Pre-Calculus Math 11

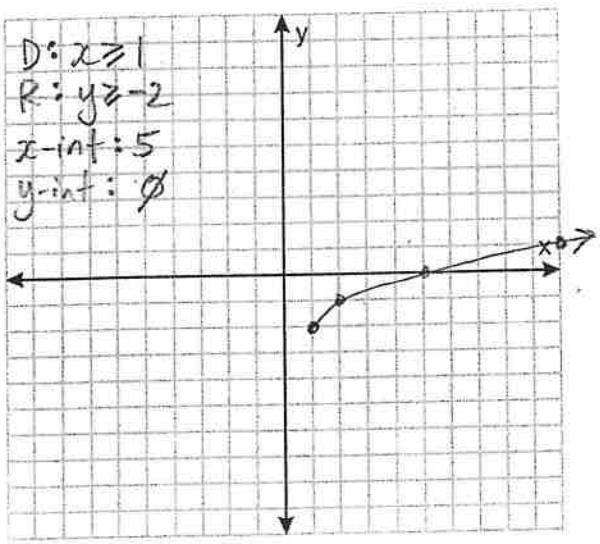
Name: Key

Functions Review Worksheet Date:

1) Graph the following radical functions and state the Domain, Range, x-int, y-int for each:

1. $f(x) = \sqrt{x-1} - 2$ vertex $(1,-2)$ over 1, up 1 4 2 9 3
2. $f(x) = \sqrt{1-x} - 2$ $y = \sqrt{-(x-1)} - 2$
3. $f(x) = \frac{1}{2}\sqrt{x+4} - 5$
4. $f(x) = -3\sqrt{-x} + 4$
5. $f(x) = -\sqrt[3]{x} + 3$
6. $g(x) = \sqrt[3]{3-x} - 2$
7. $g(x) = \sqrt{x^2 - 1}$
8. $g(x) = \sqrt{-(x+4)^2 + 4}$

1



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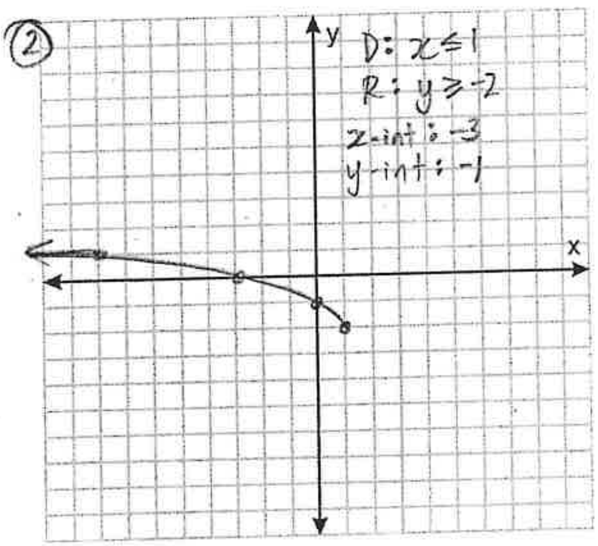

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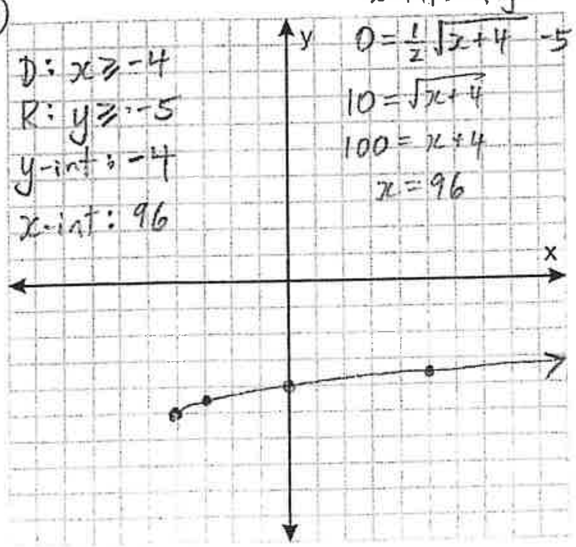
x-int: set y = 0



