Key

Geometric Sequences

definition

A **geometric sequence** is a sequence in which the ***ratio*** of consecutive terms is constant.

Warmup – Suppose you have the geometric sequence $4 \xrightarrow{\times 3} 12 \xrightarrow{\times 3} 36 \xrightarrow{\times 3} 108$, …

a) What is t_1? 4

b) What do you multiply by to get the next term (this is the ***r*** value)? 3

c) Is the sequence geometric (see the definition above)? In other words, is the ***r*** value consistent throughout the sequence? yes

d) What is t_5? Explain how you got t_5. Write a general formula for this.

$t_5 = 108 \times 3 = 324$ $\quad t_4(r) = t_5$ $\quad$ so $\quad t_n = t_{n-1} r$

e) Show how to get t_5 using only t_1 and r.

$t_5 = t_1 rrrr = t_1 r^4$

f) Show how to get t_8 using only t_1 and r.

$t_8 = t_1 r^7$

g) What do you notice about the exponent on r compared to n?

it is always one less than n

h) Write a general formula for t_n for any geometric sequence: $t_n = t_1 r^{n-1}$

Geometric Sequence formula

> The general term of a geometric sequence where n is a positive integer is:
>
> $t_n = t_1 r^{n-1}$ **OR** $t_n = t_{n-1} r$
>
> where t_1 is the first term, n is the number of terms, r is the common ratio, and t_n is a general term

common ratio

For a geometric sequence, the **common ratio (*r*)**, can be found by taking any term (except the first) and dividing that term by the preceding term. So $r = \frac{t_n}{t_{n-1}}$

Example – Are the following sequences geometric (ie. Is the ***r*** value consistent)?

a) 2, 4, 6, 8

$\frac{4}{2} = 2 \quad \frac{6}{4} = 1.5$

No

b) 4, 10, 25, 62.5

$\frac{10}{4} = 2.5 \quad \frac{25}{10} = 2.5 \quad \frac{62.5}{25} = 2.5$

✓ geometric

Example – Find t_{18} for the following: 3, -6, 12, -24, …

$t_{18} = t_1 r^{17}$ $\quad r = \frac{-6}{3} = -2$ $\quad t_1 = 3$

$t_{18} = 3(-2)^{17}$

$= -393\,216$

Example – Find t_1 if $t_5 = 567$ and $t_6 = 1701$.

$r = \frac{1701}{567} = 3$ $t_5 = t_1 r^4$ $t_1 = \frac{567}{81} = 7$

$567 = t_1 3^4$

$567 = t_1(81)$

Example – Bacteria reproduce by splitting into two. Suppose there were three bacteria originally present in a sample. How many bacteria will there be after 8 generations?

after 8 generations is $n = 9$

$t_1 = 3$

$r = 2$

$t_9 = t_1 r^8$

3, 6, 12, 24, 48

after 1 generation

$t_9 = 3(2)^8 = 3(256) = 768$

Example – Suppose a photocopier can reduce a picture to 60% of its original size. If the picture is originally 42cm long, what length will it be after five successive reductions?

$t_1 = 42$ $r = 0.6$ $t_6 = t_1 r^5$

$t_2 = 42(0.6) = 25.2$ $t_6 = 42(0.6)^5$

after 1 reduction $t_6 = 3.27\text{ cm}$

t_6 = after 5 reductions

Example – In 1990 the population of Canada was approximately 26.6 million. The population projection for 2025 is approximately 38.4 million. If this projection were based on a geometric sequence, what would be the annual growth rate?

$t_n = t_1 r^{n-1}$

$t_1 = 26\,600\,000$ $t_{36} = t_1 r^{35}$ $r = \sqrt[35]{1.4436}$

$t_{36} = 38\,400\,000$ $r = 1.01$

Find r $38\,400\,000 = 26\,600\,000\, r^{35}$

$r^{35} = 1.4436$ 1% annual growth rate.

Percentages

If a question involves percent **growth**, ***r*** must be greater than 1.

Ex. If there is 30% growth each year, what is the ***r*** value for the problem? $1 + 0.30 = 1.3$

If a question involves a percent reduction, ***r*** must be less than 1 and must represent the percent remaining (not the percent lost).

Ex. If you reduce the size of your savings by 25% per year, what is ***r***?

$r = 1 - 0.25 = 0.75$

(75% of savings remaining)