}{8} = \frac{1}{2^3}$

$2^{-4} = \frac{1}{16} = \frac{1}{2^4}$

Would this work for other bases?

Yes!

4.6A – Simplifying with Exponent Laws

Name: Notes Key
Date:

Goal: to apply all of the exponent laws to simplify expressions

Toolkit:
- Exponent Laws
- Fractional and negative exponents
- Operations with fractions, integers

Main Ideas:

Exponent Laws

Product of powers: (same base!) $x^m \cdot x^n = x^{m+n}$ ex: $x^2 \cdot x^3 = x^{2+3} = x^5$

Quotient of powers: (same base!) $x^m \div x^n = x^{m-n}$ ex: $\frac{x^4}{x^2} = x^{4-2} = x^2$

Power of a power: $(x^m)^n = x^{m\cdot n}$ ex: $(x^2)^5 = x^{2\cdot5} = x^{10}$

Power of a product: $(xy)^m = x^m y^m$ ex: $(2x)^2 = 2^2 \cdot x^2 = 4x^2$

Power of a quotient: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ ($y \neq 0$) ex: $\left(\frac{y}{3}\right)^3 = \frac{y^3}{3^3} = \frac{y^3}{27}$

Power of zero: $n^0 = 1$ ($n \neq 0$) anything to the power zero, except zero, equals 1.

Fractional exponents: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ or $(\sqrt[n]{x})^m$ ex: $x^{2/3} = \sqrt[3]{x^2}$ or $(\sqrt[3]{x})^2$

Negative exponents: $x^{-m} = \frac{1}{x^m}$, $\frac{1}{x^{-n}} = x^n$, $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Note: write all powers with POSITIVE EXPONENTS.

Which law(s) did you use?

Ex 1) Simplify by writing as a single power.

a) $0.6^2 \cdot 0.6^{-6}$ (same base!)

$= 0.6^{2+-6}$

$= 0.6^{-4}$

$= \frac{1}{0.6^4}$

b) $x^{-4} \cdot x^7$ (same base!)

$= x^{-4+7}$

$= x^3$

c) $m^7 \div m^{-2}$

$= m^{7-(-2)}$

$= m^{7+2}$

$= m^9$

d) $\frac{0.4^3}{0.4^4}$

$= 0.4^{3-4}$

$= 0.4^{-1}$

$= \frac{1}{0.4}$

e) $(n^2)^{-4}$

$= n^{2\times-4}$

$= n^{-8}$

Power of a Power first!

Simplify num/denom separately

Ex 2) Simplify by writing as a single power.

a) $\left[\left(-\frac{4}{7}\right)^{2}\right]^{-3} \div \left[\left(-\frac{4}{7}\right)^{4}\right]^{-5}$

$= \left(-\frac{4}{7}\right)^{-6} \div \left(-\frac{4}{7}\right)^{-20}$

$= \left(-\frac{4}{7}\right)^{-6-(-20)}$

$= \left(-\frac{4}{7}\right)^{-6+20}$

$= \left(-\frac{4}{7}\right)^{14}$

b) $\dfrac{(2.3^{-3})^{-5}}{2.3^{5}}$

$= \dfrac{2.3^{15}}{2.3^{5}}$

$= 2.3^{15-5}$

$= 2.3^{10}$

c) $\dfrac{8^{\frac{5}{4}} \cdot 8^{-\frac{1}{4}}}{8^{\frac{3}{4}}}$

$= \dfrac{8^{\frac{5}{4}-\frac{1}{4}}}{8^{\frac{3}{4}}}$ need common denom. to add/subtr.

$= \dfrac{8^{\frac{4}{4}}}{8^{\frac{3}{4}}}$ leave $\frac{4}{4}$ (common denom!)

$= 8^{\frac{4}{4}-\frac{3}{4}}$

$= 8^{\frac{1}{4}}$

Note: write all powers with POSITIVE EXPONENTS.

Ex 3) Simplify.

a) $(x^4y^{-2})(x^2y^3)$ all multiplied - reorder!

$= x^4 \cdot x^2 \cdot y^{-2} \cdot y^3$

$= x^{4+2} \, y^{-2+3}$

$= x^6 \cdot y^1$

$=$ or x^6y

b) $(27x^6y^9)^{\frac{1}{3}}$ $\frac{9 \times 1}{1 \cdot 3} = \frac{9}{3}$

$= 27^{\frac{1}{3}} (x^6)^{\frac{1}{3}} (y^9)^{\frac{1}{3}}$

$= \sqrt[3]{27} \; x^{\frac{6}{3}} \; y^{\frac{9}{3}}$

$= 3x^2y^3$

c) $\left(\dfrac{3a^4b^{-3}}{7a^{-2}b^2}\right)^{-2}$ Simplify inside, write neg. exp. as pos.