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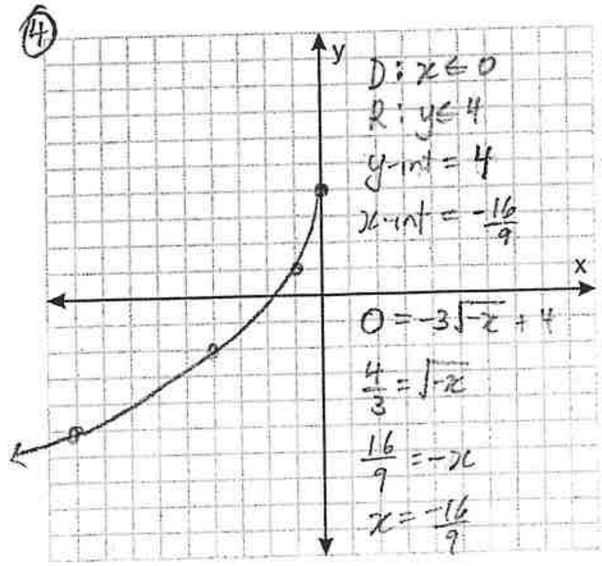
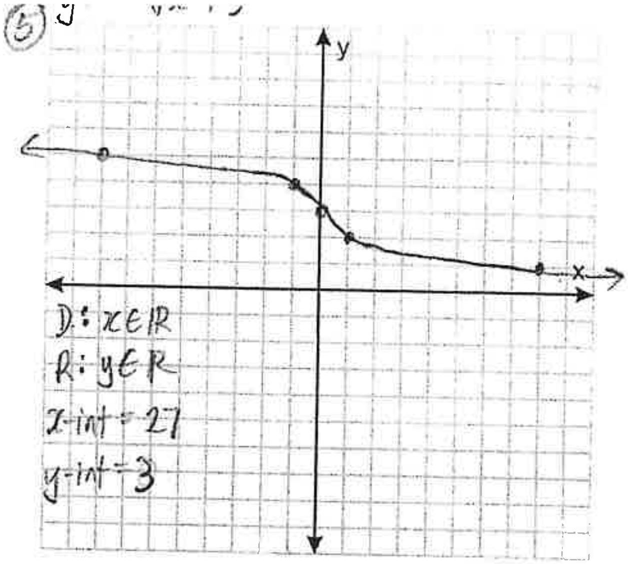
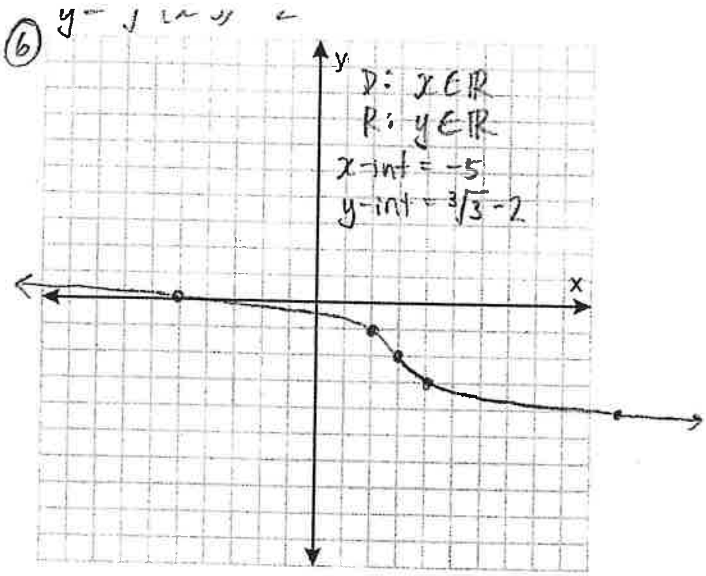


