

3.1 & 3.2 – Factors and Multiples

Name:

Date:

Goal: to determine prime factors, GCFs, and LCMs of whole numbers, and find roots using prime factors

Toolkit:

- Division
- Multiplication
- Writing repeated multiplication using powers, e.g. $2 \times 2 \times 2 \times 2 \times 2 =$

Main Ideas:

Definitions

Factor – a term which divides evenly into another term

Prime number – when a number has only 2 distinct factors (1 and itself). **Examples:**

Composite number – when a number has more than 2 factors. **Examples:**

Prime factorization – a term written as a product of prime factors

every composite number can be expressed as a product of prime factors

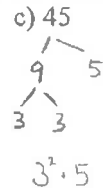
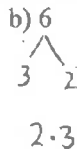
Greatest common factor (GCF) – the largest term which will divide evenly into a series of separate terms

Least (or Lowest) common multiple (LCM) – the smallest multiple which is common to series of separate terms

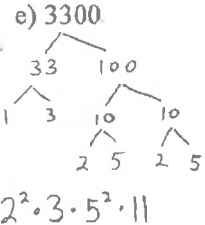
Prime Factorization

Ex1) Write the prime factorization for each of the composite numbers:

a) 3



d) 47



Perfect Squares & Cubes

What is a perfect square?

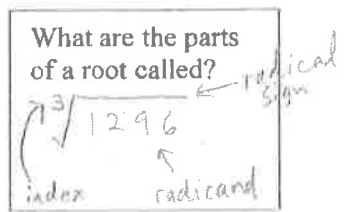
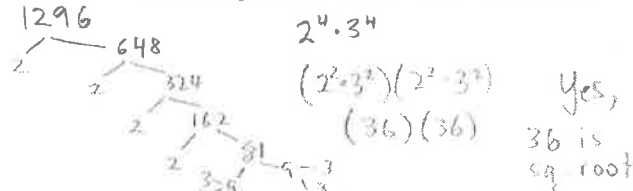
A number whose square root is a rational number

What is a perfect cube?

A number whose cube root is a rational number

Ex2) Using prime factors, is 1296 a perfect square? If so, what is the square root?

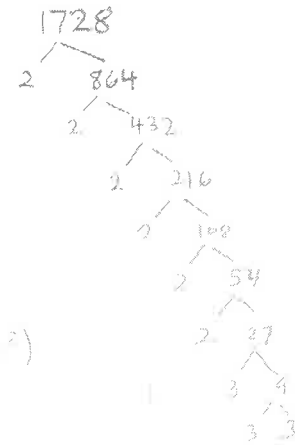
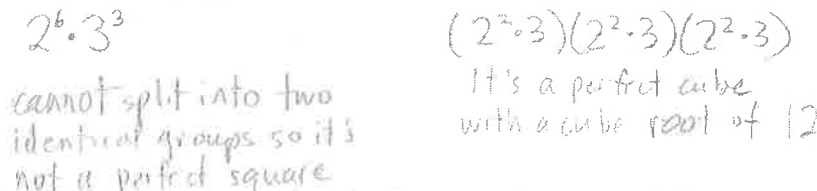
Can the prime factors be arranged into TWO identical groups?



Is 1296 a perfect cube? *Three identical groups?*

No!

Ex3) Is 1728 a perfect square? Is it a perfect cube?



Ex4) Determine the edge length of a cube with volume $64x^6$.



Finding the GCF
by listing all the factors of each number (the rainbow method)

Ex5) Determine the greatest common factor of 126 and 144
Method 1 - list all the factors and find the largest one in common (write small!)

126: 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126
144: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144
GCF = 18

Finding the GCF
by writing the prime factorization of each number

Method 2
1) write the prime factorization for each number
2) highlight the factors that they have in common
3) multiply all the common factors together to get the GCF

126: $2 \cdot 3^2 \cdot 7$
144: $2^4 \cdot 3^2$

* take the least of each prime factor from the lists

GCF = $2 \cdot 3^2 = 18$

Finding the LCM
by listing the first multiples of each number

Ex6) Find the least common multiple of 28, 42, and 63
Method 1 - list the first few multiples of each number until you find (the first, lowest) one in common

