6.1 – Slope of a Line

Name: Notes
Date: Key

Goal: Determine the slope of a line segment and a line.

Toolkit:	**Main Ideas:**
- Rate of change - Simplifying fractions	

Definitions

Rise: the vertical distance between two points.

Run: the horizontal distance between two points.

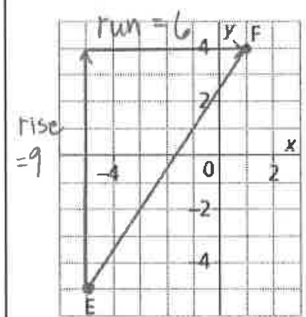
Slope: a measure of how one quantity changes with respect to the other, it can be calculated using:

$$\text{Slope: } \frac{rise}{run} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

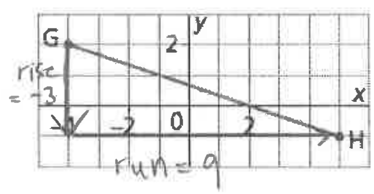
Determining the Slope of a Line Segment

Ex1) Determine the slopes of the following line segments.

Step 1: Choose two points on the line segment.
Step 2: Count the units to determine the *rise* and the *run*.
Step 3: Write the fraction in simplest form.



$slope_{EF} = \frac{rise}{run} = \frac{9}{6} = \frac{3}{2}$



$slope_{GH} = \frac{rise}{run} = \frac{-3}{9} = -\frac{1}{3}$

When a line segment goes up to the RIGHT, both *x* and *y* INCREASE. Both the rise and run are POSITIVE, so the slope of the line segment is POSITIVE.

When a line segment goes down to the RIGHT, *y* DECREASES and *x* INCREASES. The rise is NEGATIVE and the run is POSITIVE, so the slope of the line segment is NEGATIVE.

For a horizontal line segment, the change in *y* is 0. The rise is 0 and the run is positive.

$$\text{Slope} = \frac{rise}{run} = \frac{0}{run} = 0$$

For a vertical line segment, *y* INCREASES and the change in *x* is 0. The rise is positive and the run is 0.

$$\text{Slope} = \frac{rise}{run} = \frac{rise}{0} = \text{undefined}$$

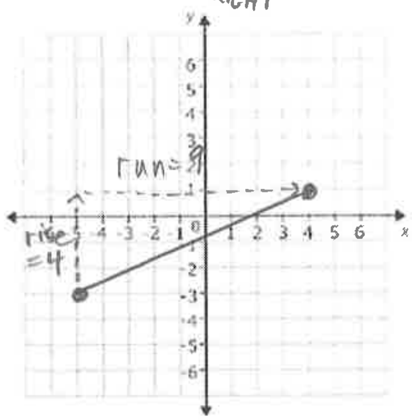
Drawing a line segment with a given slope.

Ex 2) Draw a line segment with the given slope.

a) slope = $\frac{4}{9}$ = $\frac{rise}{run}$

UP

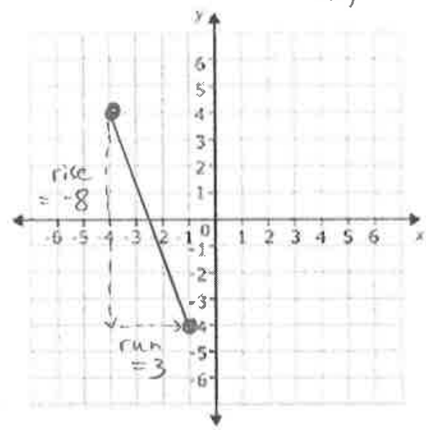
RIGHT



b) slope = $\frac{-8}{3}$

down

right



Finding slope when given two points.