$= \left(\dfrac{3a^4a^2}{7b^3b^2}\right)^{-2}$

$= \left(\dfrac{3a^6}{7b^5}\right)^{-2}$ make + (flip inside)

$= \left(\dfrac{7b^5}{3a^6}\right)^{2}$

$= \dfrac{7^2(b^5)^2}{3^2(a^6)^2}$

$= \dfrac{49b^{10}}{9a^{12}}$

d) simplify inside $\left(\dfrac{50m^2n^4}{2m^4n^2}\right)^{\frac{1}{2}}$ Can skip to here

$= \left(25m^{2-4}n^{4-2}\right)^{\frac{1}{2}}$

$= \left(25m^{-2}n^2\right)^{\frac{1}{2}}$

$= \left(\dfrac{25n^2}{m^2}\right)^{\frac{1}{2}}$

$= \dfrac{25^{\frac{1}{2}} n^{\frac{2}{2}}}{m^{\frac{2}{2}}}$

$= \dfrac{5n}{m}$

e) top first $\dfrac{\left(2x^{\frac{3}{2}}y^2\right)\left(3x^{\frac{1}{2}}y^{-1}\right)}{(4x^3y^{-1})}$

$= \dfrac{{}^{3}\cancel{6}\,x^{\frac{3}{2}+\frac{1}{2}}\,y^{2+-1}}{{}^{2}\cancel{4}\,x^3y^{-1}}$

$= \dfrac{3x^{\frac{4}{2}}y^1}{2x^3y^{-1}}$

$= \dfrac{3x^2y^1}{2x^3y^{-1}}$

$= \dfrac{3x^{2-3}y^{1-(-1)}}{2}$

$= \dfrac{3x^{-1}y^2}{2}$

$= \dfrac{3y^2}{2x}$

Reflection: How would you simplify the expression $\left(\frac{x^a}{x^3}\right)^2$ and how is it similar/different compared to the other problems we've done?

- same laws
- different because we have a variable in the exponent

$\left(\dfrac{x^a}{x^3}\right)^2 = \dfrac{x^{2a}}{x^6} = x^{2a-6}$ / or $\left(x^{a-3}\right)^2 = x^{2(a-3)} = x^{2a-6}$

4.6B – Evaluating with Exponent Laws

Name: Notes key
Date:

Goal: to apply all of the exponent laws to evaluate expressions

Toolkit:
- Exponent Laws, incl. fractional /negative
- Operations with fractions, integers
- Substitution, BEDMAS

Main Ideas:

What is the difference between "simplifying" and "evaluating"?

Simplify: (write as single base)

Ex 1) Simplify $x^{\frac{5}{3}} \cdot x^{\frac{1}{3}}$

$= x^{\frac{5}{3}+\frac{1}{3}}$

$= x^{\frac{6}{3}}$

$= x^2$

Evaluate: (write as a single number – no exponents/variables)

Ex 2) Evaluate $1.5^{\frac{5}{3}} \cdot 1.5^{\frac{1}{3}}$

$= 1.5^{\frac{5}{3}+\frac{1}{3}}$

$= 1.5^{\frac{6}{3}}$

$= 1.5^2$

$= 1.5 \times 1.5 = 2.25$

Ex 3) Evaluate each expression for $m = -1$ and $n = 2$

Step 1: Simplify the expression

Step 2: Substitute →replace letters with numeric values (use brackets!)

Step 3: Evaluate

a) $(m^2n^3)(m^3n^2)$

① $m^{2+3}n^{3+2}$

m^5n^5

② $(-1)^5(2)^5$

③ $=(-1)(32)$

$= -32$

b) $\left(\dfrac{m^{-5}n^5}{m^{-2}n^6}\right)^{-3}$

① $= (m^{-5+2}n^{5-6})^{-3}$

$= (m^{-3}n^{-1})^{-3}$

$= m^9n^3$

② $= (-1)^9(2)^3$

③ $= (-1)(8)$

$= -8$

c) $\dfrac{(m^n)^2}{m^3}$

① $= \dfrac{m^{2n}}{m^3}$

$= m^{2n-3}$

② $= (-1)^{2(2)-3}$

$= (-1)^{4-3}$

$= (-1)^1$

$= -1$

Solving Problems using the Exponent Laws

Note: (for H.W.)

$V_{cone} = \frac{1}{3}\pi r^2 h$

if r, h are equal: r=h

$V = \frac{1}{3}\pi h^2 \cdot h$

$V_{cone} = \frac{1}{3}\pi h^3$

Ex 4) A sphere has volume 600m³.

a) Write an expression for the radius in exponent form

b) What is the radius of the sphere to the nearst tenth of a metre?

a) $V = \frac{4}{3}\pi r^3$

$3 \times 600 = 3 \times \frac{4}{3}\pi r^3$

$\dfrac{1800}{4\pi} = \dfrac{4\pi r^3}{4\pi}$

$\left(\dfrac{1800}{4\pi}\right)^{1/3} = (r^3)^{1/3}$

$r = \left(\dfrac{1800}{4\pi}\right)^{1/3}$

b) $r = \sqrt[3]{\left(\dfrac{1800}{4\pi}\right)}$

$r \doteq 5.2322$

The radius is 5.2m.

Reflection: Why is it important to simplify BEFORE evaluating? You can often answer without a calculator; it's much easier; there are fewer values/operations to deal with.