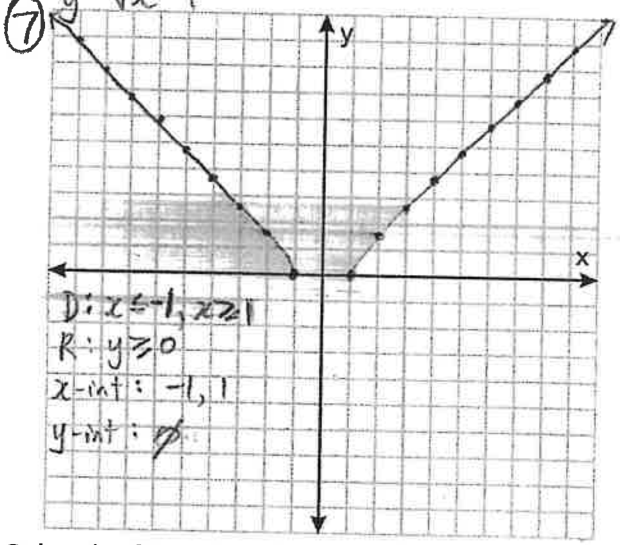
⑤ y



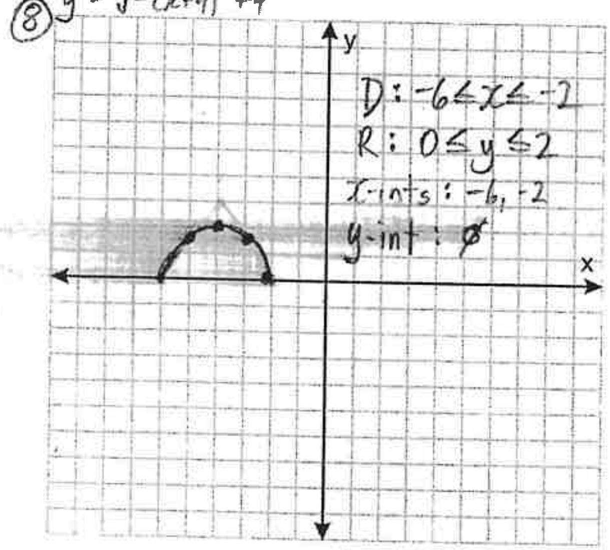
⑥ y



⑦ $y = \sqrt{x^2-1}$



⑧ $y = \sqrt{-(x+1)^2+4}$



2) Solve the following absolute value equations:

a) $2|2x+1| = 18$

$|2x+1| = 9$

$2x+1=9$	$2x+1=-9$
$2x=8$	$2x=-10$
$x=4$	$x=-5$

Checks:

LS	RS	LS	RS
$2\|2(4)+1\|$	18	$2\|2(-5)+1\|$	18
$2\|8+1\|$		$2\|-9\|$	
$2\|9\|$		$2(9)$	
$2(9)$ ✓		18 ✓	
18			

b) $|5-4x|+3=3$

$|5-4x|=0$

$5-4x=0$

$5=4x$

$x=\frac{5}{4}$

Check

LS	RS
$\|5-4(\frac{5}{4})\|+3$	3
$\|5-\frac{20}{4}\|+3$	
$\|5-5\|+3$	
$\|0\|+3$	
$0+3$ ✓	

c) $|2x-1| = 4-x$

$2x-1=4-x$	$2x-1=-(4-x)$
$3x=5$	$2x-1=-4+x$
$x=\frac{5}{3}$	$x=-3$

LS	RS	LS	RS
$\|2(\frac{5}{3})-1\|$	$4-\frac{5}{3}$	$\|2(-3)-1\|$	$4-(-3)$
$\|\frac{10}{3}-1\|$	$\frac{12}{3}-\frac{5}{3}$	$\|-6-1\|$	7
$\|\frac{7}{3}\|$	$\frac{7}{3}$ ✓	$\|-7\|$	
$\frac{7}{3}$		7 ✓	

