6.2 – Solving Systems of Equations Graphically

Linear-Quadratic

A Linear-Quadratic System of Equations is a linear equation and a quadratic equation involving the same two variables. The solution(s) to this system are the point(s) where the line intersects the parabola (if it does at all).

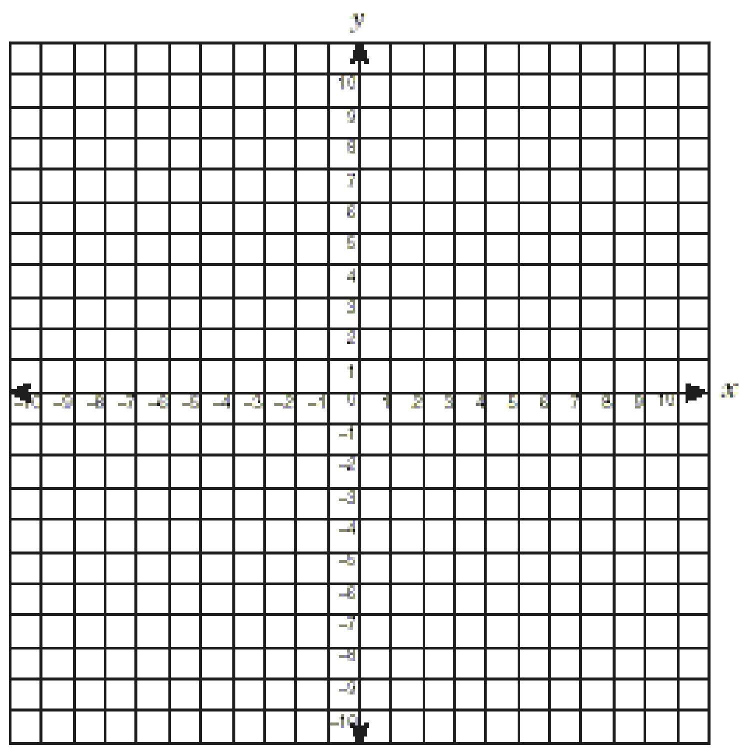
Draw pictures to represent the possible number of solutions that a linear-quadratic system can have:

Example 1 – Solve the following system of equations graphically:

1) $4x - y + 3 = 0$
2) $2x^2 + 8x - y + 3 = 0$

a) Get the linear equation into $y = mx \pm b$ form and graph.
b) Complete the square and graph the quadratic equation.
c) Identify and write down the points of intersection (the solution).
d) Verify the solution by checks.

a)



b)

c)

d)

Example 2 – Is (5, 7) a solution to the system 1) $3x^2 - 10y = 5$ and 2) $-y = x - 11$?

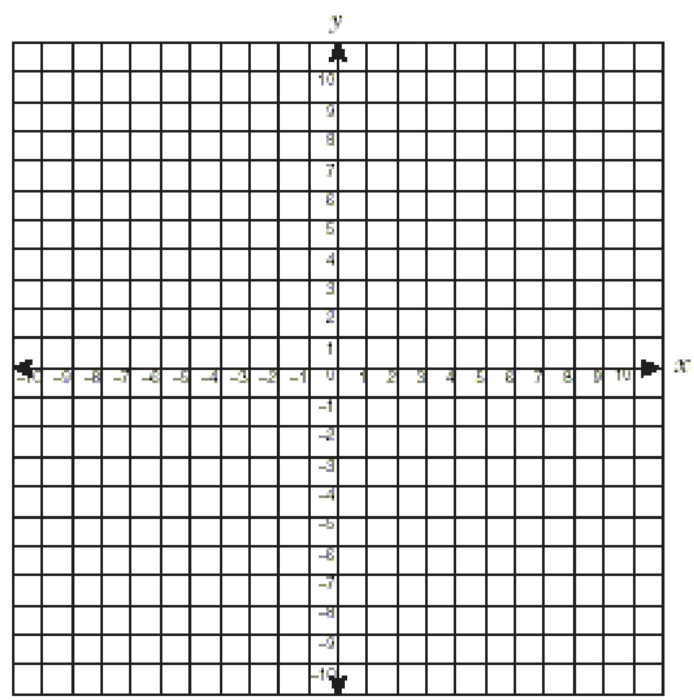
Quadratic-Quadratic

A Quadratic-Quadratic System of Equations is two quadratic equations involving the same variables. The solution(s) to this system are the point(s) where the parabola intersects the other parabola (if it does at all).