28: 28, 56, 84, 112, 140, 168, 196, 224, 252
42: 42, 84, 126, 168, 210, 252
63: 63, 126, 189, 252
LCM = 252

Finding the LCM
by writing the prime factorization of each number

Method 2
1) write the prime factors of each number
2) highlight the greatest power of each prime in ANY of the lists
3) multiply the greatest powers of each prime together to get the LCM

28: $2^2 \cdot 7$
42: $2 \cdot 3 \cdot 7$
63: $3^2 \cdot 7$

* take the most of each prime factor from the lists

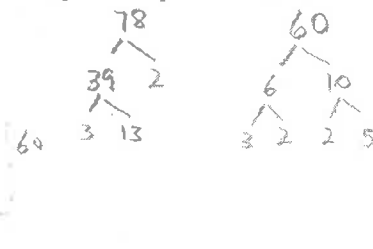
LCM = $2^2 \cdot 3^2 \cdot 7 = 252$

What types of real-world problems involve GCFs and LCMs?

Ex7) Beside each problem, write whether you would need the GCF or the LCM, then answer the question!

a) A bathroom wall (the part above the bathtub) is a rectangle that measures 78" by 60". If you wanted to cover it exactly with square tiles, what is the largest possible square tile you could use?

GCF



78: $2 \cdot 3 \cdot 13$
60: $2^2 \cdot 3 \cdot 5$

GCF = $2 \cdot 3 = 6$

6" x 6" square tile

b) You have red bungee cords that are 18cm long and green bungee cords that are 14cm long. What is the shortest length of connected bungees you can make with each colour so that they make the same length?

Red: 18
Green: 14

18: $2 \cdot 3^2$
14: $2 \cdot 7$

LCM = $2 \cdot 3^2 \cdot 7 = 126$

126cm

If you're looking for an answer:
- smaller than values given = GCF
- larger than values given = LCM

LCM



3.7 – Multiplying Polynomials

Name:

Date:

Goal: to expand monomial and binomial products (multiply out!)

Toolkit:

- Adding, subtracting, multiplying polynomials
- Multiplying powers with the same base: add the exponents
- Collecting like terms: same variable(s) with same exponents

Ex: $(x^3)(x^4) = x^{3+4} = x^7$

Ex: $2x^2 + 3x - 1x^2 + 2x + 1 = x^2 + 5x + 1$

Main Ideas:

The sign in front of a term is part of that term!

The sign in front of

Definitions

Polynomial – many terms (terms separated by add and subtract)

Monomial – 1 term

Binomial – 2 terms

Trinomial – 3 terms

$-2x(x^2 + 4)$
 $-2x^3 - 8x$

F O I L
 f i r s t s
 o u t s i d e s
 i n s i d e s
 l a s t s

$(y-2)(y-5)$
 $y^2 - 5y - 2y + 10$
 $y^2 - 7y + 10$

Ex1) Expand and simplify → translates to: distributive property (multiply) and then like terms (add/subtract)

a) $3x^2(x+3)$
 $3x^3 + 9x^2$

b) $(x+2)(x+3)$
 $x^2 + 3x + 2x + 6$
 $x^2 + 5x + 6$

c) $(2y+z)(3y-2z)$
 $6y^2 - 4yz + 3yz - 2z^2$
 $6y^2 - yz - 2z^2$

$(x-4)(x+5)$
 $x^2 + 5x - 4x - 20$
 $x^2 + x - 20$

d) $(2a-1)(2a+3) + (a-1)(a-2)$
 $4a^2 + 6a - 2a - 3 + a^2 - 2a - a + 2$
 $4a^2 + 4a - 3 + a^2 - 3a + 2$
 $5a^2 + a - 1$

Ex2) Expand and simplify:

a) $(x+3y)(x+y-3)$

$$x^2 + xy - 3x + 3xy + 3y^2 - 9y$$

$$x^2 + 3y^2 + 4xy - 3x - 9y$$

b) $(x+2)^3$

$$(x+2)(x+2)(x+2)$$

$$x^2 + 2x + 2x + 4$$

$$(x^2 + 4x + 4)(x+2)$$

$$x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$$

$$x^3 + 6x^2 + 12x + 8$$

c) $(r^2 + 3r - 1)(2r^2 - r + 2)$

$$2r^4 - r^3 + 2r^2 + 6r^3 - 3r^2 + 6r - 2r^2 + r - 2$$

$$2r^4 + 5r^3 - 3r^2 + 7r - 2$$

Ex3) Find the area of the shaded region (simplified!):



$$A_{\text{shaded}} = A_{\text{big rec}} - A_{\text{small rec}}$$

$$A_{\text{shaded}} = (2x+4)(6x+1) - 3x(x+2)$$

$$12x^2 + 2x + 24x + 4$$

$$= 12x^2 + 26x + 4 - 3x^2 - 6x$$

$$A_{\text{shaded}} = 9x^2 + 20x + 4$$

3.3 – Common Factors of a Polynomial

Name: Key
Date: _____

Goal: to determine the factors of a polynomial by identifying the GCF

Toolkit:

- Finding the GCF
- Distributive Property

Main Ideas:

Factor a binomial using the GCF

Ex 1) Factor the binomial: $3g + 6 \Rightarrow$ 2 terms : $3g, 6$
 What's the GCF of 3 and 6? 3
 Can the variable be part of the GCF? No, because the 2nd term is a constant
 $3(g + 2)$ Check: $3(g+2)$
 $3g + 6 \checkmark$

Ex 2) Factor the binomial: $-8y + 16y^2$
 Reorder from highest to lowest degree (exponent on variable)

$$\frac{16y^2}{8y} - \frac{8y}{8y}$$

$$8y(2y - 1)$$

Factor a trinomial using the GCF

Ex 3) Factor the trinomial: $\frac{3x^2}{3} + \frac{12x}{3} - \frac{6}{3}$
 $3(x^2 + 4x - 2)$

Ex 4) Factor the trinomial: $6 - 12z + 18z^2$

Reorder: $\frac{18z^2}{6} - \frac{12z}{6} + \frac{6}{6}$

$$6(3z^2 - 2z + 1)$$

Factor polynomials in more than one variable

Ex 5) Factor the trinomial: $\frac{-20c^4d}{-5cd} - \frac{30c^3d^2}{-5cd} - \frac{25cd}{-5cd}$
 $-5cd(4c^3 + 6c^2d + 5)$

* If 1st term is negative, the GCF should be negative!

Reflection: How are the processes of factoring and expanding related?

3.5 – Factoring Trinomials of the form $x^2 + bx + c$, where $a=1$

Name:

Date:

Goal: to use models and algebraic strategies to multiply binomials and to factor trinomials.

Toolkit:

- Factoring

$$ax^2 + bx + c$$

↑ coefficient for x^2 ↑ coefficient for x ↑ constant

Main Ideas:

Definitions:

Descending order: the terms are written in order from the term with the greatest exponent to the term with the least exponent

Ascending order: the terms are written in order from the term with the least exponent to the term with the greatest exponent

Steps for Factoring a Trinomial in the form: $x^2 + bx + c$, where $a=1$

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable (biggest to smallest)

Step 2: Find two numbers that multiply to equal the c term and add to equal the b term (add to the middle, multiply to the end)

Step 3: Factor into two binomials using the numbers from step 2, with the variable from the question placed first in each bracket

Multiplying two binomials

Ex 1) Expand and Simplify: $(x-1)(x-7)$ use FOIL

$$x^2 - 7x - 1x + 7$$

$$x^2 - 8x + 7$$

$b = -8$ came from adding -1 and -7
 $c = 7$ came from multiplying $(-1)(-7)$

Remember: expanding and factoring are opposite operations... they UNDO each other!

Factoring a trinomial in the form $x^2 + bx + c$

Ex 2) Factor the trinomial: $x^2 - 8x + 7$...we should end up with $(x-1)(x-7)$!

- re-order (not necessary) $a=1$ $b=-8$ $c=7$ $x7, + -8$
- two numbers that multiply to c and add to b $7, 1$
 $(-7, -1)$

③ $(x-7)(x-1)$ or $(x-1)(x-7)$

Notice that a (the number in front of the x^2) will always end up being 1 in these questions!

