Name: NOTES KEY

Date: ____________________

FACTORING UNIT

Calendar of Chapter: See the 'Homework' link on the webpage

What You'll Learn:

F.1 – multiplying binomials & polynomials
F.2 – factoring out common factors
F.3 – factoring trinomials with $a = 1$
F.4 – special cases of factoring
F.5 – all methods of factoring together

What is the opposite of multiplication?

So if we simplify $2(x + 3)$, we are multiplying (distributive property). What is the answer?

What if we want to do the opposite to end up with the original expression again?

F.1 – Multiplying Binomials & Polynomials

Focus: To expand (multiply) binomial & polynomial products.

In this set of notes (F.1), we are not yet factoring.

Warmup:

Simplify: $2x(x-7)$

$2x(x-7)$

$2x^2 - 14x$

Binomial Products

Ex1 – Expand & Simplify

$(x+2)(x+5)$

$(x+2)(x+5)$

F O I L

Firsts Outsides Insides Lasts

$x^2 + 5x + 2x + 10$

$x^2 + 7x + 10$

Ex2 – Expand & Simplify

a) $(y-4)(y+3)$
b) $(p-1)(p-6)$
c) $(3x+2)^2$
d) $(2n-4)(-n+5)$

a) $(y-4)(y+3)$

$y^2 + 3y - 4y - 12$

$y^2 - y - 12$

b) $(p-1)(p-6)$

$p^2 - 6p - 1p + 6$

$p^2 - 7p + 6$

c) $(3x+2)^2$

$(3x+2)(3x+2)$

$9x^2 + 6x + 6x + 4$

$9x^2 + 12x + 4$

d) $(2n-4)(-n+5)$

$-2n^2 + 10n + 4n - 20$

$-2n^2 + 14n - 20$

Ex3 – Expand & Simplify

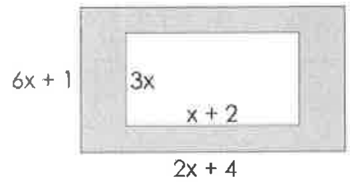
$(2a-1)(2a+3) + (a-1)(a-2)$

$4a^2 + 6a - 2a - 3 + [a^2 - 2a - 1a + 2]$

$4a^2 + 4a - 3 + a^2 - 3a + 2$

$5a^2 + a - 1$

Ex4 – Find the area of the shaded region



$A_{shaded} = A_{\substack{big \\ rec}} - A_{\substack{small \\ rec}}$

$$A_{shaded} = (2x+4)(6x+1) - 3x(x+2)$$
$$= 12x^2 + 2x + 24x + 4 - 3x^2 - 6x$$
$$= 9x^2 + 20x + 4$$

Ex5 – Expand & Simplify
a) $(x + 3)(2x^2 - 4x + 3)$
b) $(y - 2)(y^2 + 5y - 8)$

a) $(x + 3)(2x^2 - 4x + 3)$

$$2x^3 - 4x^2 + 3x + 6x^2 - 12x + 9$$
$$2x^3 + 2x^2 - 9x + 9$$

b) $(y - 2)(y^2 + 5y - 8)$

$$y^3 + 5y^2 - 8y - 2y^2 - 10y + 16$$
$$y^3 + 3y^2 - 18y + 16$$

Ex6 – Simplify
$3(n + 5)(n - 6)$

$$3(n + 5)(n - 6)$$
$$3[n^2 - 6n + 5n - 30]$$
$$3[n^2 - n - 30]$$
$$3n^2 - 3n - 90$$

F.2 – Factoring a Greatest Common Factor

Focus: To determine the factors of a polynomial by identifying the GCF.

Warmup

Simplify: 3(g + 2)

$3(g + 2)$

$3g + 6$

What if we want to do the opposite to obtain the starting expression?

Because we multiplied the 3 into the brackets, we must do the opposite to get it out. This is called **factoring**, but it is essentially dividing.