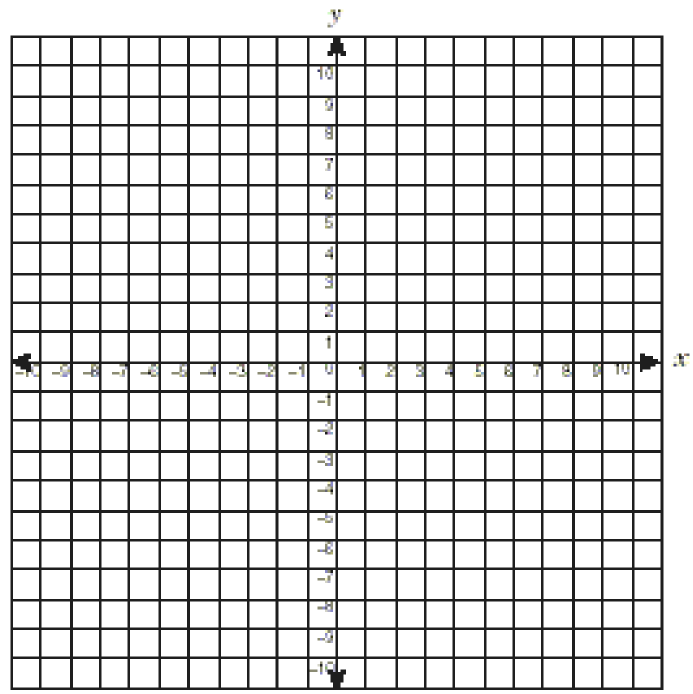
Draw pictures to represent the possible number of solutions that a quadratic-quadratic system can have:

Quadratic-Quadratic

Example 3 – Solve 1) $2x^2 - 8x + 7 - y = 0$ and 2) $y + x^2 - 4x + 2 = 0$



Example 4 – Solve the system $y - x^2 + 4 = 0$ and $-2y + 2x^2 - 8 = 0$



6.3/6.4 – Solving Systems of Equations Algebraically

Linear-Quadratic

For a Linear-Quadratic System of Equations, what are all the possible # of solutions?

Solutions can be found graphically, as in Section 6.2, or algebraically, using either substitution or elimination.

substitution

Example 1 – Solve the following linear-quadratic system using **substitution**:

1) $3x + y = -9$
2) $4x^2 - x + y = -9$

a) Solve the linear equation for y.
b) Substitute the linear equation for y in the quadratic equation.
c) Solve the quadratic equation by factoring (if you cannot factor, use the quadratic formula).
d) Substitute the resulting x value(s) into the original linear equation to determine the corresponding y values.

a)	d)
b &c)	

Example 2 – Solve by substitution: 1) $5x - y = 10$ and 2) $x^2 + x - 2y = 0$

elimination

Now, solve the same system using **elimination**:

1) $5x - y = 10$
2) $x^2 + x - 2y = 0$

a) Align the terms with the same degree. Since the squared term is the variable x, eliminate the y-term.
b) Multiply one or more of the equations if necessary to have the same coefficient for y.
c) Add or subtract the two equations to eliminate y.
d) Solve the resulting quadratic equation by factoring or the quadratic formula to find the x coordinates of the solution(s).
e) Substitute the resulting x value(s) into the original linear equation to determine the corresponding y values.

Quadratic-Quadratic

For a Quadratic-Quadratic Systems of Equations, what are all the possible # of solutions?

Example 3 – Solve the following system first by substitution, then by elimination.