Ex 3) Factor: $a^2 - 2a - 8$
 $a=1$ $b=-2$ $c=-8$

- in order ✓
- $x c, + b$ $(a-4)(a+2)$ FOIL To check: $(a-4)(a+2)$
 $x -8, + -2$ $a^2 + 2a - 4a - 8$
 $-8, 1$ $4, -2$ $a^2 - 2a - 8$
 $8, -1$ $(-4, 2)$

Factoring a trinomial written in ascending order

Ex 4a) Factor: $-30 + 7m + m^2$

- $m^2 + 7m - 30$
- $x -30 + 7$ $(10, -3)$
- $(m+10)(m-3)$

b) $x^2 - 4xy + 21y^2$

- in order
- $x 21, + -4$ $(-7, 3)$
- $(x-7y)(x+3y)$

Ex 5) Factor: $-5h^2 - 20h + 60$

Always check to see if there is a GCF you can factor out first! If there is a negative number in front of the x^2 , factor out the negative as well.

get? (-5) $-5(h^2 + 4h - 12)$
 copy (-5) and proceed!
 reorder? \checkmark
 $-5(h-2)(h+6)$

② $(+)$ to -12 $(+)$ to $+4$
 $\begin{array}{r} -3 \quad -4 \rightarrow +1 \\ 2 \quad -6 \rightarrow -1 \\ \hline +2 \quad +6 \rightarrow +12 \\ 2 \quad -6 \rightarrow -12 \end{array}$

check: For first!

$-5(h^2 + 6h - 2h - 12)$
 $-5(h^2 + 4h - 12)$
 $-5h^2 - 20h + 60 \checkmark$
 circle answer!

Ex 6) Factor: $-12g^2 - 9g + 3g^2$

reorder/get: $3g^2 - 9g - 12$
 $3(g^2 - 3g - 4)$
 $3(g+1)(g-4)$

③ to -4 $(+)$ to -3
 $\begin{array}{r} -1 \quad -4 \rightarrow -3 \\ 1 \quad -4 \rightarrow -3 \\ \hline 1 \quad -4 \rightarrow -3 \\ 2 \quad -2 \rightarrow -4 \end{array}$

check: For first!

$3(g^2 - 4g + 1g - 4)$
 $3(g^2 - 3g - 4)$
 $3g^2 - 9g - 12 \checkmark$
 circle answer!

Ex 7) Factor: $2x^2 - 6x - 80$

reorder? get?
 $2(x^2 - 3x - 40)$
 $2(x-8)(x+5)$

need
 ③ to -40 $(+)$ to -3
 is -1×40 smart to check?
 not really!
 $-1 \times 40 = -40$
 try: $(-8 \times 5) \rightarrow (-3)$
 $8 \times -5 \rightarrow -40$

check: For first!

$2(x^2 + 5x - 8x - 40)$
 $2(x^2 - 3x - 40)$
 $2x^2 - 6x - 80 \checkmark$

Ex 8) Factor: $x^2 + x - 2$

③ to -2 $(+)$ to $+1$
 $\begin{array}{r} 1 \quad -2 \rightarrow -1 \\ -1 \quad 2 \rightarrow +1 \end{array}$
 $(x-1)(x+2)$

Reflection: Does the order in which the binomial factors are written affect the solution? Explain.

$(x-1)(x+2)$
 $= x^2 + 2x - 1x - 2$
 $= x^2 + x - 2$

$(x+2)(x-1)$
 $= x^2 - 1x + 2x - 2$
 $= x^2 + x - 2$

No! You get the same product regardless of the order you multiply in

Goal: to extend the strategies for multiplying binomials and factoring trinomials

Toolkit:

- Multiplying binomials
- Factoring

Main Ideas:

Factoring by Decomposition: (needed when the $a \neq 1$ in $ax^2 + bx + c$)

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable (biggest to smallest)

Step 2: Find two numbers that multiply to equal ac and add to equal b (add to the middle, multiply to product of first and last)

Step 3: Re-write the expression but split or decompose the b term using the two numbers from step 2.

Step 4: Now the expression has FOUR terms, so we can factor by grouping the first two terms and the last two terms.

Step 5: When fully factored, the remaining two brackets need to be identical! These are now a common factor, and can be factored out, and what is left becomes the components of the second bracket.

