7.2 – Solving Systems with Graphs

Name:
Date:

Goal: to use the graphs of linear equations to solve linear systems

Toolkit:	**Main Ideas:**

Definitions:

Linear System – two or more linear equations together is called a **linear system.**

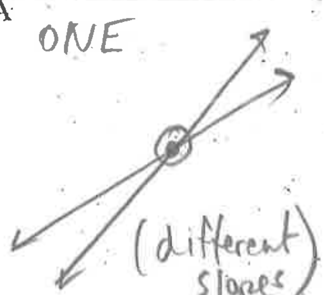
Solving a System – to solve a linear system, find the coordinates where the two lines intersect (the point where the lines cross). You will have an x-value and a y-value!

Steps for solving systems graphically:

1. Change each equation to a form that is easy to graph ($y = mx + b$ or $Ax + By = C$)
2. Graph each line
3. Write the solution (state the point where the lines cross)
4. Check the solution by substituting into each original equation (point must "satisfy" both lines)

What are the three possibilities when two lines are graphed?

A



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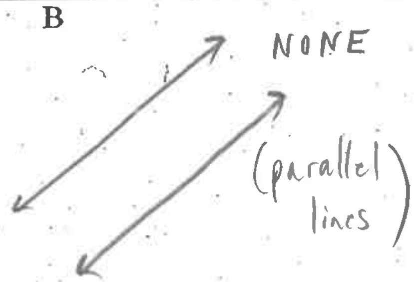

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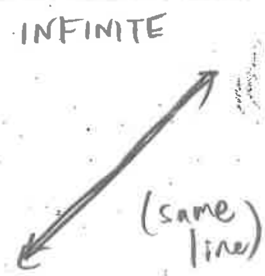
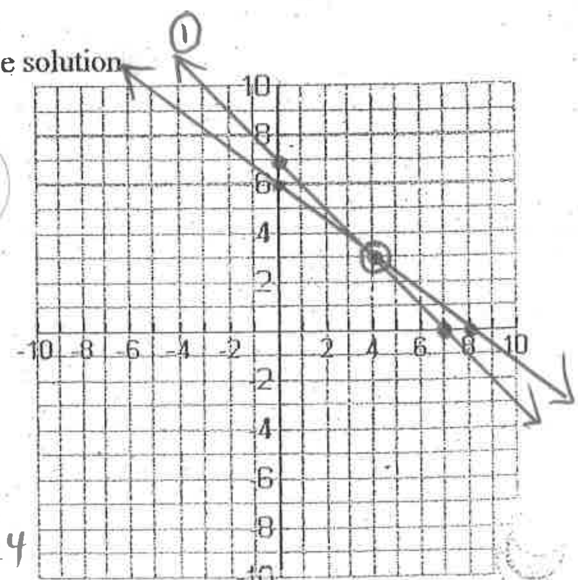


Ex1) Solve the system graphically and check the solution

$\boxed{1}$ x + y = 7
$\boxed{2}$ 3x + 4y = 24



Solution: (4, 3)

$y = \frac{-x}{1} + 7$

1) $x + y = 7$
x-int = 7
y-int = 7

2) $3x + 4y = 24$
x-int = 8
y-int = 6

Check:
1) $x + y = 7$
$4 + 3 = 7$ ✓

2) $3x + 4y = 24$
$3(4) + 4(3) = 24$
$12 + 12 = 24$ ✓

What if you just need to "check"?