1) $6x^2 - x - y = -1$
2) $4x^2 - 4x - y = -6$

Substitution:

Elimination:

$$6x^2 - x - y = -1$$

$$4x^2 - 4x - y = -6$$

Example 4 – A Canadian cargo plane drops a crate of emergency supplies to aid-workers on the ground. The crate drops freely at first before a parachute opens to bring the crate gently to the ground. The crate's height, h, in metres, above the ground t seconds after leaving the aircraft is given by the following two equations. $h = -4.9t^2 + 900$ represents the height of the crate during freefall. $h = -4t + 500$ represents the height of the crate with the parachute open.

a) How long after the crate leaves the aircraft does the parachute open? Express your answer to the nearest hundredth of a second.
b) What height above the ground is the crate when the parachute opens? Express your answer to the nearest metre.
c) Verify your solution.

7.2A – Linear Inequalities in Two Variables

Warmup

How do we read these inequalities (from left to right)? $5 > 2$ $-3 < -1$

What does each symbol mean? $>$ $<$ $\geq$ $\leq$

How do you say this aloud? $x \geq 4$

What are some possible answers?

What is the primary difference between an **equation** and an **inequality**?

Example 1 - Solve the following inequality: $3x - 7 < -5$

Example 2 – What are some possible answers to $-2x < 6$?

How is solving an inequality like solving an equation? How is it different?

Find some solutions to $3y - 2x \geq 6$

There is a more efficient way to find the range of solutions for the inequality above.

steps

1. Rearrange the inequality so it's in $mx \pm b$ form. Don't forget to flip the inequality if you multiply or divide by a negative number.

2. Decide whether to use a solid line or dotted line:

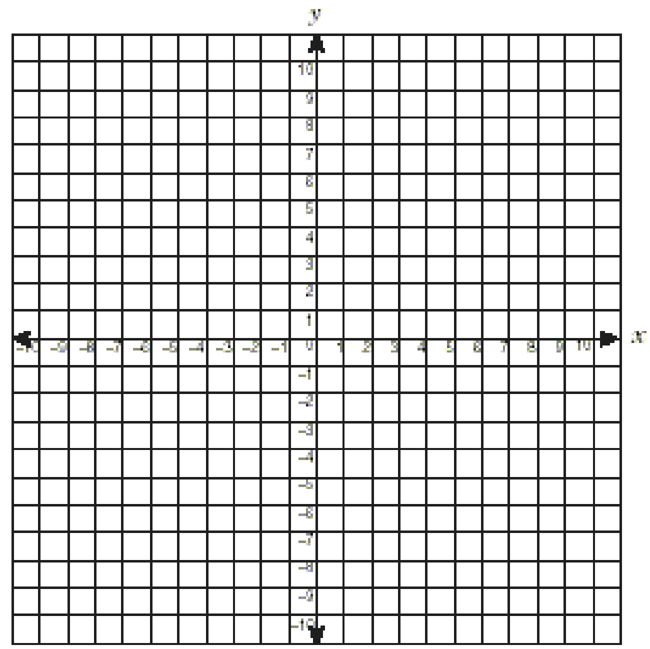
- If the inequality is $\leq$ or $\geq$, points on the line are included in the solution (due to the 'equals to' line under the sign), so we keep the line solid.
- If the inequality is $<$ or $>$, points on the line are not included in the inequality, so we draw a dotted line.

3. Graph the line using slope and y-intercept. The line is called the **boundary**.

4. For $y > mx + b$ or $y \geq mx + b$, solutions to the inequality are all of the points **above** the line, so shade above. For $y < mx + b$ or $y \leq mx + b$, shade **below** the line. The shading represents the **solution region**: all of the points that satisfy the inequality.

5. CHECK: Pick a **test point** in the shaded region and test its coordinates in the inequality. If it satisfies the inequality, you've been successful. If it doesn't satisfy the inequality, you either shaded the incorrect region, or the boundary line has been graphed incorrectly.