d) $|x-3| = x-3$

true as long as $x-3 \ge 0$

or $x \ge 3$

LS	RS
$\|(3)^2-3+3\|$	9
$\|9-3+3\|$	
$\|9\|$	
9 ✓	

e) $-2|x+3|+3=5$

$-2|x+3| = 2$

$|x+3| = -1$

No solutions! ∅

f) $|x^2-x+3| = 9$

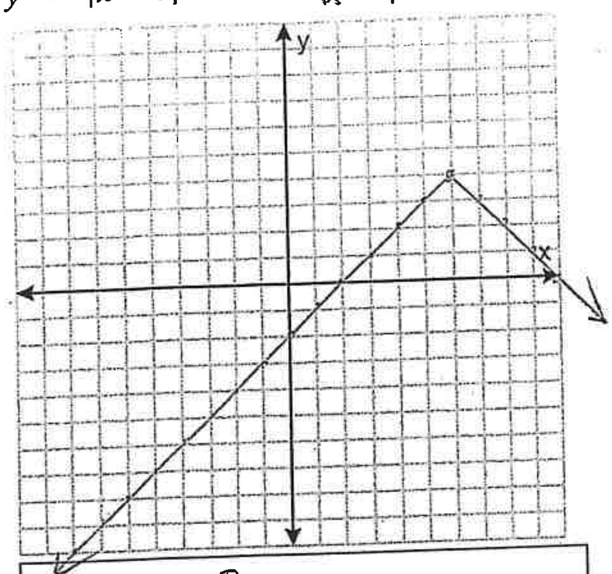
$x^2-x+3=9$	$x^2-x+3=-9$
$x^2-x-6=0$	$x^2-x+12=0$
$(x-3)(x+2)=0$	can't factor so try quad formula
$x=-2, 3$	$x=\frac{1\pm\sqrt{-47}}{2}$
	NO solution

Checks:

LS	RS
$\|(-2)^2-(-2)+3\|$	9
$\|4+2+3\|$	
$\|9\|$	
9 ✓	

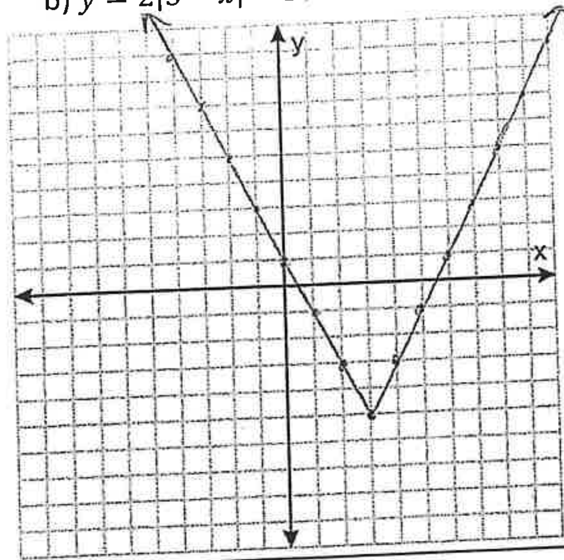
3) For each absolute value function, graph the function, state the domain and range, and write as a piecewise function.

$y = -|x-6| + 4$ vertex $(6,4)$ $a=-1$



Domain: $x \in \mathbb{R}$
Range: $y \leq 4$
Piecewise:
$$y = \begin{cases} x-2 & \text{when } x<6 \\ -x+10 & \text{when } x \geq 6 \end{cases}$$