Ex 3) Determine the slope of the line that passes through E(4,-5) and F(8,6).

x_1 y_1 x_2 y_2

Slope of a line = $\frac{y_2-y_1}{x_2-x_1}$	$A(x_1,y_1)$ $B(x_2,y_2)$

$$\text{slope}_{EF} = \frac{rise}{run} = \frac{y_2-y_1}{x_2-x_1} = \frac{6-(-5)}{8-4} = \frac{11}{4}$$

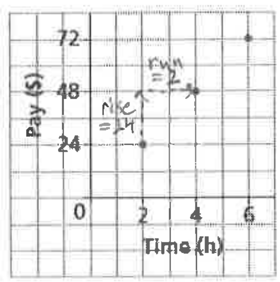
How else could we have found the slope? By graphing and counting

Interpreting the slope of a line

Ex 4)

Tom has a part-time job. He recorded the hours he worked and his pay for 3 different days. Tom plotted these data on a grid.

Graph of Tom's Pay



a) What is the slope of the line through these points?

b) What does the slope represent?

c) How can the answer to part b be used to determine:

i) how much Tom earned in $3\frac{1}{2}$ hours?

ii) the time it took Tom to earn $30?

a) slope $= \frac{\text{rise}}{\text{run}} = \frac{\$24}{2\text{h}} = \frac{\$12}{1\text{h}}$

b) Tom's wage of \$12/h

c) \$12 × 3.5 = \$42

d) $\frac{\$30}{\$12}$ = 2.5 hours

Reflection: How is the slope of a line related to rate of change?

6.4 – Slope-Intercept Form of the Equation for a Linear Function

Name: Key
Date:

Goal: to relate the graph of a linear function to its equation in slope-intercept form.

Toolkit:

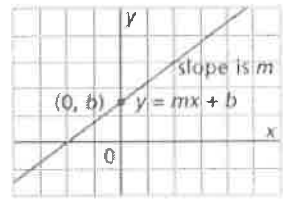
- Slope of a line (m) = $\frac{y_2-y_1}{x_2-x_1} \rightarrow \frac{rise}{run}$
- The y-intercept (vertical intercept) of a line is b

Main Ideas:

What is Slope-Intercept Form of the Equation of a Linear Function

The equation of a linear function can be written in the form $y=mx+b$, where m is the slope of the line and b is its y-intercept (with coordinates $(0,b)$).

$$y = mx + b$$



Writing an Equation Given Slope and y-intercept

Ex. 1) The graph of a linear function has a slope $\frac{3}{5}$ and y-intercept of -4. Write an equation for this function.

$y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{\text{y-intercept}}$

$y = \frac{3}{5}x - 4$

Graphing a Linear Function Given the Equation in $y = mx + b$

Ex. 2) Graph the linear functions with the following equations:

put negative on top!

a) $y = \frac{1}{2}x + \underbrace{3}$

b) $y = \frac{-3}{4}x \underbrace{- 1}$

① $b = 3$ (y-int)

- The line crosses the y axis at 3
- put a dot 'up 3'

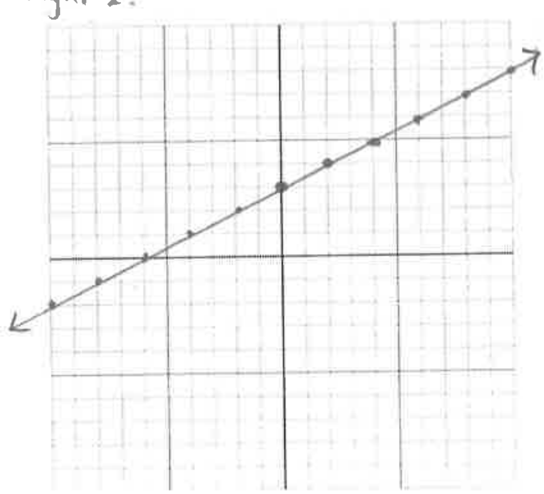
② $m = \frac{1}{2}$ (slope)

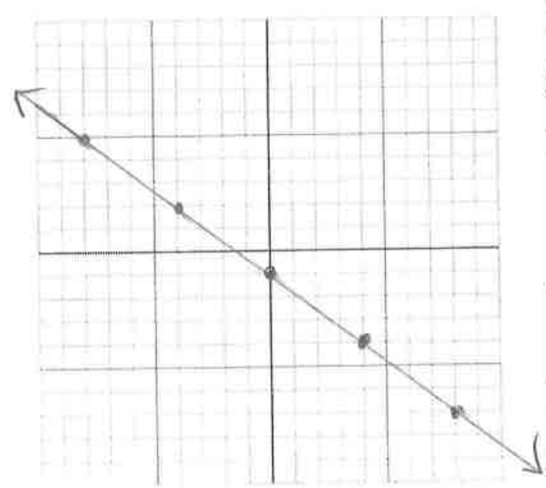
$\frac{1}{2} = \frac{rise}{run}$ ← up 1, ← right 2

from the y-int, count up 1 and right 2.