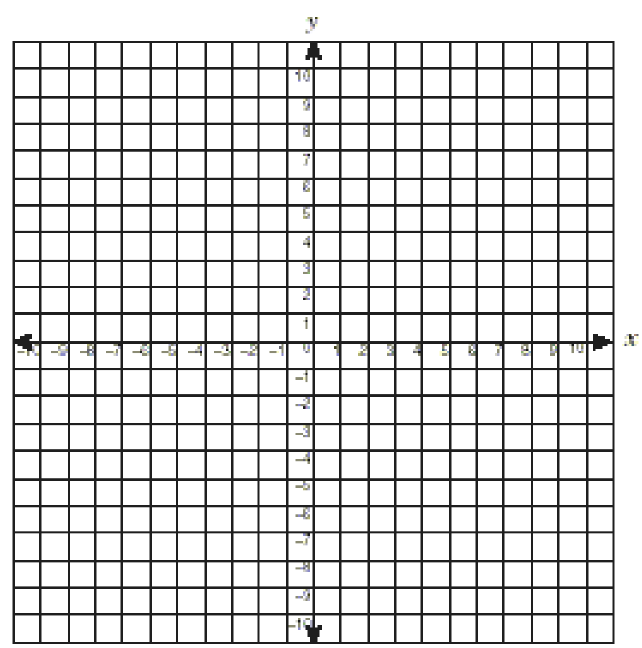
Example 1 – Solve the inequality by graphing $3y - 2x \geq 6$.



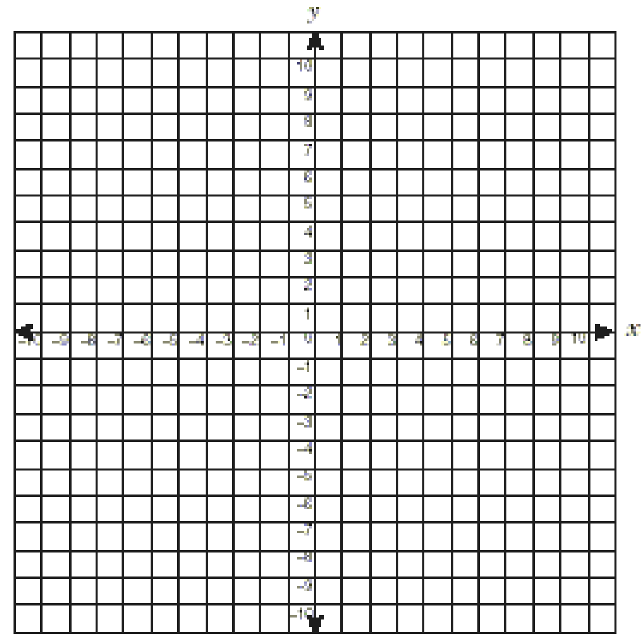
CHECK:

***When you read an inequality for shading purposes, it must be in *y = mx + b* form!**

Example 2 – Solve $4x - 2y > 10$. Determine if (1, 3) is part of the solution.