b) $y = 2|3-x| - 5$ $y = 2|-(x-3)|-5$

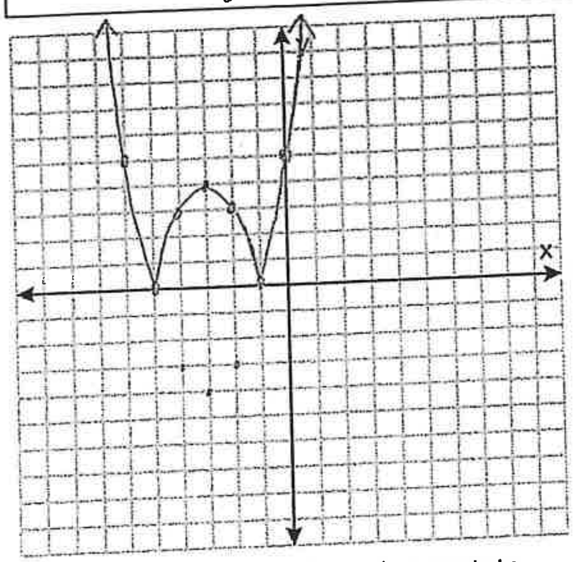


Domain: $x \in \mathbb{R}$
Range: $y \geq -5$
Piecewise:
$$y = \begin{cases} -2x+1 & \text{when } x<3 \\ 2x-11 & \text{when } x \geq 3 \end{cases}$$

Graph $f(x) = -(x+3)^2 + 4$ in pencil and then use it to graph $g(x) = |-(x+3)^2 + 4|$

$f(x) = -(x+3)^2 + 4$
vertex $(-3,4)$
$a=-1$

to get $g(x)$:
reflect all $f(x)$ values that are below the x axis to above the x axis.



Domain: $x \in \mathbb{R}$
Range: $y \geq 0$

4) Given the following rational expressions, determine the vertical asymptotes, horizontal asymptote, x-intercepts, and y-intercept.

Equation	*V.A*	*H.A.*	*x-intercept*	*y-intercept*
$f(x) = \frac{1}{x+2}$	$x=-2$	$y=0$	$\varnothing$	$\frac{1}{2}$
$f(x) = \frac{x-2}{x^2-1}$	$x=\pm 1$	$y=0$	2	2
$f(x) = \frac{x+3}{x^3-25x}$	$x=-5,0,5$	$y=0$	-3	$\varnothing$
$f(x) = \frac{-x}{4-x^2}$	$x=\pm 2$	$y=0$	0	0

$f(x)=\frac{x^3+x}{x+2}$	$x=-2$	$\varnothing$	0	0
$f(x)=\frac{2-3x^2}{2x^2}$	$x=0$	$y=\frac{-3}{2}$	$\pm\frac{\sqrt{6}}{3}$	$\varnothing$
$f(x)=\frac{x-1}{x^2-1}$	$x=-1$	$y=0$	$\varnothing$	1
$f(x)=\frac{1}{x+2}+3$	$x=-2$	$y=3$	$\frac{-7}{3}$	$3\frac{1}{2}$ or $\frac{7}{2}$

5) The following exercise is designed to get you to determine whether the rational function has **HOLES** or **VERTICAL ASYMPTOTES.** If the function has a vertical asymptote, state the equation as you did in the above exercise. If the function has a hole, then state the coordinates of the hole.

Equation	***V.A***	***Holes coordinate***
$f(x)=\frac{1}{x+5}$	$x=-5$	$\varnothing$
$f(x)=\frac{x^2-4}{x+2}$	$\varnothing$	$(-2,-4)$
$f(x)=\frac{x+3}{x^2-9}$	$x=3$	$(-3,-\frac{1}{6})$
$f(x)=\frac{x-3}{x^2-5x+6}$	$x=2$	$(3,1)$
$f(x)=\frac{x^2-1}{3x^3-3x}$	$x=0$	$(1,\frac{1}{3})$ & $(-1,\frac{1}{3})$

$y=\frac{x^2-1}{3x^3-3x}$

$y=\frac{(x+1)(x-1)}{3x(x^2-1)}$

$y=\frac{(x+1)(x-1)}{3x(x+1)(x-1)}$

$y=\frac{1}{3x}$

6) When graphing a rational function, if the x-values are approaching a vertical asymptote then the values of y will approach $\pm\infty$. When the values of x become very large or very small ($x\to\pm\infty$) then the values of y will approach the horizontal asymptotes. Remember that the values can approach from above or below the horizontal asymptote. Complete the statements below so show how the function approaches the asymptotes.

