Name: NOTES KEY

Date:

CHAPTER 9 NOTES – Scale Diagrams & Similarity

Calendar of Chapter: See the 'Homework' link on the webpage

What You'll Learn

9.1 – draw and interpret enlargement scale diagrams
9.1 – draw and interpret reduction scale diagrams
9.2/9.3 – learn and apply properties of similar shapes
Imperial Unit Conversions – learn how to convert units involving the Imperial System

What are some careers that require either the construction or analysis of scale diagrams?

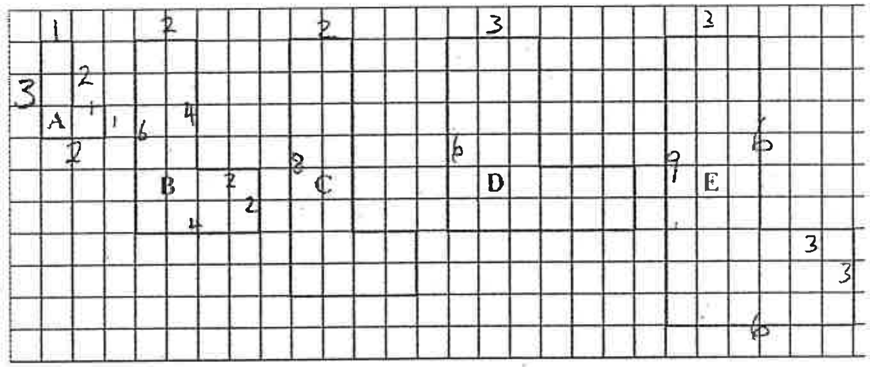
Here is some useful information for Similarity (Ch 9):

- *If the sides of a shape have the same dash mark(s), they're equal in length*
- *An isosceles triangle has two equal sides and two equal angles*
- *Sides are described with two capital letters   ex. AB*
- *Angles are described with three capital letters   ex. <ABC*
- *If you know two sides of a right triangle, you can find the 3rd side using Pythagoras*
- *The three angles in a triangle always add to 180 degrees*

9.1A– Scaling: Enlargements

Focus: Draw and interpret scale diagrams that represent enlargements.

Warmup:
Shape **A** is the original.
Shapes **B, C, D, & E** are all possible enlargements.
a) Which one(s) are enlargements of A?
b) How can you be certain?
c) Show, using numbers, that your choices are correct.



a) B, E (b) B: every length has been doubled
E: every length has been tripled
so shapes are same shape, different size

c) B: $\frac{2}{1}, \frac{6}{3}, \frac{4}{2}, \frac{2}{1}, \frac{2}{1}, \frac{4}{2}$
all equal 2

E: $\frac{3}{1}, \frac{9}{3}, \frac{6}{2}, \frac{3}{1}, \frac{3}{1}, \frac{6}{2} \Rightarrow$ all equal 3

What does **proportional** mean?

PROPORTIONAL means 'same shape, different size'

What is a **scale diagram**?

A proportional enlargement or reduction of an original

What are **corresponding lengths**?

Lengths on each of the original and scale diagram that 'match up' ex left side of original with left side of scale diagram etc.

What is a **scale factor**?

$$\text{SCALE FACTOR} = \frac{\text{length on scale diagram}}{\text{length on original}} = \frac{SC}{OR}$$

What are the two ways to write a scale factor?

As a fraction or a decimal ex $\frac{3}{2}$; 1.5

What is an **enlargement**, and how can you tell an enlargement by the scale factor?

An enlargement is when the scale diagram is larger than the original. The scale factor will be an improper fraction, or a decimal larger than 1

When are scale diagram enlargements used in society?

- Plans for microchips or jewelry
- billboards
- microscopes

Ex1 – Look at the **'Act 1'** picture of ***Gulliver's Desk***.

a) What questions come to mind?

b) What information do we need?

c) Find the **scale factor** compared to human (original) size.

d) Find the height of the chair if it is human (original) size.

a) What is the scale factor of the desk + chair?

b) - measurements of Gulliver's desk + chair
- measurements of a standard desk + chair

c) height of Gulliver's desk (scale diagram) = approx 180cm × 5.5 = 990 cm

height of human size desk (original) = 76cm

Scale Factor = $\frac{990\,cm}{76\,cm} = 13$

d) height of Gulliver's Chair (approx): 7 × 180 = 1260cm

Sc Factor = $\frac{SC}{OR}$ $\quad 13 = \frac{1260}{x}$ so $x = 97\,cm$

Ex2 - A scale factor for a filing cabinet is $\frac{8}{3}$.