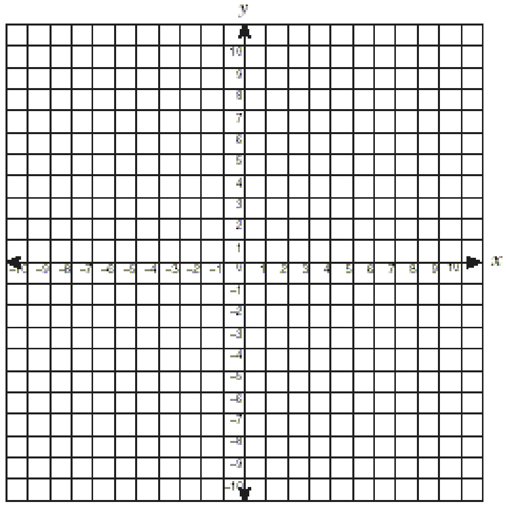
Example 3 – Solve $x \leq 4$



7.2B – Systems of Linear Inequalities in Two Variables

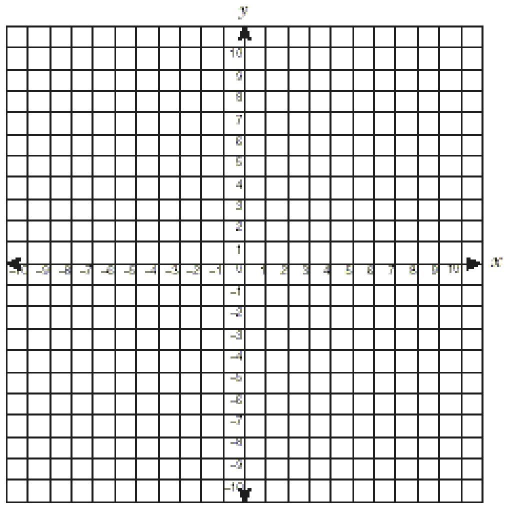
A **system** of linear inequalities is:

Example 1 – Solve the system: $y \geq 2x + 5 \; and \; y < -x - 5$



Pick one possible solution and perform a check:

Example 2 – Solve the system: $3x + 2y > -6 \; and \; -3 \leq y \leq 3$



Check:

STEPS: 1. Rearrange each inequality into ***mx + b*** form.

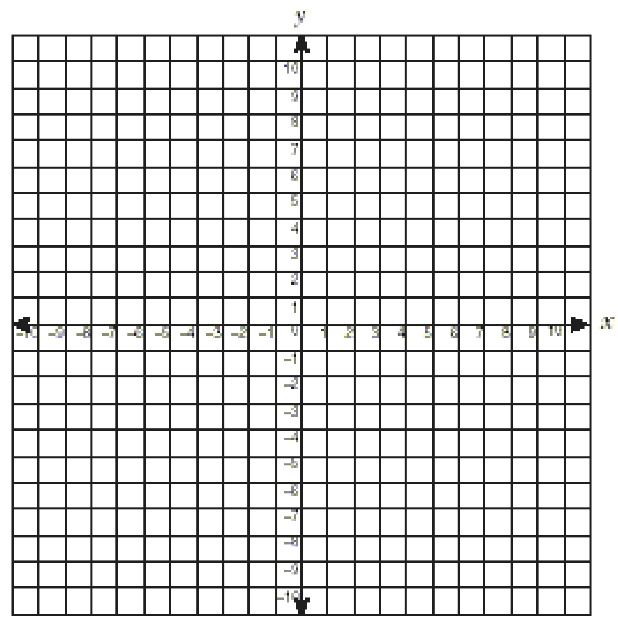
2. Graph each line, using dashed (>, <) or solid (≥, ≤) lines.

3. To find the solution region (shaded region), look to see whether to shade above or below the first line, then above or below the second line (read the inequality in ***mx + b*** form).

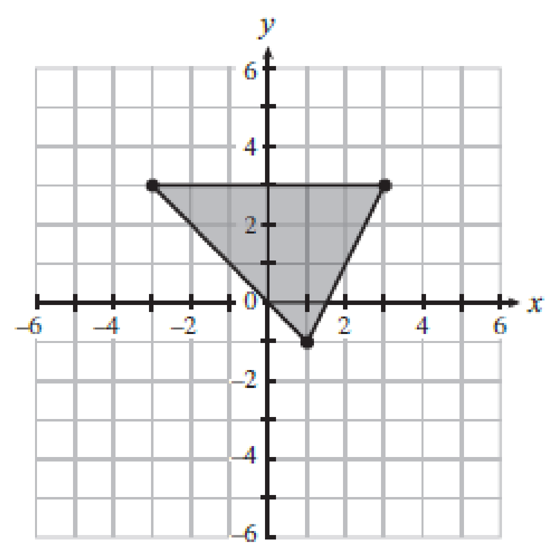
4. Check your solution by picking a point in your solution and testing it in each of the two original inequalities. It must satisfy both inequalities. If it doesn't, an error was made at some point, so try to find out what it is, or redo the question.