a. Given $f(x)=\frac{1}{x+2}$

i. As $x\to -2^+$ then $y\to$ ∞ ex $\frac{1}{-1.99+2}$

ii. As $x\to -2^-$ then $y\to$ $-\infty$ ex $\frac{1}{-2.01+2}$

b. Given $f(x)=\frac{1}{x+2}$

i. As $x\to +\infty$ then $y\to$ 0^+ ex $\frac{1}{102}$ if $x=100$

horiz asymp at $y=0$

ii. As $x\to -\infty$ then $y\to$ 0^- ex $\frac{1}{-98}$ if $x=-100$

c. Given $f(x)=\frac{x-3}{x^2-3x+2}$

i. As $x\to 1^-$ then $y\to$ $-\infty$

$y=\frac{x-3}{(x-2)(x-1)}$

ii. As $x\to 2^-$ then $y\to$ ∞

horiz asymp at $y=0$

iii. As $x\to -\infty$ then $y\to$ 0^-

d. Given $g(x)=\frac{6x^2-4}{2x^2+1}$

i. As $x\to +\infty$ then $y\to$ 3^-

ii. As $x\to -\infty$ then $y\to$ 3^-

horiz asymp at $y=3$

7) Graph & state asymptote equations, *D*, *R*, *x-int*, & *y-int* and any hole coordinates (if applicable).