a) Is it an enlargement or a reduction?

b) If the **scale diagram** has a length that is 60cm, what is the actual length of the original?

a) enlargement as fraction is improper

ex if original length is 3cm, scale length is 8cm

b) Sc Factor = $\frac{SC}{OR}$

$\frac{8}{3} = \frac{60}{x}$ ← scale diagram length

CROSS-MULTIPLY:

$\frac{8}{3} \times \frac{60}{x}$

"Multiply the pair, divide the spare."

$x = \frac{3 \times 60}{8} = 22.5\,cm$

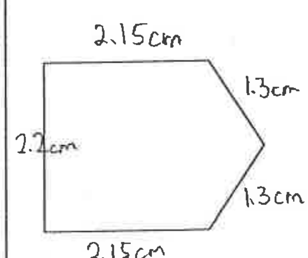
Ex3 - A scale factor for a computer chip is is 9:1 (same as $\frac{9}{1}$).

If the **length of the original** chip is 6mm, what is the length on the scale model?

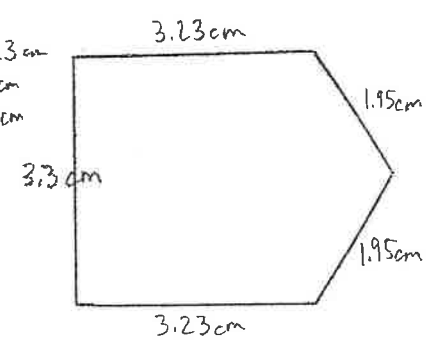
$\frac{9}{1} = \frac{x}{6\,mm}$ ← original

$\frac{9}{1} \times \frac{x}{6}$ $\quad x = \frac{9 \times 6}{1} = 54\,mm$

Ex4 - Draw a scale diagram of the drawing. Use a **scale factor of 1.5**.



2.15 × 1.5 = 3.23 cm
2.2 × 1.5 = 3.3cm
1.3 × 1.5 = 1.95cm



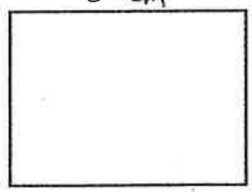
9.1B – Scaling: Reductions (DISCOVERY LESSON)

Focus: Draw and interpret scale diagrams that represent reductions.

Warmup:
Find the scale factor of the enlargement:

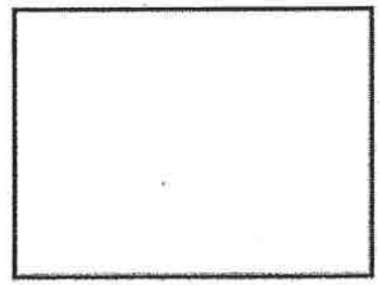
*Use the same units
For all measurements!

3.1cm



Original

4.7cm



Scale Drawing

$\frac{SC}{OR} = \frac{4.7cm}{3.1cm} = 1.5$

How are all scale factors written (review of last class)?

SCALE FACTOR = $\frac{\text{Scale Diagram}}{\text{Original}}$

What is a **reduction**, & how can you tell a reduction by the scale factor?

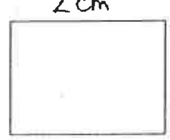
A scale diagram that is smaller than the original.
The scale factor $\left(\frac{SC}{OR}\right)$ will be a proper fraction,
or a decimal less than 1.

When are scale diagram reductions used in society?

maps, building blueprints etc.

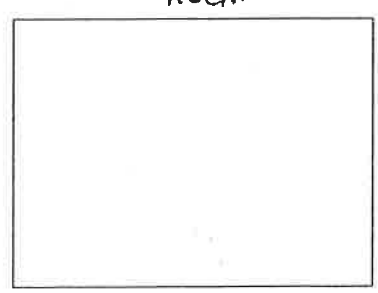
Ex1
Use a ruler
to determine the scale factor in decimal form.