Example 3 – Solve the system of linear inequalities

$x \geq -3, \;\; y > -4, \;\; y > 2x - 4, \;\; x + 6y \leq 15$



Example 4 – Write the system of inequalities for the following solution set.



word problem

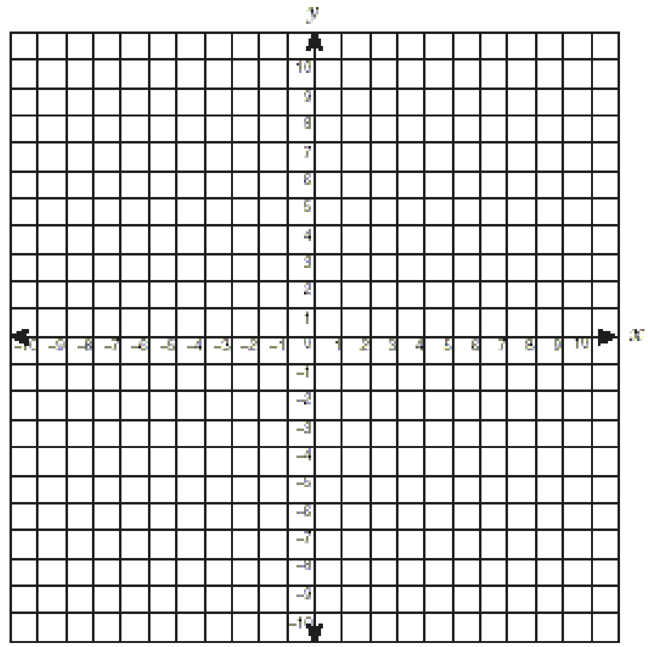
Example 5 –The Canucks have 8 games left to play and need 10 points to make the playoffs. A win is worth 2 points and an overtime loss is worth 1 point. Write and graph a system of linear inequalities to see all the possible ways the Canucks can make the playoffs.

Let x =

Let y =

inequalities:

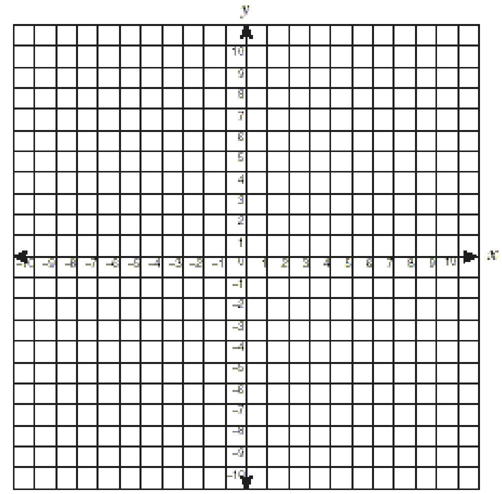
rearranged:



Ways to make the playoffs:

7.3 – Solving Quadratic Inequalities

In Grade 9, you learned to solve linear inequalities such as $x - 4 > 0$. One way to solve them was to isolate x, so in this case add 4 to both sides and you get: $x > 4$.
Another way to solve these is to use a graph:
$x - 4 > 0$



We wouldn't regularly solve a linear inequality using a graph, because it is much easier to just solve it algebraically, as we did at first.
However, to solve a quadratic inequality, the graphing method is often the preferred method, as it is quicker. However, we will learn both the graphing method and the algebraic method.

Example 1 – Solve $x^2 + 2x > 8$ **by graphing**, **and then using algebra (test intervals)**. Graph the solution on a number line.