Ex2) Is $(2, -1)$ a solution to the following system? $\boxed{1}$ $3x + 5y = 1$ $\boxed{2}$ $2x - 2y = 5$

$x \; y$

① $3x + 5y = 1$

$3(2) + 5(-1) = 1$

$6 + -5 = 1$ ✓

② $2x - 2y = 5$

$2(2) - 2(-1) = 5$

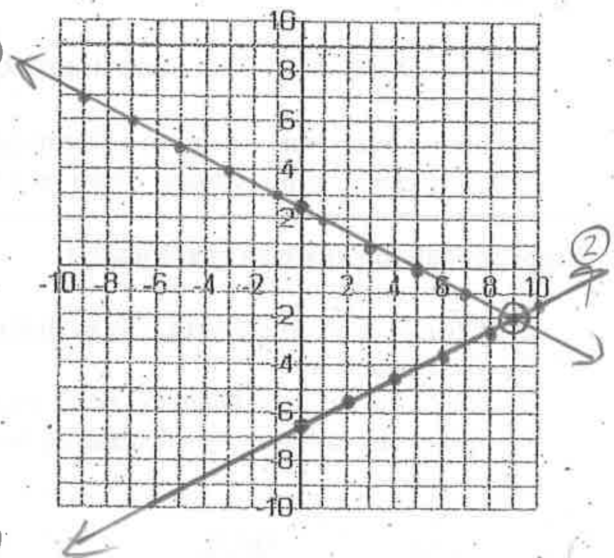
$4 + 2 = 5$ ✗

$(2,-1)$ is NOT the solution to this system

Ex3) Solve the system by graphing. Explain whether the solution is exact or approximate.

$\boxed{1}$ $x + 2y - 5 = 0$

$\boxed{2}$ $x - 2y - 13 = 0$



① $x + 2y = 5$

x-int = 5

y-int = 2.5

slope = $\frac{-1}{2}$ ← down, ← right

② $x - 2y = 13$

x-int = 13

y-int = -6.5

slope = $\frac{1}{2}$ ← up, ← right

Solution: $(9, -2)$

exact

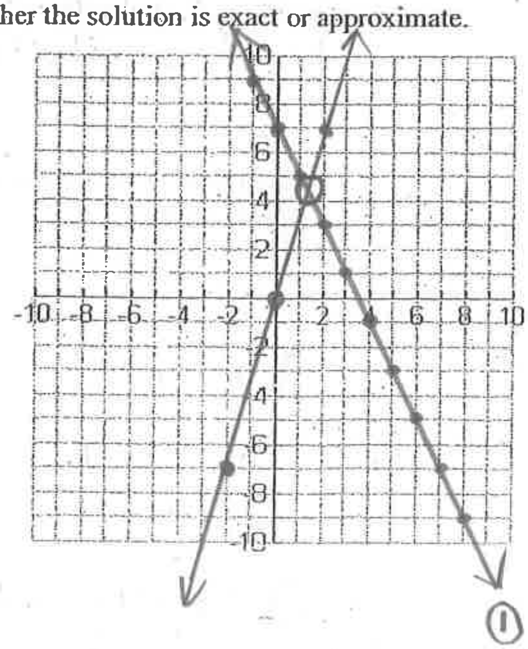
Ex4) Solve the system by graphing. Explain whether the solution is exact or approximate.

$\boxed{1}$ $y = -2x + 7$

$\boxed{2}$ $7x - 2y = 0$

① $y = \frac{-2x}{1} + 7$ ← down, ← right, ← y-int

approx Solution: $(1.3, 4.4)$



② $7x - 2y = 0$

x-int = 0

y-int = 0

slope = $\frac{7}{2}$ ← up, ← right

Reflection: What is a disadvantage to solving a system using GRAPHING?

7.4 – Solving Systems Algebraically using Substitution

Name:

Date:

Goal: to use the substitution of one variable to solve a linear system

Toolkit:	**Main Ideas:**

Linear systems can be solved without graphing. One method is by substitution.