2cm



Scale Drawing

4.8cm



Original

$\frac{SC}{OR} = \frac{2}{4.8} = 0.42$

$$S.F. = \frac{SC}{OR}$$

Ex2
A scale factor for a reduction of a chair is 1: 8 (same as $\frac{1}{8}$).
If the height of the actual chair is 75cm, what is the height on the scale drawing?

$$\frac{1}{8} = \frac{x}{75} \leftarrow \text{original.}$$

$$\frac{1}{8} = \frac{x}{75} \qquad x = \frac{1 \times 75}{8} = 9.375\text{cm}$$

Ex3 – As a class, watch **'Act 1'** of ***Splittime***.

a) What questions come to mind?

a) If he went the same speed, what would his split time be on the indoor track?

b) What do we need to know?

b) The distance of each track!

c) Do the math!

$$S.F. = \frac{SC}{OR} \qquad \frac{160}{400} = \frac{x}{75s} \leftarrow \qquad 1:15 = 75 \text{ seconds}$$

d) Watch **'Act 3'** to get the answer.

$$x = \frac{160 \times 75}{400} = 30 \text{ seconds!}$$

e) How many laps of the indoor track equal one lap of the outdoor track?

$$75 = 30 + 30 + 15$$

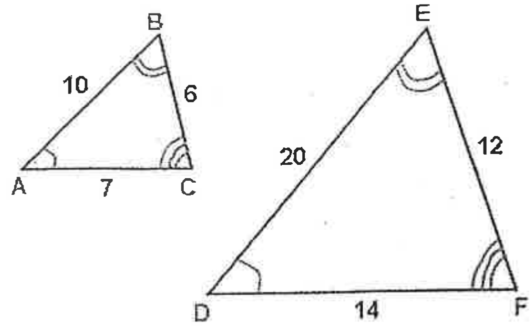
$$1 + 1 + 0.5$$

2.5 laps!

9.2/9.3 – Similar Triangles (and Other Polygons)

Focus: Recognize, draw, & solve problems for similar triangles (and other polygons).

Warmup:
What is alike about the two triangles, and what is different?



SAME SHAPE, DIFFERENT SIZE

SAME SHAPE MEANS CORRESPONDING ANGLES ARE EQUAL!
DIFFERENT SIZE MEANS CORRESPONDING SIDES NOT EQUAL!

What does **similar** mean in geometry?

Similar = Proportional = same shape, different size!

What are **similar triangles?**

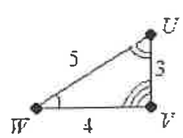
Triangles that have the same shape but different size

Using the **similar triangles** from the **warmup**, describe the relationships between the angles, and the relationships between the sides.

- Corresponding Angles are equal
- Fractions made from each set of corresponding sides are equal

Ex $\frac{10}{20} = \frac{6}{12} = \frac{7}{14}$

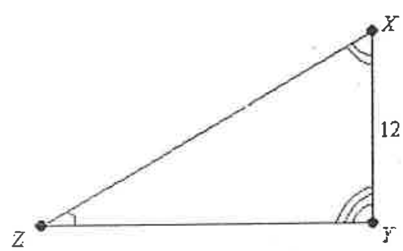
Ex1
a) The triangles are similar. Write a statement to represent this.
b) Solve for sides XZ & side ZY.



$\triangle UVW \sim \triangle XYZ$

↑ is similar to

(corresponding angles must be written in the same order)



$\frac{3}{12} = \frac{5}{XZ}$ $\qquad$ $\frac{3}{12} = \frac{4}{ZY}$

$XZ = \frac{12 \times 5}{3} = 20$ $\qquad$ $ZY = \frac{12 \times 4}{3} = 16$

Ex2 – How can we find the ***height of the lamppost*** without measuring it directly? Answer to the nearest tenth.

Measure the shadows, and the height of the person. Then set up a cross-multiply to find the height of the lamp post.

