Geometric Sequences

definition

A **geometric sequence** is a sequence in which the ***ratio*** of consecutive terms is constant.

Warmup – Suppose you have the geometric sequence 4, 12, 36, 108, …

a) What is t_1?
b) What do you multiply by to get the next term (this is the ***r*** value)?
c) Is the sequence geometric (see the definition above)? In other words, is the ***r*** value consistent throughout the sequence?
d) What is t_5? Explain how you got t_5. Write a general formula for this.

e) Show how to get t_5 using only t_1 and r.

f) Show how to get t_8 using only t_1 and r.

g) What do you notice about the exponent on r compared to n?

h) Write a general formula for t_n for any geometric sequence:

Geometric Sequence formula

The general term of a geometric sequence where n is a positive integer is:

OR

where t_1 is the first term, n is the number of terms, r is the common ratio, and t_n is a general term

common ratio

For a geometric sequence, the **common ratio (*r*)**, can be found by taking any term (except the first) and dividing that term by the preceding term. So $r = \frac{t_n}{t_{n-1}}$

Example – Are the following sequences geometric (ie. Is the ***r*** value consistent)?

a) 2, 4, 6, 8

b) 4, 10, 25, 62.5

Example – Find t_{18} for the following: *3, -6, 12, -24, …*

Example – Find t_1 if $t_5 = 567$ and $t_6 = 1701$.

Example – Bacteria reproduce by splitting into two. Suppose there were three bacteria originally present in a sample. How many bacteria will there be after 8 generations?

Example – Suppose a photocopier can reduce a picture to 60% of its original size. If the picture is originally 42cm long, what length will it be after five successive reductions (to the nearest hundredth)?

Example – In 1990 the population of Canada was approximately 26.6 million. The population projection for 2025 is approximately 38.4 million. If this projection were based on a geometric sequence, what would be the annual growth rate?

Percentages

If a question involves percent **growth**, ***r*** must be greater than 1.

Ex. If there is 30% growth each year, what is the ***r*** value for the problem?

If a question involves a percent reduction, ***r*** must be less than 1 and must represent the percent remaining (not the percent lost).

Ex. If you reduce the size of your savings by 25% per year, what is ***r***?