① $b = -1$

② $m = \frac{-3}{4}$ ← down 3, ← right 4

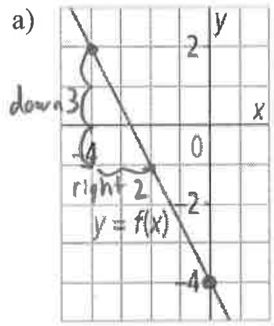




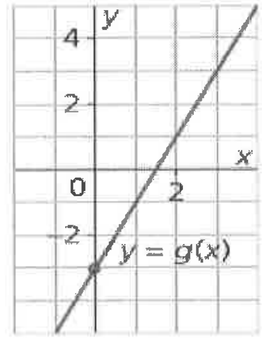
Writing the Equation of a Linear Function Given Its Graph

Ex. 3) Write equations to describe the following functions. Verify the equation.

a)



b)



y-int = -4 (b value)

$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{down } 3}{\text{right } 2} = \frac{-3}{2}$ (m value)

$y = mx + b$

$y = \frac{-3}{2}x - 4$

$b = -3$

$m = \frac{\text{up } 2}{\text{right } 1} = \frac{2}{1} = 2$

$y = 2x - 3$

Using an Equation of a Linear Function to Solve a Problem

Ex. 4) The student council sponsored a dance. A ticket cost \$5 and the cost for the DJ was \$300.

a) Write an equation for the profit, P, on the sale of t tickets.

$P = 5t - 300$

b) Suppose 123 people bought tickets. Find the profit.

$P = 5(123) - 300$
$= 615 - 300$
$= \$315$

c) Suppose the profit was \$350. How many people bought tickets?

$P = 5t - 300$

$350 = 5t - 300$
$+300 \quad\quad +300$

$\frac{650}{5} = \frac{5t}{5}$

$130 = t$

130 tickets were sold

d) Could the profit be exactly \$146? Justify the answer.

$146 = 5t - 300$
$+300 \quad\quad +300$

$\frac{446}{5} = \frac{5t}{5}$

$t = 89.2$

No, as profit must be divisible by 5

Reflection: How do the values of m and b in the linear equation $y=mx+b$ relate to the graph of the corresponding linear function? Use examples to help.

6.6 – General Form of the Equation for a Linear Relation

Name: KEY
Date:

Goal: to relate the graph of a linear function to its equation in general form.

Toolkit:	**Main Ideas:**
• Slope-Intercept form→ $y = mx + b$ • Rearranging Equations	

What is General Form of the Equation of a Linear Relation?

How is Standard Form similar?

GENERAL FORM of the Equation of a Linear Relation:

$$Ax + By + C = 0$$

where A is a whole number (not negative!), and B and C are integers.

STANDARD FORM of the Equation of a Linear Relation:

$$Ax + By = C$$

Rewriting an Equation in General Form

Ex. 1) Write each equation in general form and standard form:

a) $y = -\frac{2}{3}x + 4$

$-\frac{2}{3}x = -\frac{2}{3}\left(\frac{x}{1}\right) = \frac{-2x}{3}$

$y = \frac{-2x}{3} + 4$

$(3)y = -\frac{2x}{3}(3) + 4(3)$

$3y = -2x + 12$

$+2x \quad +2x$

$2x + 3y = 12$ and $2x + 3y - 12 = 0$

standard | General

b) $y - 1 = \frac{3}{5}(x + 2)$

$y - 1 = \frac{3x}{5} + \frac{6}{5}$

$(5)y - 1(5) = \frac{3x}{5}(5) + \frac{6}{5}(5)$

$5y - 5 = 3x + 6 + 5$

$-5y \quad +5 \quad -5y$

$0 = 3x - 5y + 11$ $3x - 5y = -11$

general | standard

Graphing a Line in General Form

Ex. 2) a) Determine the x- and y-intercepts of the line whose equation is $3x + 2y - 18 = 0$