a. $f(x) = -\frac{1}{x-3} + 2$

H.A. $y=2$
V.A. $x=3$

x	y
2	3
4	1
10	$1\frac{6}{7}$
-10	$2\frac{1}{13}$
3^-	∞
3^+	$-\infty$

x-int

$0 = \frac{-1}{x-3} + 2$

$-2 = \frac{-1}{x-3}$

$-2(x-3) = -1$

$-2x+6 = -1$

$-2x = -7$

$\boxed{x = \frac{7}{2}}$

y-int

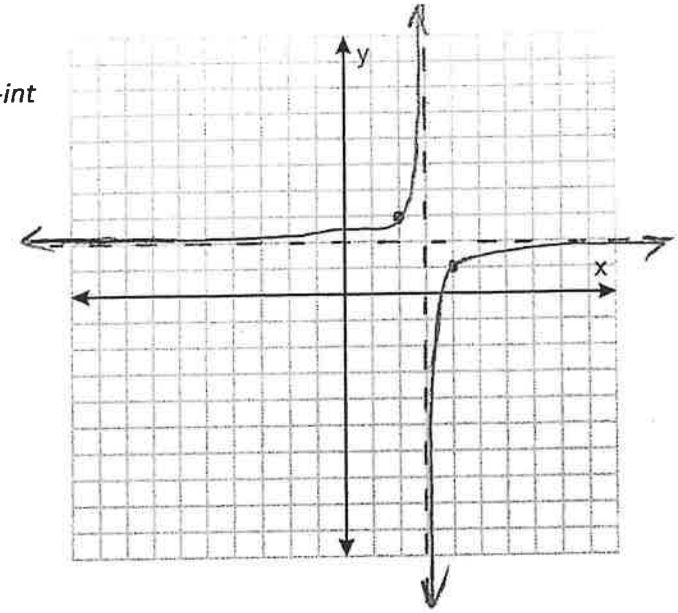
$y = \frac{-1}{0-3} + 2$

$y = \frac{1}{3} + 2$

$\boxed{y = 2\frac{1}{3}}$

D: $x \neq 3$

R: $y \neq 2$



b. $f(x) = \frac{x}{x^2-9} = \frac{x}{(x+3)(x-3)}$

H.A. $y=0$
V.A. $x = \pm 3$

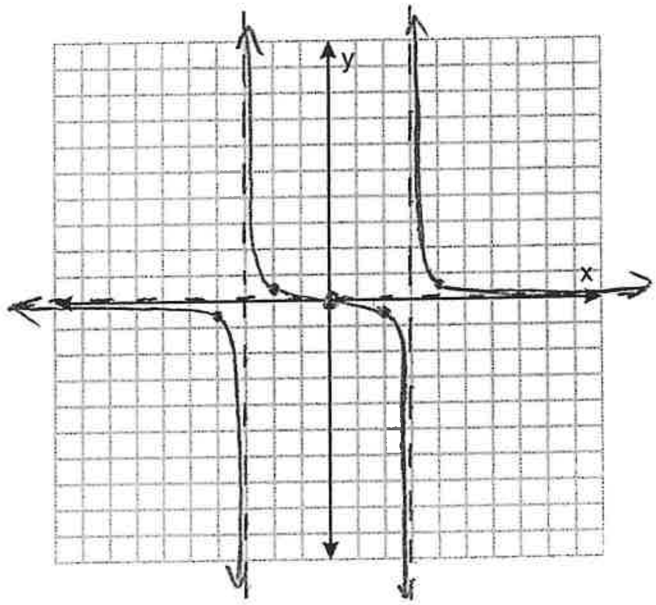
x	y
4	$\frac{4}{7}$
-4	$-\frac{4}{7}$
0	0
2	$-\frac{2}{5}$
-2	$\frac{2}{5}$

D: $x \neq \pm 3$

R: $y \in \mathbb{R}$

x-int : 0

y-int : 0



c. $f(x) = \frac{x-3}{x^2-x-6} = \frac{\cancel{x-3}}{(\cancel{x-3})(x+2)} = \frac{1}{x+2}$

H.A. $y=0$
V.A $x=-2$

Hole $(3, \frac{1}{5})$

$y = \frac{1}{3+2} = \frac{1}{5}$

x	y
-1	1
-3	-1
-10	$-\frac{1}{8}$
10	$\frac{1}{12}$
$(-2)^+$	∞
$(-2)^-$	$-\infty$

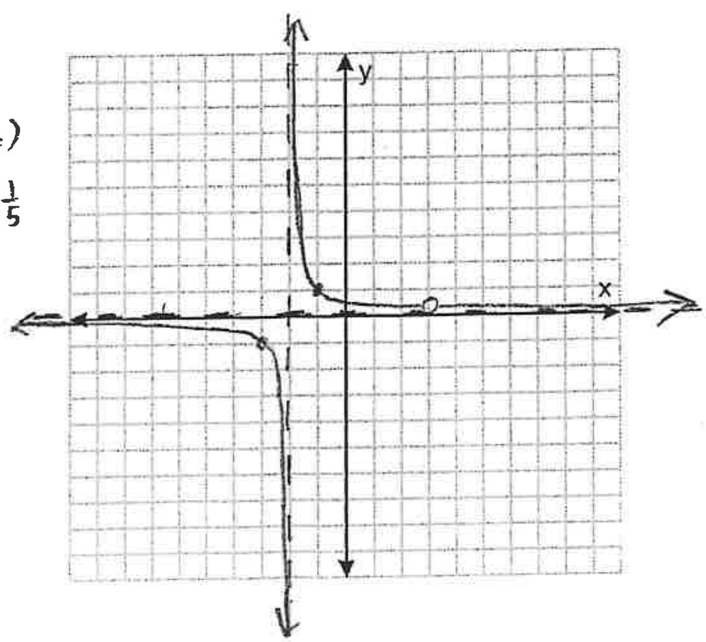
D: $x \neq -2, 3$

R: $y \neq 0, \frac{1}{5}$

x-int : $\emptyset$

y-int : $\frac{1}{2}$

$y = \frac{1}{0+2} = \frac{1}{2}$



d. $f(x) = \frac{x^2-4}{x^2-16} = \frac{(x+2)(x-2)}{(x+4)(x-4)}$ H.A. $y=1$ V.A. $x=\pm 4$