$\frac{3g}{3} + \frac{6}{3}$

$3(g + 2)$

Ex1 – Factor out a GCF

a) $2x - 10$
b) $4w + 14$
c) $5w - 10y$
d) $6x^2 - 9x$

a) $\frac{2x}{2} - \frac{10}{2}$

$2(x - 5)$

b) $\frac{4w}{2} + \frac{14}{2}$

$2(2w + 7)$

c) $\frac{5w}{5} - \frac{10y}{5}$

$5(w - 2y)$

d) $\frac{6x^2}{3x} - \frac{9x}{3x}$

$3x(2x - 3)$

Ex2 – Factor out a GCF

a) $-8y + 16y^2$
b) $3x^2 - 12x - 6$
c) $4 - 6p + 8p^2$
d) $3w^2 - 7w$

a) $\frac{-8y}{-8y} + \frac{16y^2}{-8y}$

$-8y(1 - 2y)$

b) $\frac{3x^2}{3} - \frac{12x}{3} - \frac{6}{3}$

$3(x^2 - 4x - 2)$

c) $\frac{4}{2} - \frac{6p}{2} + \frac{8p^2}{2}$

$2(2 - 3p + 4p^2)$

d) $\frac{3w^2}{w} - \frac{7w}{w}$

$w(3w - 7)$

Ex3 – Factor out a GCF

a) $\dfrac{-20c^4d}{-5cd} - \dfrac{30c^3d^2}{-5cd} - \dfrac{25cd}{-5cd}$

$-5cd(4c^3 + 6c^2d + 5)$

b) $\dfrac{18x^2y^2z^3}{18x^2yz^2} - \dfrac{36x^3yz^2}{18x^2yz^2} + \dfrac{54x^2y^4z^3}{18x^2yz^2}$

$18x^2yz^2(yz - 2x + 3y^3z)$

c) $\dfrac{40m^4n^3}{8m^4} - \dfrac{24m^7n^4}{8m^4} - \dfrac{8m^5}{8m^4}$

$8m^4(5n^3 - 3m^3n^4 - m)$

d) $\dfrac{16fg^3}{fg^2} - \dfrac{24f^2g^2}{fg^2} + \dfrac{fg^2}{fg^2}$

$fg^2(16g - 24f + 1)$

F.3 – Factoring Trinomials with $a = 1$

Focus: To factor trinomials with $a = 1$ into a binomial product

Warmup:

Simplify:

$(x + 2)(x + 3)$

$(x + 2)(x + 3)$

$x^2 + 3x + 2x + 6$

$x^2 + 5x + 6$

'FOILing' is simply multiplying. So what is the opposite process?

factoring

How did the last term (the constant) come about?

by multiply 2×3

How did the middle term come about?

by adding $2x + 3x$

For most trinomials, it is good practice to arrange the terms in descending order (by degree). Then, the coefficient of the first term is ***a***, the coefficient of the second term is ***b***, and the third term is ***c*** (quite often it's a constant).

Example: $\boldsymbol{ax^2 \pm bx \pm c}$

Using what we learned in the **Warmup**, we can devise a procedure to factor a trinomial back into its binomial products when the ***a*** value is 1 (that's how it will always be this year):

Find two numbers that multiply to 'c' and add to 'b'.

Ex1 – Factor

a) $x^2 + 5x + 6$

b) $y^2 - 2y - 15$

a) $x^2 + 5x + 6$

$a=1$, $b=5$, $c=6$

Find two numbers that multiply to 'c', which is 6, and add to 'b', which is 5

Factors of 6: 1, 6   2, 3

add to 5

$(x+2)(x+3)$

b) $y^2 - 2y - 15$

$a=1$ $b=-2$, $c=-15$

-15: $-1, 15$ $1, -15$ $-3, 5$ $3, -5$

add to -2

$(y+3)(y-5)$

Ex2 – Factor

a) $4p - 21 + p^2$

b) $18 + w^2 - 9w$

How can you check your answer? Do a check for (a):