Graphing Steps

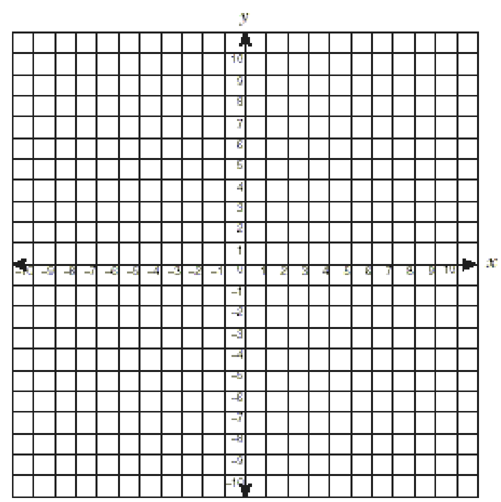
1. Get everything to the left side so that zero is on the right.
2. Find the roots (x-intercepts).
3. Sketch a graph and use the visual to solve the inequality.

→ if the quadratic is > 0, find the domain where the graph is above the x-axis
→ if the quadratic is < 0, find the domain where the graph is below the x-axis

1)

2)

3)



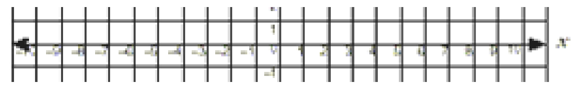
Solution: Solution on Number Line:

Algebraic (Test Interval) Steps

1. Find the critical numbers (the zeros) of the inequality.
2. Make an *x*-axis diagram of the resulting test intervals.
3. Test a value from each interval using the original inequality.

1)

2)



3)

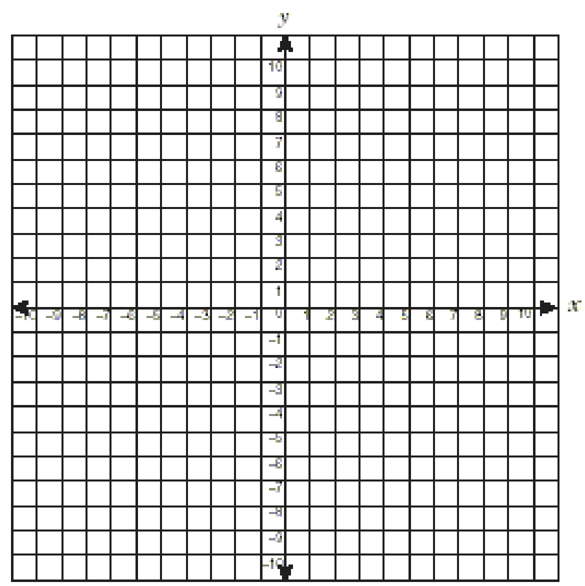
Solution:

Example 2 – Solve $x^2 - 10x + 16 \leq 0$ using both methods and graph the solution on a number line

*if the quadratic is ≥ 0, find the domain where the graph is **above or on** the x-axis

*if the quadratic is ≤ 0, find the domain where the graph is **below or on** the x-axis

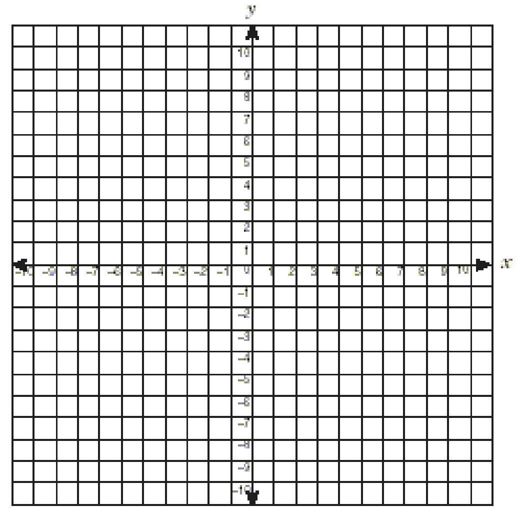
Graphing:



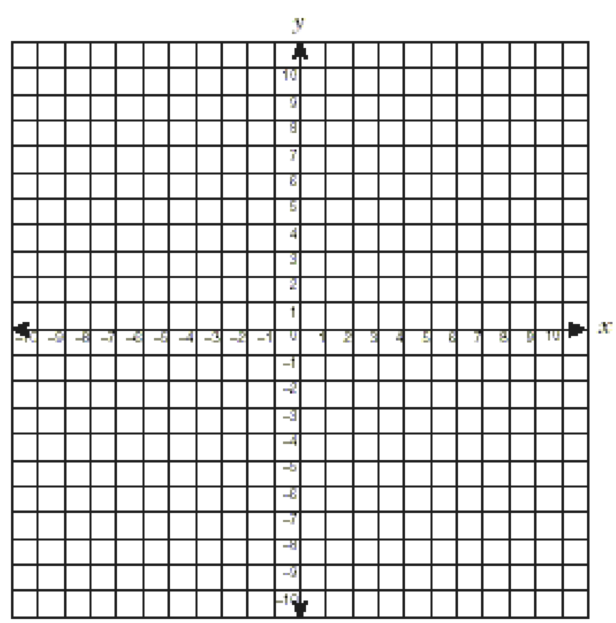
Solution: Solution on Number Line:

Test Intervals:

Example 3 – Graph the quadratic function $f(x) = x^2 - 6x + 9$. What is the solution to:
a) $x^2 - 6x + 9 \geq 0$ b) $x^2 - 6x + 9 > 0$ c) $x^2 - 6x + 9 \leq 0$ d) $x^2 - 6x + 9 < 0$



Example 4 – Solve $x^2 - 2x > 2$. Then graph the solution on a number line.

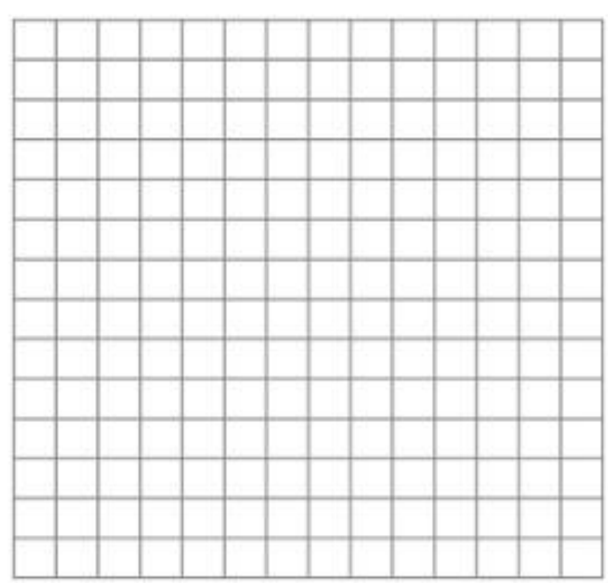


Solution:	Solution on Number Line:

7.5 – Applications of Systems & Systems of Inequalities

Example 1 – A certain website offers online interactive puzzles, but the puzzle-makers present the following problem for entry to their site. “Determine two integers such that the sum of the smaller number and twice the larger number is 46. Also, when the square of the smaller number is decreased by three times the larger, the result is 93. By determining the smaller and larger numbers, use it as a password to gain access to the site.

Example 2 – An automobile storage area can fit at most 100 cars & trucks on its lot. A car covers 100 sq feet and a truck 200sq ft of space on a lot that is 12 000 sq ft. What are all the possibilities of cars & trucks that can be on the lot at any one time?



Example 3 - The height in metres of a projectile shot from the top of a building is given by $h(t) = -16t^2 + 60t + 25$, where t represents the time in seconds the projectile is in the air.

a) Find the time the projectile is in the air before hitting the ground, to the nearest thousandth.
b) Find the time interval that the projectile is above 25m, to the nearest hundredth.

**Enrichment:* Example 4 – The price a stereo will be sold for is given by $S(x) = 200 - 0.1x$, $0 \leq x \leq 2000$, where x is the number of stereos produced each day. It costs $18 000 per day to operate the factory and $15 for material to produce each stereo.

a) Find the daily revenue. (b) Find the daily cost of producing stereos. (c) Find the interval that produces a profit.