to get x-intercept, set $y = 0$

$3x + 2(0) - 18 = 0$

$3x + 0 - 18 = 0$

$3x - 18 = 0$

$+18 \quad +18$

$\frac{3x}{3} = \frac{18}{3}$

$x = 6$

$(6, 0)$ is a point on the line

to get y-int, set $x = 0$

$3(0) + 2y - 18 = 0$

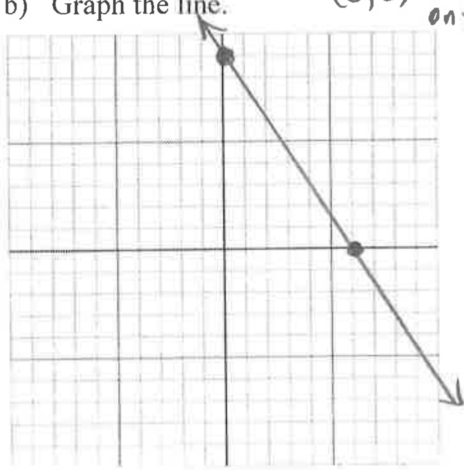
$2y - 18 = 0$

$+18 \quad +18$

$\frac{2y}{2} = \frac{18}{2}$ $y = 9$

$(0, 9)$ is on the line

b) Graph the line.



c) Verify that the graph is correct

choose a point on the graph:

$(2, 6)$

test in equation

$3x + 2y - 18 = 0$

$3(2) + 2(6) - 18 = 0$

$6 + 12 - 18 = 0$

$18 - 18 = 0$ ✓

Determining the Slope of a Line Given Its Equation in General Form

(switch to Standard!)

Ex. 3) a) Determine the slope of the line with the equation $3x - 2y - 16 = 0$

Two Methods

Method 1: change to slope y-int form:

$3x - 2y - 16 = 0$

$3x - 2y = 16$

$-3x \qquad -3x$

$\frac{-2y}{-2} = \frac{-3x}{-2} + \frac{16}{-2}$

$y = \frac{3}{2}x - 8$ slope is $\frac{3}{2}$

Method 2: Shortcut!

In General or Standard Form,

slope $= -\frac{A}{B}$, ALWAYS!!

$\underset{A}{3x}\underset{B}{-2y} - 16 = 0$

slope $= -\frac{A}{B} = -\frac{3}{-2} = \frac{3}{2}$

b) Determine the slope of the line with the equation $5x - 2y + 12 = 0$

Method 1:

$\cancel{5x} - 2y + \cancel{12} = 0$

$-5x \qquad -12 \qquad -5x \quad -12$

$\frac{-2y}{-2} = \frac{-5x}{-2} \frac{-12}{-2}$

$y = \frac{5}{2}x + 6$ slope is $\frac{5}{2}$

Method 2: slope $= -\frac{A}{B}$

$\underset{A}{5x}\underset{B}{-2y} + 12 = 0$

slope $= -\frac{5}{-2} = \frac{5}{2}$

c) Determine the slope AND the y-intercept of the line with the equation $4x - 6y = 0$, then graph the line.

Method 1: get into slope y-int form:

$\cancel{4x} - 6y = 0$

$-4x \qquad -4x$

$\frac{-6y}{-6} = \frac{-4x}{-6}$

$y = \frac{4}{6}x$

$y = \frac{2}{3}x + 0$

slope $= \frac{2}{3}$

y-int $= 0$

Method 2:

slope $= -\frac{A}{B}$

$\underset{A}{4x}\underset{B}{-6y} = 0$

slope $= -\frac{4}{-6}$

$= \frac{4}{6}$

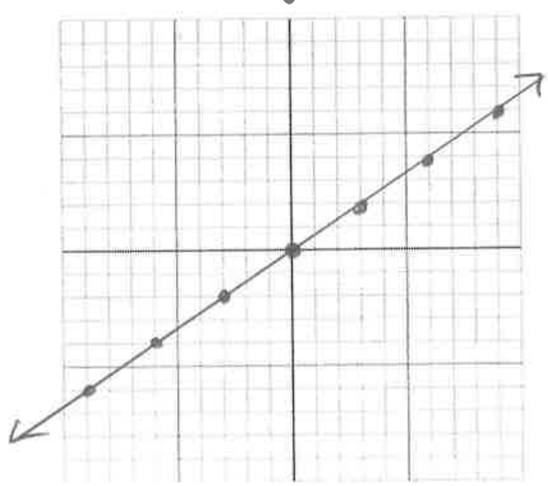
$= \frac{2}{3}$

y-int : set $x = 0$

$4(0) - 6y = 0$

$\frac{-6y}{-6} = \frac{0}{-6}$

$y = 0$



Reflection: Why can't you use intercepts to graph the equation $4x - y = 0$? (where $C = 0$)

6.7 – Graphing Linear Functions in Two Forms Name: Key
Date:

Goal: to recognize the two different forms of linear functions, & to graph them using the easiest method

Toolkit:	**Main Ideas:**
• Slope/y-intercept form • General & Standard form	

What is the best way to graph an equation in $y=mx+b$ form?

Ex 1) Label each linear equation as either "$y = mx + b$", or "*standard*":

$y = -3x + 5$	$2x + 3y = 9$	$2x - y = -4$	$y = \frac{1}{2}x - \frac{3}{4}$	$y = 0.4x - 0.15$
$y=mx+b$	standard	standard	$y=mx+b$	$y=mx+b$

Ex2) Graph the equation $y = -\frac{3}{2}x + 6$

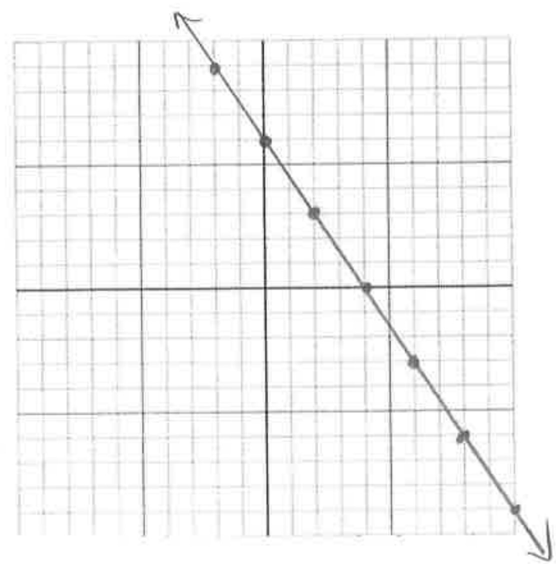
Step 1: decide what form it is in: $y=mx+b$ state $m= \frac{-3}{2}$ and $b = 6$

Step 2: for $y = mx + b$, put a **point** on the y-axis at "b"

Step 3: use the slope $\left(m = \frac{-3}{2} = \frac{rise}{run}\right)$ to count up/down 3 and over (right) 2 to a **new point**

Step 4: connect the dots!

$y = mx + b$
- start at b
- go up/down and over using slope
- connect the dots!



Hint: if you like y = mx + b, you can change any function to y = mx + b form and use this method!

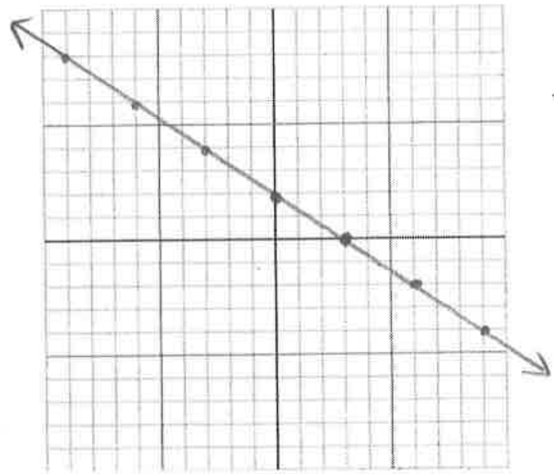
Ex 3) Change to $y = mx + b$ form: $3x - 2y = 8$