FOIL

$(x+2)(x+3)$

$x^2+3x+2x+6$

x^2+5x+6 ✓

a) $4p - 21 + p^2$

$p^2 + 4p - 21$

$a=1,\ b=4,\ c=-21$

$-21:\ -1,21 \quad 1,-21 \quad 7,-3 \quad -7,3$

add to 4

$(p+7)(p-3)$

b) $18 + w^2 - 9w$

$w^2 - 9w + 18$

$b=-9 \quad c=18$

$18:\ 1,18 \quad -1,-18 \quad 9,2 \quad -9,-2 \quad 6,3 \quad -6,-3$

$(w-6)(w-3)$

Ex3 – Factor

a) $x^2 - 3xy - 10y^2$

b) $m^2 - mn - 2n^2$

a) $x^2 - 3xy - 10y^2$

$b=-3 \quad c=-10$

$-5,2$

$(x-5y)(x+2y)$

b) $m^2 - 1mn - 2n^2$

$b=-1 \quad c=-2$

$-2,1$

$(m-2n)(m+1n)$

$(m-2n)(m+n)$

Sometimes, you can factor a GCF out of a trinomial, and then further factor it if a = 1.

Ex4 – Factor

a) $-12 - 9g + 3g^2$

b) $-5x^2 - 20x + 60$

a) $-12 - 9g + 3g^2$

$\frac{3g^2}{3} - \frac{9g}{3} - \frac{12}{3}$

$3(g^2 - 3g - 4)$

$b=-3 \quad c=-4$

$-4,1$

$3(g-4)(g+1)$

b) $\frac{-5x^2}{-5} - \frac{20x}{-5} + \frac{60}{-5}$

$-5(x^2+4x-12)$

$b=4 \quad c=-12$

$6,-2$

$-5(x+6)(x-2)$

Ex5 - Factor

$y^2 - 6y - 8$

$y^2 - 6y - 8$

$b=-6 \quad c=-8$

$-8:\ -1,8 \quad 1,-8 \quad -2,4 \quad 2,-4$

none add to -6

cannot factor

F.4 – Special Cases of Factoring

Focus: To investigate some special factoring patterns.

Warmup:

Factor: $x^2 + 8x + 16$

$x^2 + 8x + 16$

$b=8 \quad c=16$

$4, 4$

$(x+4)(x+4)$

What is another way you can write the binomial product?

$(x+4)^2$

Trinomials that can be factored into two identical binomials are called '**Perfect Square Trinomials**'.

Ex1 – Factor

a) $-4y + y^2 + 4$

b) $49 + w^2 - 14w$

a) $-4y + y^2 + 4$

$y^2 - 4y + 4$

$b=-4 \quad c=4$

$-2, -2$

$(y-2)(y-2)$

$(y-2)^2$

b) $49 + w^2 - 14w$

$w^2 - 14w + 49$

$b=-14 \quad c=49$

$-7, -7$

$(w-7)(w-7)$

$(w-7)^2$

Ex2 – Simplify

$(x-2)(x+2)$

$(x-2)(x+2)$

$x^2 \underbrace{+2x}\underbrace{-2x} - 4$

$x^2 - 4$

What happened to the middle terms? they add to 0, so no more middle term

Why did this happen?

They are opposites of each other

How can you recognize when this will happen by looking at the binomial product?

Same binomials except one has a '+' and the other '–'

Ex3 – Factor

$x^2 - 4$

$x^2 - 4$

$(x+2)(x-2)$

What is this type of factoring called?

DIFFERENCE OF SQUARES

How can you recognize when to use this method of factoring?

2 terms, subtract in the middle, each term is a perfect square

Ex4 – Factor

a) $p^2 - 25$
b) $m^2 - 81$
c) $n^2 - 64$

a) $p^2 - 25$ b) $m^2 - 81$ c) $n^2 - 64$

$(p+5)(p-5)$ $(m+9)(m-9)$ $(n+8)(n-8)$

Why is this method called '***difference of squares***'?