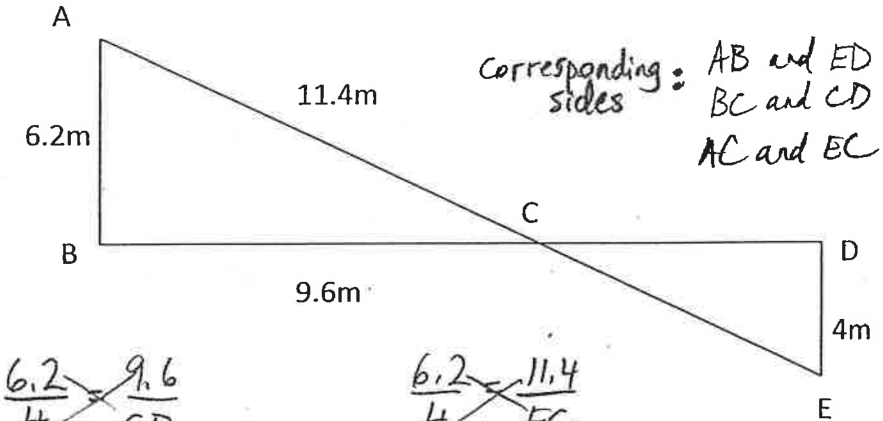
Shadow Fraction : $\frac{168}{506}$ Lamp/Person Fraction : $\frac{180}{x}$

$\frac{168}{506} \times \frac{180}{x}$

$x = \frac{506 \times 180}{168} = 542.1\text{cm}$

Neat way to measure the lamp post without having to climb it!

Ex3
The two triangles are similar. Find the missing lengths.



Corresponding sides: AB and ED, BC and CD, AC and EC

$\frac{6.2}{4} \times \frac{9.6}{CD}$ $\frac{6.2}{4} \times \frac{11.4}{EC}$

$CD = \frac{4 \times 9.6}{6.2} = 6.19m$ $EC = \frac{4 \times 11.4}{6.2} = 7.35m$

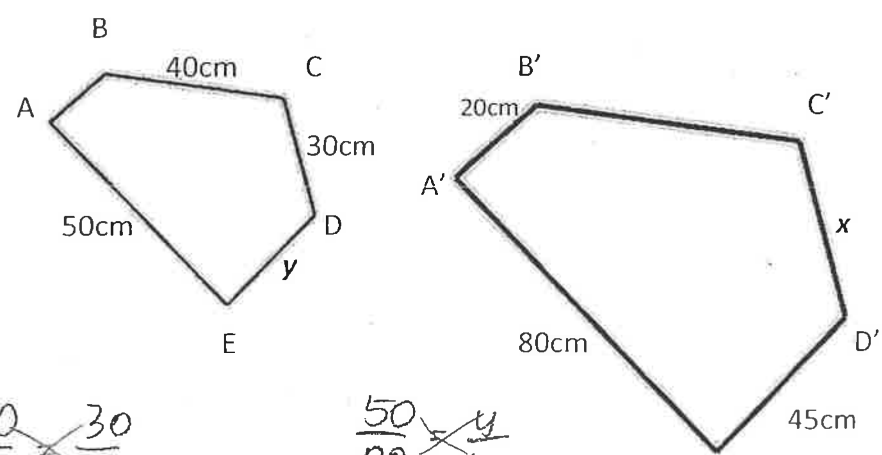
Similar polygons have the same characteristics as similar triangles.

What is a **polygon**?

A polygon is a closed shape composed of straight lines.

Ex4 – The polygons are similar. Find sides $\boldsymbol{x}$ and $\boldsymbol{y}$.

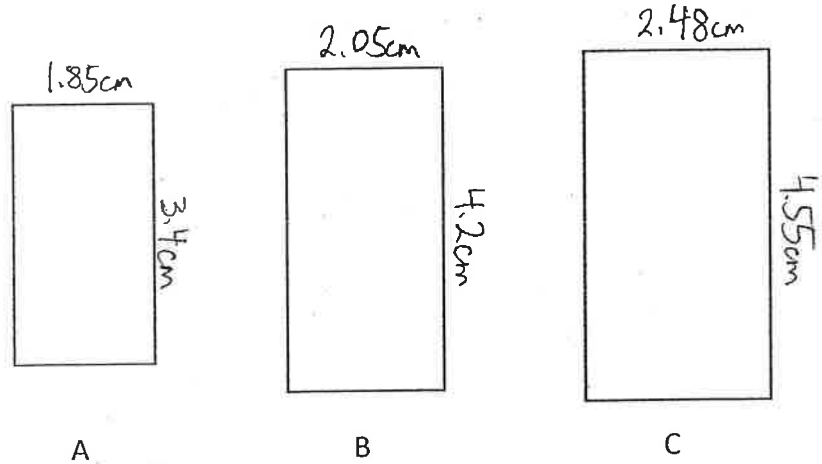
Keep the scale factor as a fraction and use cross-multiply.