$\cancel{3x} - 2y = 8$
$-\cancel{3x} \quad -3x$

$\frac{-2y}{-2} = \frac{-3x}{-2} + \frac{8}{-2}$

$y = \frac{3}{2}x - 4$

What is the best way to graph an equation in general/standard form?	Ex 4) Graph the equation $\overset{\text{general}}{2x + 3y - 6 = 0} \rightarrow \overset{\text{standard}}{2x+3y=6}$ Step 1: decide what form it is in: standard note: *slope* = $\frac{-A}{B} = -\frac{2}{3}$ Step 2: for *standard form*, find the intercepts (cover *x* to get *y*, cover *y* to get *x*) *x-int* = 3 *y-int* = 2 Step 3: plot *x*- and *y*-intercepts (3,0) and (0,2) Step 4: connect the dots! (Can check slope)



standard form
- get intercepts
- plot intercepts
- connect the dots!

x -int
set $y=0$

$2x+3y=6$
$2x+3(0)=6$
$\frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2}$
$x=3$
$(3,0)$

y-int
set $x=0$

$2x+3y=6$
$2(0)+3y=6$
$\frac{\cancel{3}y}{\cancel{3}} = \frac{6}{3}$
$y=2$
$(0,2)$

Reflection: Which form of equation do you prefer to graph?

Would you change every equation to your preferred form, or use the different methods for the different ones? (You may want to try a few in the homework before you answer!)

6.2 – Slopes of Parallel and Perpendicular Lines

Name:
Date:

Goal: to use slope to determine whether two lines are parallel or perpendicular.

Toolkit:	**Main Ideas:**
- Slope - Simplifying fractions - Reciprocals	

Identifying Parallel Lines

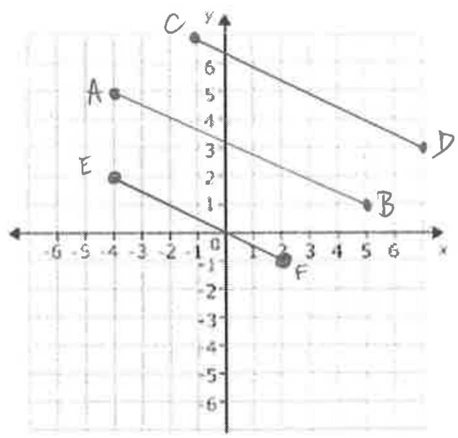
Lines that have THE SAME SLOPE **are parallel.**

$x_1\ y_1 \qquad x_2\ y_2$

Ex 1) Line EF passes through E(-4,2) and F(2,-1).
Line CD passes through C(-1,7) and D(7,3).
Line AB passes through A(-4,5) and B(5,1).
Sketch the lines. Are they parallel?

$slope = \frac{y_2 - y_1}{x_2 - x_1}$

(*on formula sheet)



$slope_{EF} = \frac{-1-2}{2-(-4)} = \frac{-3}{6} = \frac{-1}{2}$

$slope_{CD} = \frac{3-7}{7-(-1)} = \frac{-4}{8} = \frac{-1}{2}$

$slope_{AB} = \frac{1-5}{5-(-4)} = \frac{-4}{9}$

'parallel to'

$EF \parallel CD$

Identifying perpendicular lines

The slopes of two perpendicular lines are NEGATIVE RECIPROCALS **; that is a line with a slope $a, a \neq 0$, is perpendicular to a line with slope $-\frac{1}{a}$**

$x_1\ y_1 \qquad x_2\ y_2$

Ex 2) Line ST passes through S(-2,7) and T(2,-5). Line UV passes through U(-2,3) and V(7,6).

Are these lines parallel, perpendicular or neither? Calculate the slopes, and then sketch the lines to verify your answer.