Steps:

1. Solve one equation for either x or y (get either x or y by itself).
   Let's say you get y by itself in this case.
2. Substitute the equation into the second equation
3. Solve the second equation for the other variable (in this case x)
4. Now that you have the solution to one variable (in this case x), substitute it into one of the original two equations to get y
5. Write the solution
6. Check that the solution satisfies each equation

Ex1) Solve by substitution and check

$\boxed{1}$ $3x + y = 3$

$\boxed{2}$ $7x - 2y = 20$

① $3x + y = 3$
$\quad -3x \qquad -3x$

$y = -3x + 3$

② $7x - 2y = 20$

$7x - 2(-3x+3) = 20$

$7x + 6x - 6 = 20$

$13x - 6 = 20$
$\quad +6 \quad +6$

$13x = 26$

$x = 2$

here's what we know so far:

$(2, \_)$

① $3x + y = 3$

$3(2) + y = 3$

$6 + y = 3$

$y = -3$

Solution: $(2, -3)$

Check:

① $3x + y = 3$

$3(2) + (-3) = 3$

$6 + -3 = 3$ ✓

② $7x - 2y = 20$

$7(2) - 2(-3) = 20$

$14 + 6 = 20$ ✓

Equations solved for y	Ex2) Solve by substitution
Equations with fractions	Ex3) Solve by substitution

Ex2) Solve by substitution

$\boxed{1}$ $y = 3x + 2$

$\boxed{2}$ $y = -x - 14$

$3x + 2 = -\cancel{x} - 14$

$+x \qquad +\cancel{x}$

$4x + \cancel{2} = -14$

$-\cancel{2} \qquad -2$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{-16}{4}$

$x = -4$

① $y = 3x + 2$

$y = 3(-4) + 2$

$y = -12 + 2$

$y = -10$

$(-4, -10)$

Ex3) Solve by substitution

$$\begin{cases} -\frac{x}{5} + \frac{y}{3} - \frac{2}{15} = 0 \\ \frac{x}{7} + y = 0 \end{cases}$$

* To clear fractions from an equation, ask: What would be the common denominator, and then multiply every term by that value.

① $\frac{-x(15)}{5} + \frac{y(15)}{3} - \frac{2(\cancel{15})}{\cancel{15}} = 0(15)$

$-3x + 5y - 2 = 0$

② $\frac{x(\cancel{7})}{\cancel{7}} + y(7) = 0(7)$

$x + 7y = 0$

$x = -7y$

① $-3x + 5y - 2 = 0$

$-3(-7y) + 5y - 2 = 0$

$21y + 5y - 2 = 0$

$26y - 2 = 0$

$26y = 2$

$y = \frac{2}{26} = \frac{1}{13}$

$(\_, \frac{1}{13})$

$x + 7y = 0$

$x + \frac{7}{1}(\frac{1}{13}) = 0$

$x + \frac{7}{13} = 0$

$x = \frac{-7}{13}$

$(\frac{-7}{13}, \frac{1}{13})$

Reflection: When you have a system with fractions in it, and you want to write an equivalent system without fractions, how do you decide what number to multiply by?

Name:
Date:

Goal: to use the elimination of one variable to solve a linear system

Toolkit:	**Main Ideas:**

Linear systems can be solved without graphing. One method is by elimination.

Steps:

1. **May not be necessary** Multiply both sides of one or both equations by a constant to get either the same x or the same y coefficient in both equations to get an **"equivalent system"**
2. Add or subtract the two equations to eliminate either x or y
3. Solve the resulting equation for the remaining variable
4. Substitute the value obtained in step 3 back into one of the original equations to get the other variable
5. Write the solution
6. Check that the solution satisfies each equation

Ex 1) Solve the system by elimination and check

$\boxed{1}$ $3x - 5y = -9$
$\boxed{2}$ $4x + 5y = 23$

① $3x - 5y = -9$
\+ ② $(4x + 5y = 23)$

$7x = 14$

$x = 2$

$(2, \_)$

① $4x + 5y = 23$
$4(2) + 5y = 23$
$8 + 5y = 23$
$-8 \qquad -8$
$5y = 15$
$y = 3$

$(2, 3)$

① $3(2) - 5(3) = -9$
$6 - 15 = -9$ ✓

② $4(2) + 5(3) = 23$
$8 + 15 = 23$ ✓

How do you know when to add the eq'ns or subtract the eq'ns in step 2?