Factor by Grouping

Ex 1) Factor the following by grouping: no gcf for all 4, but each pair has a gcf!

a) $3x^2 - 3x - 2x + 2$

gcf: $3x$
 $3x(x-1) - 2(x-1)$
 $(x-1)(3x-2)$
 "Can check!"

b) $2x^2 - 4x + x - 2$

gcf: 2
 $2x(x-2) + 1(x-2)$
 $(x-2)(2x+1)$

"take out common bracket"

Factoring a trinomial of the form $ax^2 + bx + c$

Ex 2) Factor the trinomial: $4g^2 + 11g + 6$ by decomposition

- Need to find two numbers that multiply to $ac = 24$ and add to $b = 11$.
- ① to 24: $1 \times 24 \rightarrow 25$
 - $2 \times 12 \rightarrow 14$
 - $3 \times 8 \rightarrow 11$ ✓
- ③ split into $3g$ and $8g$
- ④ grouping

① $4g^2 + 11g + 6$

② $4g^2 + 3g + 8g + 6$

③ $g(4g+3) + 2(4g+3)$

④ $(4g+3)(g+2)$

notice that a (the number in front of x^2) is not = 1 in any of these questions!

check:
 $(4g+3)(g+2)$
 $4g^2 + 8g + 3g + 6$
 $4g^2 + 11g + 6$ ✓

Ex 3) Factor the trinomial: $-7m - 10 + 6m^2$

reorder: $6m^2 - 7m - 10$

Need to find two numbers that multiply to $ac = -60$ and add to $b = -7$.

- ① to -60 : $1 \times -60 \rightarrow -59$
- $2 \times -30 \rightarrow -28$
- $3 \times -20 \rightarrow -17$
- $4 \times -15 \rightarrow -11$
- $5 \times -12 \rightarrow -7$ ✓

③ split into $5m$ and $-12m$

④ grouping

$m(6m+5) - 2(6m+5)$

$(6m+5)(m-2)$

Ex 3) Factor: $8p^2 - 18pq - 5q^2$

in order ✓

$a=8 \quad b=-18 \quad c=-5$
 $\times ac, +b \Rightarrow \times -40, +18$

$(-20, 2)$

$$8p^2 - 18pq - 5q^2$$

$$8p^2 + 2pq - 20pq - 5q^2$$

$$2p(4p+q) - 5q(4p+q)$$

$$(4p+q)(2p-5q)$$

Ex 4) Factor: $6x^2 + 14x - 12$

GCF $2(3x^2 + 7x - 6)$

$a=3, b=7, c=-6$
 $\times -18, +7$

$(9, -2)$

$$2(3x^2 + 7x - 6)$$

$$3x^2 + 9x - 2x - 6$$

$$3x(x+3) - 2(x+3)$$

$$2(x+3)(3x-2)$$

If you can make a trinomial have $a=1$ by removing a G.C.F., then you can use "the simple way"!

Ex 5) Factor: $3x^2 + 6x - 9$

GCF $3(x^2 + 2x - 3)$

$a=1 \quad b=2 \quad c=-3$

remember, if $a=1$, can factor easier!

$$\begin{array}{r} \times c \quad + b \\ \times -3 \quad + 2 \end{array}$$

$(3, -1)$

$$3(x+3)(x-1)$$

Ex 6) Find an integer to replace \square so that the trinomial can be factored. How many integers can you find?

$$4x^2 + \square x + 9$$

$$\times ac, +b$$

$$\times 36, +b$$

factors of 36:

$$36, 1 \Rightarrow 37$$

$$-36, -1 \Rightarrow -37$$

$$18, 2 \Rightarrow 20$$

$$-18, -2 \Rightarrow -20$$

$$12, 3 \Rightarrow 15$$

$$-12, -3 \Rightarrow -15$$

$$9, 4 \Rightarrow 13$$

$$-9, -4 \Rightarrow -13$$

$$6, 6 \Rightarrow 12$$

$$-6, -6 \Rightarrow -12$$

any of these integers!

10 different integers in total!

Reflection: Will decomposition work if the a value of a trinomial is 1? Do an example to prove this.

Yes!

$$x^2 + 2x - 3$$

$$x^2 - 1x + 3x - 3$$

$$x(x-1) + 3(x-1)$$

$$(x-1)(x+3) \checkmark$$

$$\begin{array}{r} \times ac \quad + b \\ \times -3 \quad + 2 \\ 3, -1 \end{array}$$

3.8 - Factoring Special Polynomials

Name: Key
Date: _____

Goal: to investigate perfect square trinomials and difference of squares

Toolkit:

- Finding a square root
- Finding GCF
- Multiplying Polynomials

Main Ideas:

Definitions:

Perfect Square Trinomial: a trinomial of the form $m^2 + 2mn + n^2$; it can be factored as $(m + n)^2$
or of the form $m^2 - 2mn + n^2$; it can be factored as $(m - n)^2$

Difference of Squares: a binomial of the form $m^2 - n^2$; it can be factored as $(m - n)(m + n)$