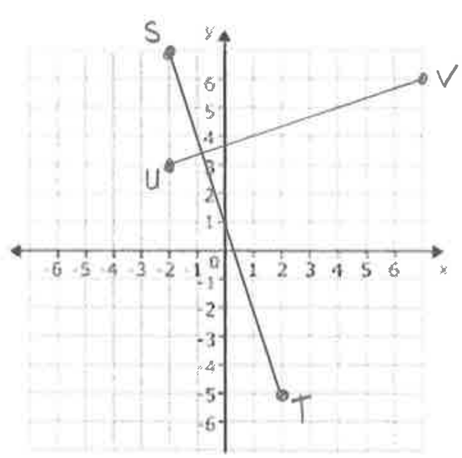
$slope_{ST} = \frac{-5-7}{2-(-2)} = \frac{-12}{4} = \frac{-3}{1}$

$slope_{UV} = \frac{6-3}{7-(-2)} = \frac{3}{9} = \frac{1}{3}$

$\frac{-3}{1}$ is neg recips with $\frac{1}{3}$

so $ST \perp UV$

'perpendicular to'



Identifying a line perpendicular to a given line.

Ex 3)

a) Determine the slope of a line that is perpendicular to the line through G(-2,3) and H(1,-2).

x_1 y_1 x_2 y_2

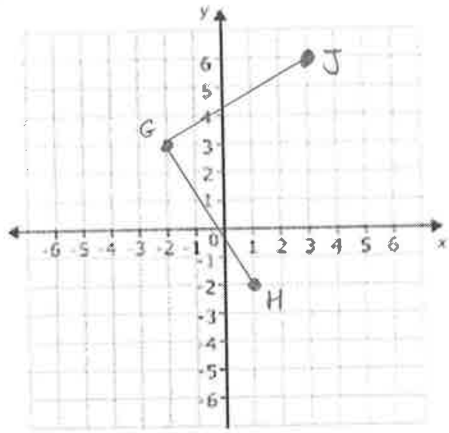
$\text{slope}_{GH} = \dfrac{-2-3}{1-(-2)} = \dfrac{-5}{3}$ need neg recip So slope of our line $= \dfrac{3}{5}$

b) Determine the coordinates of J so that line GJ is perpendicular to line GH.

GJ must have a slope of $\frac{3}{5}$

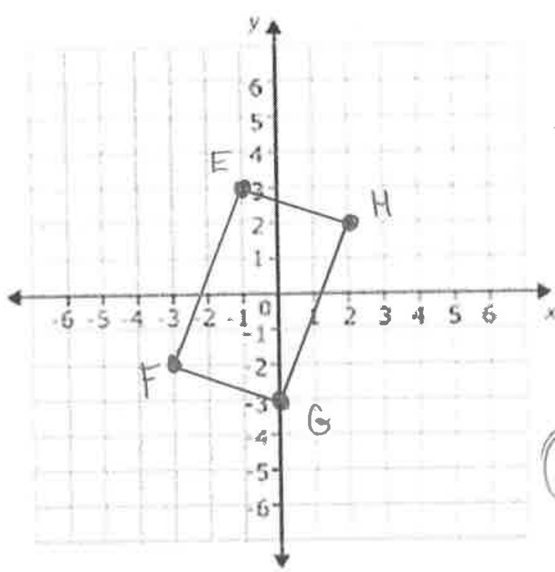
so, from point G, go $\frac{3 \leftarrow \text{up}}{5 \leftarrow \text{right}}$

One possible answer: J(3,6)



Using slope to identify a polygon.

Ex 4) Is EFGH a parallelogram? Is it a rectangle?



E(-1,3) F(-3,-2)
G(0,-3) H(2,2)

$\text{slope}_{EF} = \frac{5}{2}$ slope EH $= \frac{-1}{3}$

slope GH $= \frac{5}{2}$ slope FG $= \frac{-1}{3}$

(a) a parallelogram has opposite sides that are parallel (same slopes) so EFGH <u>is</u> a parallelogram!

(b) a rectangle has four 90° angles so sides are perpendicular, so slopes EF and EH should be neg recips! - they are <u>not</u>, so EFGH <u>not</u> a rectangle.

Reflection: What have you learned about parallel and perpendicular lines?