For the terms being eliminated, if signs different, ADD. If signs same, SUBTRACT

$3-(-7)$

$3+7$

Ex 2) Solve by elimination

$\boxed{1}$ $4x + 3y = 5$

$\boxed{2}$ $4x - 7y = 15$

① $4x + 3y = 5$

$-$ ② $(4x - 7y = 15)$

$10y = -10$

$y = -1$

① $4x + 3y = 5$

$4x + 3(-1) = 5$

$4x - \cancel{3} = 5$

$+3 \quad +3$

$4x = 8$

$x = 2$

$(2, -1)$

Ex 3) Solve by elimination

$\boxed{1}$ $(2x + 5y = 11) \times 3$

$\boxed{2}$ $(3x - 2y = 7) \times 2$

① $6x + 15y = 33$

$-$ ② $(6x - 4y = 14)$

$19y = 19$

$y = 1$

① $2x + 5y = 11$

$2x + 5(1) = 11$

$2x + 5 = 11$

$-5 \quad -5$

$2x = 6$

$x = 3$

$(3, 1)$

Equations with fractions

Ex 4) Solve by elimination

$$\begin{cases} \frac{3}{4}x - y = 2 \\ \frac{1}{8}x + \frac{1}{4}y = 2 \end{cases}$$

① $\frac{3(4)}{4}x - y^{(4)} = 2(4)$

① $3x - 4y = 8$

② $\frac{1}{8}^{(8)}x + \frac{1}{4}^{(8)}y = 2(8)$

② $x + 2y = 16$

① $3x - 4y = 8$

② $(x + 2y = 16) \times 2$

① $3x - 4y = 8$

$+$ ② $(2x + 4y = 32)$

$5x = 40$

$x = 8$

② $x + 2y = 16$

$8 + 2y = 16$

$-8 \quad -8$

$2y = 8$

$y = 4$

$(8, 4)$

Reflection: Which method do you prefer for solving linear systems AND WHY: graphing, substitution, or elimination?

7.6 – Properties of Systems

Name:
Date:

Goal: to recognize systems that will have each of the three different types of solutions

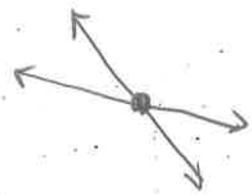
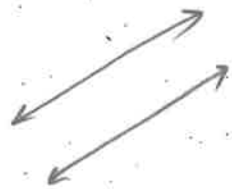
Toolkit:
- So far, all of the linear systems we've solved have given one solution (one intersection)
- Rearranging equations

Main Ideas:

Three types of solutions:

$y = mx + b$

slope y-int

A ONE solution	B NO solution	C INFINITE solutions
Sketch: 		
Description: different slopes	Same slopes, different y-ints	same slopes, same y-ints

How can you predict how many solutions a system will have without graphing?

start with slopes

different → ONE SOLUTION

same, so check y-ints

different → NO SOLUTIONS

same → INFINITE SOLUTIONS

Ex1) Predict how many solutions each system has:

a) $\begin{cases} ① \; y = 2x + 3 \\ ② \; y = \frac{6}{3}x + 3 \end{cases}$

slopes: ① 2 ② 2

y-ints: ① 3 ② 3

INFINITE SOLUTIONS

b) $\begin{cases} ① \; 1x - 1y = 4 \\ ② \; 1x + 1y = 5 \end{cases}$

slopes: ① 1 ② -1

different!

ONE SOLUTION

c) $\begin{cases} y = -\frac{1}{2}x + 7 \\ y = -\frac{1}{2}x + 2 \end{cases}$

slopes: ① $-\frac{1}{2}$ ② $-\frac{1}{2}$

Same!

y-ints: ① 7 ② 2

different

NO SOLUTIONS

Standard form "shortcut": start off like elimination—try to get x or y coefficients to match by multiplying the whole equation by a constant

Ex 2) How many solutions? $\begin{cases} 2x-5y=15 \\ 4x-10y=6 \end{cases}$

A) If x and y coefficients DO NOT BOTH match, then you have ONE solution

B) If x and y coefficients BOTH match, but the constants DO NOT, then you have NO solution

C) If x and y coefficients BOTH match, and the constants match, then you have INFINITE solutions

Ex 3) How many solutions does each system have?

a) $\begin{cases} ① \ 7x-y=10 \\ ② \ 14x-2y=20 \end{cases}$

slopes: ① 7 ② 7 same!

y-ints: ① −10 ② −10 same!

INFINITE SOLUTIONS

Solve ⓐ

① $7x-y=10$

② $14x-2y=20$

① $y=7x-10$

② $14x-2(7x-10)=20$

$14x-14x+20=20$

$20=20$

Both variables disappear, left with same number on each side = INF SOLNS

b) $\begin{cases} ① \ 4x-3y=12 \\ ② \ 8x-6y=30 \end{cases}$

slopes: ① $\frac{4}{3}$ ② $\frac{4}{3}$ same!

y-int: ① −4 ② −5 different

NO SOLUTIONS

Solve ⓑ

① $8x-6y=24$

② $(8x-6y=30)$

$0+0=-6$

$0=-6$

Both variables disappear, left with diff numbers on each side = NO SOLNS

c) $\begin{cases} ① \ 5x+y=16 \\ ② \ 2x-3y=3 \end{cases}$

slopes: ① −5 ② $\frac{2}{3}$ different

ONE SOLUTION

Reflection: Use examples and/or diagrams to explain why there cannot be exactly 2 solutions to a linear system.

7.7 – Applications of Systems Part I

Name:
Date:

Goal: to model situations and answer problems using a system of linear equations

Toolkit:
- total, sum, greater than **all mean** +
- difference, less than **mean** –
- times, product **mean** ×
- to change % to decimal, move decimal two places to the left
- remember units!!!

Main Ideas:

These word problems involve two unknowns. We need two equations to solve for two unknowns, so it will be your job to create the system of two equations and solve it!

Steps:
1. Define your two variables. You may use x and y, but it is also good to practise working with other variables (such as t for time). Use "let" statements (e.g. let x be the number of…) *Usually, they are the two things you need in order to answer the problem.*
2. Build your two equations.
3. Solve the system using elimination, substitution, or graphing.
4. Write a sentence answer.
5. Check.

Ex 1) The sum of two numbers is 53. The first is 7 greater than the second. What are the numbers?

Let x = the first number (bigger number)
Let y = the second number (smaller number)

① $x + y = 53$

② $x = y + 7$

① $(y+7) + y = 53$

$y + 7 + y = 53$

$2y + \cancel{7} = 53$
$\quad\ \ \cancel{-7} \quad -7$

$2y = 46$

$y = \frac{46}{2} = 23$

① $x + y = 53$

$x + 23 = 53$

$x = 30$

$(30, 23)$

The two numbers are 30 and 23.

Ex 2) For a basketball game, 1600 tickets were sold. Some tickets cost \$3 and the rest cost \$2. If the total receipts were \$4000, how many of each kind were sold?

Let x = amount of \$3 tix

Let y = amount of \$2 tix

① $x + y = 1600$

② $3x + 2y = 4000$

① $2x + 2y = 3200$

② $-(3x + 2y = 4000)$

$-1x = -800$

$x = 800$

① $x + y = 1600$

$800 + y = 1600$

$y = 800$

$(800, 800)$

800 \$3 tix sold, and

800 \$2 tix sold!

Ex 3) Isaac borrowed \$2100 for his college tuition. Part of it he borrowed from a government student fund at 5% annual interest. The rest he borrowed from a bank at 6.5% annual interest. If the total annual interest is \$114, how much did he borrow from each source?

Let x = amount borrowed from govt.

Let y = " " " bank.

① $x + y = 2100$

② $0.05x + 0.065y = 114$

① $x = 2100 - y$

② $0.05(2100 - y) + 0.065y = 114$

$105 \underbrace{- 0.05y + 0.065y} = 114$

$105 + 0.015y = 114$

$-105 \qquad -105$

$0.015y = 9$

$y = 600$

① $x + y = 2100$

$x + 600 = 2100$

$x = 1500$

\$1500 borrowed from govt, \$600 from bank

…flection: Would you ever need to solve for 3 variables? Think of a scenario and (no need to solve!) …ain WHAT you would need in order to be able to solve for 3 variables.

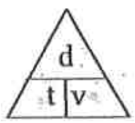
7.8 – Applications of Systems Part II

Name:
Date:

Goal: to continue to model situations and answer problems using a system of linear equations

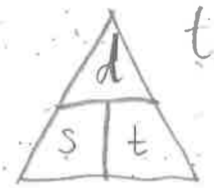
Toolkit:

- sum (+), difference (–), product (×)
- to change % to dec, move decimal two places to the left
- remember units!!!
- $speed = \frac{dist.}{time}$ OR (tv in the basement)



Main Ideas:

$s = \frac{d}{t}$



$ts = d$

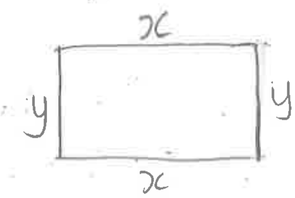
$t = \frac{d}{s}$

These word problems involve two unknowns. We need two equations to solve for two unknowns, so it will be your job to create the system of two equations and solve it!

Steps:

1. Define your two variables. You may use x and y, but it is also good to practise working with other variables (such as t for time). Use "let" statements (e.g. let x be the number of…) *Usually, they are the two things you need in order to answer the problem.*
2. Build your two equations.
3. Solve the system using elimination, substitution, or graphing.
4. Write a sentence answer.
5. Check.

Ex 1) The perimeter of a rectangle is 46 cm. What are its dimensions if the length is 4cm less than twice the width?



Let x = length
Let y = width

① $2x + 2y = 46$
② $x = 2y - 4$

$2(2y - 4) + 2y = 46$

$4y - 8 + 2y = 46$

$6y - 8 = 46$
$\quad +8 \quad +8$

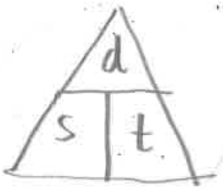
$\frac{6y}{6} = \frac{54}{6}$

$y = 9\text{cm}$

$x = 2y - 4$
$x = 2(9) - 4$
$x = 18 - 4$
$x = 14\text{ cm}$

The dimensions are 14cm × 9cm.

Ex 2) Flying with the wind, an airplane travels 4256km in 3.5h. Flying against the same wind, the airplane makes the return trip in 3.8h. Find the speed of the airplane in still air and the speed of the wind (assume both speeds are constant for the round trip).



Whenever you're doing a word problem with speed, distance, and time, it helps to set up a table like the one below:

Let x = speed of plane in still air

Let y = speed of wind

Direction	Distance (km)	Speed (km/h)	Time (h)	Equations
With the wind	4256	$x+y=\frac{4256}{3.5}$	3.5	$x+y=1216$
Against the wind	4256	$x-y=\frac{4256}{3.8}$	3.8	$x-y=1120$

① $x+y=1216$
$+$ ② $(x-y=1120)$

$2x = 2336$

$x=\frac{2336}{2}=1168\ \frac{km}{h}$

① $x+y=1216$

$1168+y=1216$

$y=48$ km/h

The speed of the plane in still air is 1168 km/h and the speed of the wind is 48 km/h

lection: How will YOU remember the relationship among distance, speed, and time?