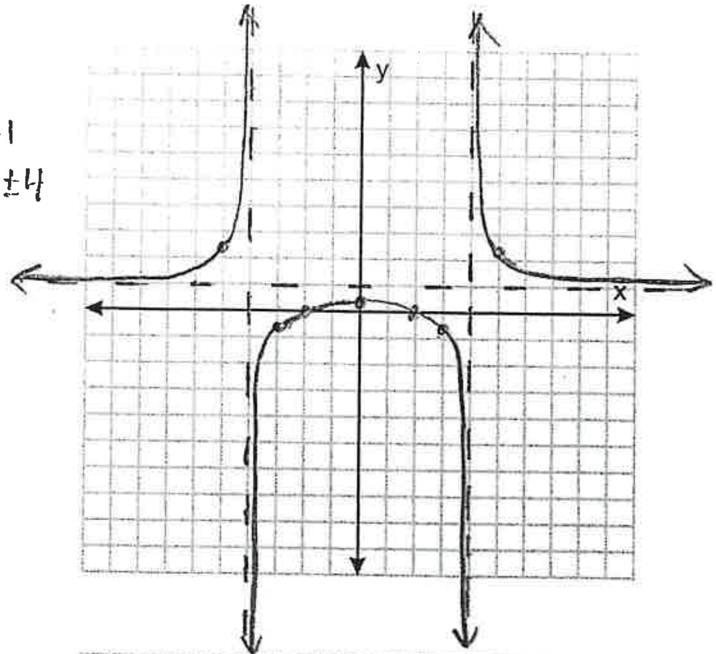
x	y
5	$\frac{21}{9} = 2.\overline{3}$
-5	$\frac{21}{9} = 2.\overline{3}$
0	$\frac{1}{4}$
-3	$\frac{-5}{7}$
3	$\frac{-5}{7}$
-2	0
2	0

D: $x \neq \pm 4$

R: $y > 1,\ y \leq \frac{1}{4}$

y-int: $\frac{1}{4}$

x-ints: ± 2



e. $f(x) = \frac{x^2-9}{x^2+4} = \frac{(x+3)(x-3)}{x^2+4}$ H.A $y=1$ V.A. $\emptyset$

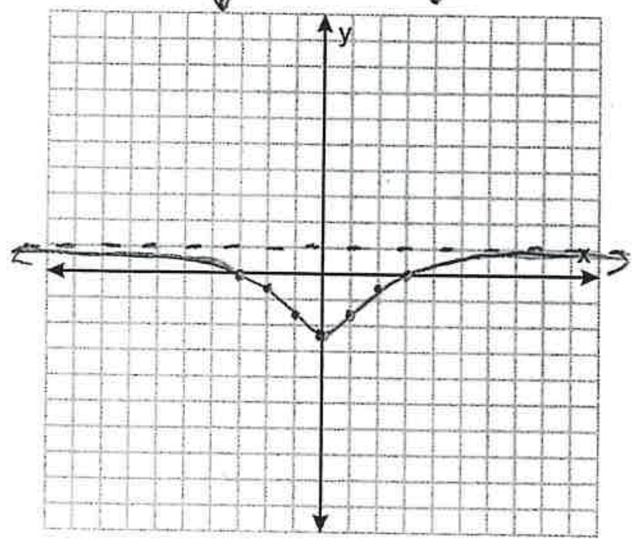
x	y
0	$\frac{-9}{4}$
1	$\frac{-8}{5}$
-1	$\frac{-8}{5}$
2	$\frac{-5}{8}$
-2	$\frac{-5}{8}$
3	0
-3	0

D: $x \in \mathbb{R}$

R: $-\frac{9}{4} \leq y < 1$

x-ints: ± 3

y-int: $-\frac{9}{4}$



f. $f(x) = -\frac{x^2-2x}{x-2} = -\frac{x(\cancel{x-2})}{\cancel{x-2}} = -x$

x	y

H.A. $\emptyset$

V.A $\emptyset$

$y = \frac{-1^{\leftarrow \text{down}}}{1_{\leftarrow \text{right}}} x + 0 \leftarrow$ y-int

Hole $(2, -2)$

$y = -(2)$

D: $x \neq 2$

R: $y \neq -2$

x-int: 0

y-int: 0