6.8 – Equations of Parallel and Perpendicular Lines

Name: Notes
Date: key

Goal: to recognize the different forms of linear functions, and to graph them using the easiest method

Toolkit:

- slopes of parallel lines are equal!
- slopes of perpendicular lines are negative reciprocals! (change sign) (flip)
- to find the equation of a line, you need:
  - slope
  - a point
- passing through → sub in!

Main Ideas:

Ex 1) For a line with the slope 0.7, $m = \frac{7}{10}$ what is the **slope** of a line that is

a) Parallel?

$m = 0.7$ or $\frac{7}{10}$

b) Perpendicular?

Flip, Change sign! $m_\perp = \frac{-10}{7}$

Ex 2) State the **slopes** of lines that are:

a) parallel to the line $3x + 2y - 4 = 0$

$m_{line} = -\frac{3}{2}$

$m_{\parallel} = -\frac{3}{2}$

b) perpendicular to $y = \frac{1}{2}x - \frac{3}{4}$

$m = \frac{1}{2}$

$m_\perp = -2$

Ex 3) For this pair of slopes, what is the value of *k* if the lines are...

$$\frac{4}{k}, 2$$

a) Parallel? (equal)

$\frac{4}{k} = \frac{\boxed{2}}{1}$

$k = \frac{4 \times 1}{2}$

$k = 2$

b) Perpendicular? (neg. rec.

$\frac{4}{k} = \frac{\boxed{-1}}{2}$

$k = \frac{4 \times 2}{-1}$

$k = -8$

Ex 4) Are the pairs of lines parallel, perpendicular, or neither?

Check slopes!

a) $2x+3y+9=0$, $y=\frac{3}{2}x+6$

$m=\frac{-2}{3}$ $\quad m=\frac{3}{2}$

neg. rec.

∴ perpendicular!

b) $y+1=\frac{3}{4}(x+2)$, $6x-8y+3=0$

$m=\frac{3}{4}$ $\quad m=\frac{+6 \div 2}{8 \div 2}=\frac{3}{4}$

equal

∴ parallel!

Ex 5) Find the equation of the line (in $y = mx + b$ form) that is parallel to the line $2x+3y+9=0$ and has the same y-intercept as the line $y = 2x + 4$.

① write formula
② fill in what you know
③ sub-in a point to solve for what's missing

// to $2x+3y+9=0$ ⇒ slope!

$m=-\frac{2}{3}$ $\quad m_{//}=-\frac{2}{3}$

y-int = same as $y=2x+4$, $b=4$

$m=-\frac{2}{3}$

$b=4$

$y=mx+b$

$y=-\frac{2}{3}x+4$

Ex 6) Find the equation of the line (in $Ax + By + C = 0$ form) that is perpendicular to $y = -3x + 4$ and passes through the point (6, 3).

$m=\frac{1}{3}$

pt (6,3)

perp. to $y=-3x+4$ ⇒ Slope is -3, our slope is $+\frac{1}{3}$

passes through (6,3) (sub in!)

Choose eq'n: (only do one ☺)

$y=mx+b$

$y=\frac{1}{3}x+b$ $\quad$ sub in (6,3)

$3=\frac{1}{3}(6)+b$

$3=2+b$

$b=1$

$\boxed{y=\frac{1}{3}x+1}$

Rewrite!

$(3)y=\frac{1}{3}(3)x+1(3)$

$3y=x+3$

$\boxed{0=x-3y+3}$

OR

$Ax+By=C$

$1x-3y=C$

$1(6)-3(3)=C$

$6-9=C$

$-3=C$

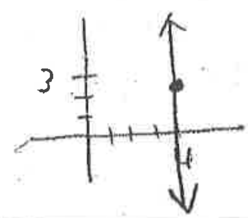
$1x-3y=-3$

$\boxed{x-3y+3=0}$

Ex 7) Find the equation of the line that is perpendicular to the x-axis and passes through the point (4 , 3)

⊥ x-axis ↕ vert. lines are perp.! always $x=$ number

must be $x=4$



Reflection: What short-cuts have you picked up this unit to make answering the questions faster?

→ $Ax+By=C$ $\quad m=-\frac{A}{B}$ $\qquad$ others?

→ if they give you y-int., use it!