-difference means subtraction
-Both terms are perfect squares

Ex5 – Factor

a) $16x^2 - 36y^2$
b) $9b^2 - 100c^2$

a) $16x^2 - 36y^2$ b) $9b^2 - 100c^2$

$(4x+6y)(4x-6y)$ $(3b+10c)(3b-10c)$

Ex6 – Factor

$m^2 + 25$

$m^2 + 25$ addition: called a 'sum of squares'
cannot factor

Ex7 - Factor

a) $2x^2 - 32$
b) $12 - 27w^2$
c) $8p^2 - 18q^2$
d) $3y^2 + 12$

Sometimes, you can factor a GCF out first!

a) $\frac{2x^2}{2} - \frac{32}{2}$

$2(x^2-16)$

$2(x-4)(x+4)$

b) $\frac{12}{3} - \frac{27w^2}{3}$

$3(4-9w^2)$

$3(2+3w)(2-3w)$

c) $\frac{8p^2}{2} - \frac{18q^2}{2}$

$2(4p^2-9q^2)$

$2(2p+3q)(2p-3q)$

d) $\frac{3y^2}{3} + \frac{12}{3}$

$3(y^2+4)$

cannot factor further
(<u>sum</u> of squares)

F.5 – Factoring Synthesis

Focus: To be able to factor expressions using the appropriate method.

FACTORING FLOW CHART

STEP 1 Take out COMMON FACTORS (**GCF**)

STEP 2 Ask: How many terms are there?

TWO

Test for **difference of squares:**

*You need **subtraction** ("difference") and each term must be a **perfect square**

If you don't have
perfect squares, check
to see if you can factor out a GCF.

$a^2 - b^2 = (a + b)(a - b)$

Example:
$4x^2 - 9$
$(2x + 3)(2x - 3)$

Example:
$2m^2 - 32n^2$
$2(m^2 - 16n^2)$
$2(m + 4n)(m - 4n)$

Example:
$4w^2 + 9y^2$
Cannot factor
As it is a SUM
of squares*

THREE

Factoring **trinomials:** $ax^2 + bx + c$

Is the trinomial in order?

Can you factor out a GCF?

<u>a = 1</u>
Example: $x^2 - 3x + 2$
Ask: what ADDS to "b" (here –3)
& MULTIPLIES to "c" (here +2)
Answers: –1, –2
Write factors: $(x - 1)(x - 2)$

Example: $y^2 - 5y - 3$
Ask: what ADDS to -5 &
MULTIPLIES to -3?
There are no numbers that do this.
Cannot factor the expression.

Ex1 - Factor
a) $2x^2 - 98$
b) $8p^2 - 4p^5 + 8p^3$

a) $\frac{2x^2}{2} - \frac{98}{2}$

$2(x^2 - 49)$

$2(x+7)(x-7)$

b) $\frac{8p^2}{4p^2} - \frac{4p^5}{4p^2} + \frac{8p^3}{4p^2}$

$4p^2(2 - p^3 + 2p)$

Ex2 – Factor

a) $24 + 11m + m^2$
b) $5y^2 - 20y - 60$
c) $64a^2 - b^2$
d) $3m + 15$

a) $24 + 11m + m^2$

$m^2 + 11m + 24$

$(m+8)(m+3)$

b) $\frac{5y^2}{5} - \frac{20y}{5} - \frac{60}{5}$

$5(y^2 - 4y - 12)$

$5(y-6)(y+2)$

c) $64a^2 - b^2$

$(8a+b)(8a-b)$

d) $\frac{3m}{3} + \frac{15}{3}$

$3(m+5)$

Ex3 - Factor

a) $10k + k^2 + 25$
b) $2x^2 - 14x - 12$
c) $8x^2 + 32y^2$
d) $100 - 25p + p^2$

a) $10k + k^2 + 25$

$k^2 + 10k + 25$

$(k+5)(k+5)$

$(k+5)^2$

b) $\frac{2x^2}{2} - \frac{14x}{2} - \frac{12}{2}$

$2(x^2 - 7x - 6)$

cannot factor further

c) $8x^2 + 32y^2$

$8(x^2 + 4y^2)$

cannot factor further

(sum of squares)

d) $100 - 25p + p^2$

$p^2 - 25p + 100$

$(p-20)(p-5)$