Factoring a perfect square trinomial

Warmup: Factor the trinomial $4x^2 - 4x + 1$ using decomposition.

$$ac = (4)(1) = 4$$

$$b = -4$$

$$-2, -2$$

$$4x^2 - 2x - 2x + 1$$

$$2x(2x-1) - 1(2x-1)$$

$$(2x-1)(2x-1) = (2x-1)^2$$

Decomposition works, but it is time consuming. Test to see if the trinomial is a perfect square! If so, it will be quicker to factor. $4x^2 - 4x + 1$

Step 1: Is the trinomial in order? Yes Can you factor out a GCF? No

Step 2: Are the first and last terms perfect squares? Yes

Step 3: Make two brackets, and write the square roots into each. Then, figure out if the brackets should have a '+' or '-' in between the terms.

$$(2x-1)(2x-1) = (2x-1)^2$$

Step 4: Now test that the middle terms (the 'O' and 'I' of FOIL) add to the middle term of the original polynomial. If so, the trinomial is a perfect square.

$$-2x - 2x = -4x \quad \checkmark \text{ Yes.}$$

Ex 1) Factor the trinomial: $36x^2 + 12x + 1$

In order? Yes
GCF? No

$$(6x+1)(6x+1)$$

Check middle term:
 $6x + 6x = 12x \quad \checkmark$

$$(6x+1)^2$$

Ex 2) Factor the trinomial: $18x^2 - 48xy + 32y^2$

In order? Yes
GCF? Yes

$$2(9x^2 - 24xy + 16y^2)$$

$$2(3x-4y)(3x-4y)$$

Check middle term:
 $-12xy - 12xy = -24xy \quad \checkmark$

$$2(3x-4y)^2$$

Ex 3) Factor the trinomial: $25c^2 - 29cd + 4d^2$

In order? Yes
GCF? No

$$(5c-2d)(5c-2d)$$

$$25c^2 - 29cd + 4d^2$$

$$25c^2 - 25cd - 4cd + 4d^2$$

Check middle term:

$$-10cd - 10cd = -20cd \quad \times$$

$$25c(c-d) - 4d \quad d$$

$$(c-d)(25c-4d)$$

Therefore, not a perfect square trinomial. Must factor by decomposition.

Factoring a Difference of Squares

Difference of Squares is only possible if you have a binomial. The binomial must have a **SUBTRACT** (difference) in between two **PERFECT SQUARES** (of squares).

Ex 4) Factor the binomial: $81m^2 - 49$

Step 1: Is there a subtract in the middle? *Yes*

Step 2: Is each term a perfect square? *Yes*

Step 3: If not, is there a GCF to factor out? *No*

Step 4: Make two brackets, one with a '+' and one with a '-'.

Step 5 Square root each term and put into the appropriate position in each bracket.

$$(9m + 7)(9m - 7)$$

CHECK: $(9m + 7)(9m - 7)$
 $= 81m^2 - 63m + 63m - 49$
 $= 81m^2 - 49 \checkmark$

Ex 5) Factor: $m^2 - 36$

$$(m + 6)(m - 6)$$

Why is one bracket '+' and one '-'?

This will cause the middle terms to be opposites, thereby adding to zero.

Ex 6) Factor: $32v^2 - 2w^2$

$$2(16v^2 - w^2) = 2(4v + w)(4v - w)$$

Ex 7) Factor: $\frac{x^2}{25} - \frac{y^2}{4}$

$$\left(\frac{x}{5} + \frac{y}{2}\right)\left(\frac{x}{5} - \frac{y}{2}\right)$$

Ex 8) Factor: $x^2 + 9$

↑ not a difference of squares

A sum of squares CANNOT be factored.

Ex 9) Factor: $2x^4 - 162$

$$2(x^4 - 81)$$

sum of squares, cannot factor.

$$2(x^2 - 9)(x^2 + 9)$$

another difference of squares

$$2(x + 3)(x - 3)(x^2 + 9)$$

*If you have a 4th power variable, there is a good chance there will be TWO LAYERS of factoring to complete.

Reflection: Does a sum of squares factor? Explain.

3.9 – Factoring Synthesis

Name: *Key*
Date:

FACTORING FLOW CHART

STEP 1 Take out COMMON FACTORS (GCF)

STEP 2 Ask: How many terms are there?