$\frac{50}{80} \times \frac{30}{x}$ $\frac{50}{80} \times \frac{y}{45}$

$x = \frac{80 \times 30}{50} = 48\text{cm}$ $y = \frac{50 \times 45}{80} = 28.125\text{cm}$

Ex5
Which, if any, rectangles are similar? Use a ruler and make fractions to confirm.



Test for Similarity: A & B

$\frac{1.85}{2.05} = 0.90 \quad \frac{3.4}{4.2} = 0.81$ ✗ A and B **NOT** similar

Test for similarity: A & C

$\frac{1.85}{2.48} = 0.75 \quad \frac{3.4}{4.55} = 0.75$ ✓ A and C **ARE** similar

Therefore B and C cannot be similar

9.UC – Imperial Unit Conversions

Focus: Learn how to convert from one unit to another in imperial & metric

Warmup:

Early in the course we learned how to do metric conversions. Let's review this:

kilo	hecto	deca	BASE	deci	centi	milli
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
k	h	da	—	d	c	m

Convert: 7.83 kg to 783 dag   92.1 mm to 0.0921 m base

2 jumps RIGHT   3 jumps LEFT

3.61×10^4 dL to 36.1 hL 2.4×10^{-3} s to 0.24 cs

36100. 3 jumps LEFT   .0024 base 2 jumps RIGHT

What makes the metric system so convenient?

EASY CONVERSIONS.

Before the metric system existed, weights and measures were dominated by the IMPERIAL SYSTEM. For the imperial system, the conversions were awkward. For example: 1 mile = 1760 yards

Why do you suppose it was like this?

IT EVOLVED BIT BY BIT, NOT WITH A SYSTEM IN MIND

Here are the common conversions for the Imperial System, and common conversions between Imperial and Metric:

	Common Imperial	Imperial and Metric
Length	1 mile = 1760 yards 1 mile = 5280 feet 1 yard = 3 feet 1 yard = 36 inches 1 foot = 12 inches	1 mile ≈ 1.609 km 1 yard ≈ 0.9144 m 1 foot ≈ 0.3048 m 1 inch ≈ 2.54 cm
Mass (Weight)	1 ton = 2000 pounds 1 pound = 16 ounces	1 pound ≈ 0.454 kg 1 ounce ≈ 28.35 g
Common Abbrevia-tions	mile = mi yard = yd ton = ton feet = ' or ft inch = " or in pound = lb ounce = oz	

We will convert using a method called UNIT ANALYSIS:

Example 1: Convert 7 feet to inches

$$7 \text{ ft} \times \frac{12 \text{ in}}{1 \cancel{\text{ft}}} = 84 \text{ in}$$

Example 2: Convert 112 ounces a) pounds b) grams

a) $112\,\cancel{oz.} \times \frac{1 \text{ pound}}{16\,\cancel{oz}} = 7 \text{ pounds}$

b) $112\,\cancel{oz} \times \frac{28.35g}{1\,\cancel{oz}} = 3175.2\,g$

Example 3: Convert 8422 feet to a) miles b) metres (nearest tenth)

a) $8422\,\cancel{ft} \times \frac{1 \text{ mile}}{5280\,\cancel{ft}} = 1.6 \text{ mi}$

b) $8422\,ft \times \frac{0.3048\,m}{1\,ft} = 2567.0\,m$

Example 4: A marathon is 26.2 miles. What is this distance in km? (nearest tenth)

$26.2\,\cancel{mi} \times \frac{1.609\,km}{1\,\cancel{mi}} = 42.2\,km$

Sometimes, to convert, you need to do a two-step equation.
Example 5: Convert 0.43 miles to inches (nearest tenth)

$0.43\,\cancel{mi} \times \frac{5280\,\cancel{ft}}{1\,\cancel{mi}} \times \frac{12\,in}{1\,\cancel{ft}} = 27\,244.8\,in$

Example 6: Convert 6000 grams to pounds (nearest tenth)

$6000g \times \frac{1\,oz}{28.35g} \times \frac{1 \text{ pound}}{16\,oz} = 13.2 \text{ pounds}$

Example 7: Convert 7.5 yards to centimetres (nearest tenth)

$7.5\,yd \times \frac{36\,\cancel{in}}{1\,yd} \times \frac{2.54\,cm}{1\,\cancel{in}} = 685.8\,cm$