TWO

Test for difference of squares:

*You need subtraction ("difference") and each term must be a perfect square

If you don't have perfect squares, check to see if you can factor out a GCF.

$$a^2 - b^2 = (a + b)(a - b)$$

Example:

$$4x^2 - 9 = (2x + 3)(2x - 3)$$

Example:

$$2m^2 - 32n^2 = 2(m^2 - 16n^2) = 2(m + 4n)(m - 4n)$$

Example:

$$4w^2 + 9y^2$$

*cannot factor
As it is a SUM of squares*

THREE

Factoring trinomials: $ax^2 + bx + c$
Is the trinomial in order?
Can you factor out a GCF?

Type 1: $a = 1$

Example: $x^2 - 3x + 2$
Ask: what ADDS to "b" (here -3) & MULTIPLIES to "c" (here +2)
Answer: -1, -2
Write factors: $(x - 1)(x - 2)$

Type 2: $a \neq 1$

Is it a perfect square trinomial?
Are first and last terms perfect squares?
Is the middle term correct?
Example: $4x^2 - 12x + 9$
Factor using square roots:
 $(2x - 3)(2x - 3)$
Middle term: $-6x - 6x = -12x$

If it isn't a perfect square trinomial, factor using DECOMPOSITION.

Example: $2x^2 - x - 1$
Ask: what ADDS to "b" (here -1) & MULTIPLIES to "ac" (here $2(-1) = -2$)
Answer: -2, 1

Use these to split (decompose) the middle term into two separate terms:

$$2x^2 - x - 1 = 2x^2 - 2x + 1x - 1$$

Factor using grouping:

$$2x(x - 1) + 1(x - 1)$$

See if two brackets are the same.
Factor the bracket out front as a GCF, & the 'leftovers' make up the 2nd bracket
 $(x - 1)(2x + 1)$

STEP 3 Ask: FF? Look inside each factor (bracket) and see if you can FACTOR FURTHER.
*If the original question has an x^4 term, there is a good chance there will be 2 layers of factoring!

Practice factoring expressions using the flowchart for assistance.

Ex 1) Factor: $2x^2 - 22x + 60$

GCF $2(x^2 - 11x + 30)$

$2(x-6)(x-5)$

$a=1$ now so two numbers that $\times c$ and $+b$
 $\times 30, + -11$

$-6, -5$
can go right to brackets.

Ex 2) Factor: $p^2 - 25q^2$

diff of squares

$(p+5q)(p-5q)$

Ex 3) Factor: $3y^2 - 7y - 6$

No GCF
No perfect squares
thus decomposition

$\times ac, +b$
 $\times -18, + -7$
 $-9, 2$

$3y^2 - 7y - 6$
 $3y^2 - 9y + 2y - 6$
 $3y(y-3) + 2(y-3)$
 $(y-3)(3y+2)$

Ex 4) Factor: $4m^2 + 12m - 56$

GCF $4(m^2 + 3m - 14)$

$a=1$, so two numbers that $\times c, +b$
 $\times -14, + 3$ NOT POSSIBLE
so can't factor further

Ex 5) Factor: $9x^2 - 42xy + 49y^2$

Perfect squares!

$(3x-7y)(3x-7y)$

$(3x-7y)^2$

test middle term: $-2 \times xy - 2 \times xy = -4xy$ ✓

Ex 6) Factor: $8b^2 + 2c^2$

GCF $2(4b^2 + c^2)$

sum of squares so cannot factor further

Ex 7) Factor: $8x^2 + 40x + 18$

GCF $2(4x^2 + 20x + 9)$

perfect squares!

$2(2x+3)(2x+3)$

must factor by decomp:

$\times ac, +b$ so $\times 36, + 20 \Rightarrow 18, 2$

$2[4x^2 + 2x + 18x + 9]$

$2[2x(2x+1) + 9(2x+1)]$

test middle term: $6x + 6x = 12x$ X

$2(2x+1)(2x+9)$

Ex 8) Factor: $32x^2 - 50y^2$

GCF $2(16x^2 - 25y^2)$ Diff of squares!

$2(4x+5y)(4x-5y)$

Ex 9) Factor: $3n^3 - 48$

GCF $3(n^3 - 16)$ diff of sq!

$3(n^2+4)(n^2-4)$

$3(n^2+4)(n+2)(n-2)$

sum of squares

diff of squares!

Reflection: