5.1 – Representing Relations

Name: Notes Key
Date:

Goal: to discuss the concept of a relation and to represent relations in different ways

Toolkit:

– anything you know about "relations"
– " " " " graphs
– word math

Main Ideas:

Definitions:

Set – a set is a collection of distinct objects

Element – an element of a set is one object in the set

Relation – a relation associates the elements of one set with the elements of another set

There are many ways to represent a relationship between two sets. Be prepared to recognize these terms and match them to the different representations:

Words, Table, Diagram, Arrow Diagram, Bar Graph, Ordered Pairs, (Line Graph) Later

Ex1)

Ex1) When we talk about a Gulf Islands community, we may want to know on which island it is located.

Community	Gulf Island
Fulford Harbour	Salt Spring Island
Gillies Bay	Texada Island
Sturdies Bay	Galiano Island
Long Harbour	Salt Spring Island
Blubber Bay	Texada Island
Vesuvius	Salt Spring Island

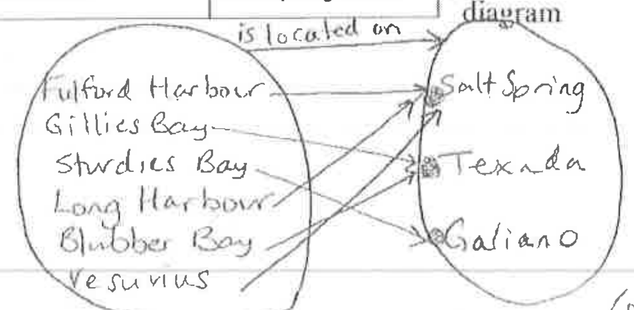
a) What type of relation is presented?

a table

b) Describe the relation in words

community "is located on" island.

c) Represent the relation as an arrow diagram



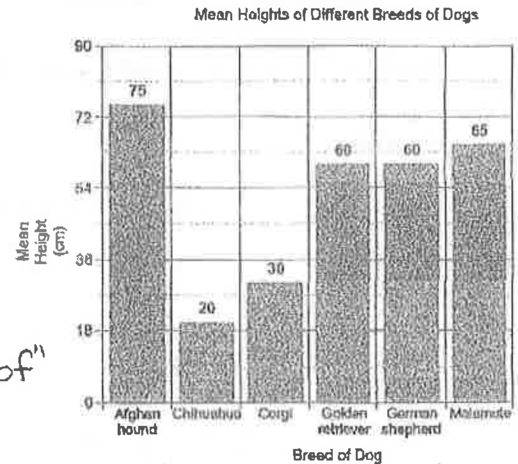
d) Represent the relation as a set of ordered pairs (first element, 2nd element)
community, island

(Fulf. H, SSpring), (Gillies B, Tex.), (Sturdies B, Gal.), (Long H, SSpring), (Blubber B, Tex.), (Vesuvius, SSpring).

Note: could this be made into a bar graph? No – need numbers somewhere.

Ex2)

Ex2) This bar graph shows the relationship between different dog breeds and their mean (average) heights.



Represent this relation

a) In words

b) As a table

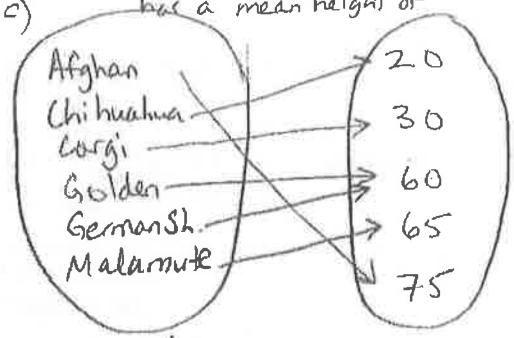
c) As an arrow diagram

a) ______ "has a mean height of" ______
dog ... height (cm)

b)

breed	Mean height
Afghan	75
Chihuahua	20
Corgi	30
Golden Ret.	60
German Sh.	60
Malamute	65

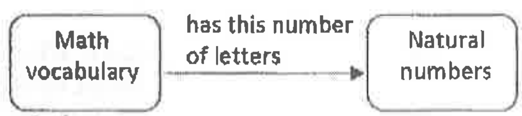
c) has a mean height of



*Note: arrow diagram - #s are smallest to largest; no repeats

Ex3)

Ex3) In the diagram,



a) Describe the relation in words

b) List 2 ordered pairs that belong to the relation

a) ______ "has this number of letters in it": ______
this vocab word ... #

b) (denominator, 11), (polynomial, 10)
etc.

Reflection: Which method of representing a relation makes the most sense to you? Why? List its advantages and disadvantages.

5.2A – Properties of Functions

Name:
Date:

Goal: to develop the concept of a function and to be able to recognize functions

Toolkit:	**Main Ideas:**

Definitions

Domain – The set of first elements of a relation is called the **domain**

Range – The set of second elements of a relation is called the **range**

Function – A **function** is a special type of relation where each element in the domain is associated with exactly one element in the range (OR a set of ordered pairs in which no two ordered pairs have the same first co-ordinate) * no repeats in the domain

Independent Variable – An **independent variable** is a variable whose value is not determined by the value of another variable

Dependent Variable – A **dependent variable** is a variable whose value is determined by the value of another (the independent) variable

Ex1)

Ex1) State the domain and range for each relation:

a)

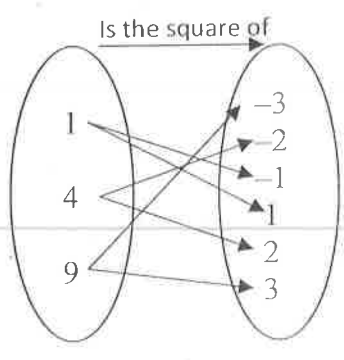
1st set Domain Ind Vars	2nd set Range Dep Vars
Animal	**# of legs**
Chicken	2
Dog	4
Cat	4
Spider	8
Ladybug	6
Eagle	2

D: {chicken, dog, cat, spider, ladybug, eagle}

R: {2, 4, 6, 8}

smallest to largest, no repeats

b)



D: {1, 4, 9}

R: {–3, –2, –1, 1, 2, 3}

c) { (–2, 4), (–1,1), (1, 1), (2,4), (3,9) }

D: {–2, –1, 1, 2, 3}

R: {1, 4, 9}

Functions

How do we determine whether a relation is also a **function**?

For a table of values or ordered pairs:

Yes, it's a function
*no repeats in domain

Animal	# of legs
Chicken	2
Dog	4
Cat	4
Spider	8
Ladybug	6
Eagle	2

Yes, a function!
*no repeats in domain!

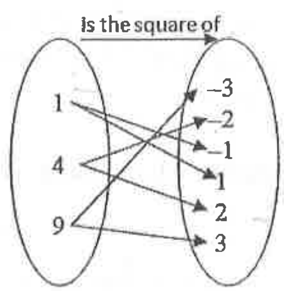
{ (–2, 4), (–1,1), (1, 1), (2,4), (3,9) }

repeat in domain!

{ (1, 1), (1,2), (3, 3), (3,4) }

No, not a function

For an arrow diagram:



Ordered Pairs
(1, –1)
(1, 1)
(4, –2)
(4, 2)
(9, –3)
(9, 3)

Not a function!

*If more than 1 arrow leaves from a domain element, there are repeats in domain, so NOT a function.

Ex2)

Ex2) Students are doing a "nickel drive" fund raiser. The amount of money they raise will **depend on** the number of nickels turned in.
a) label the domain/range, independent/dependent variables
b) is this relation a function, or not a function?

1st set domain ind var — 2nd set range dep var

Number of nickels, n	Amount raised , A ($)
0	0
50	2.50
100	5.00
150	7.50
200	10.00
Would this pattern continue?	

Yes

Yes, a function because a nickel is ALWAYS worth $0.05

Reflection: Complete the Frayer model on the next page.

5.2B – Function Notation

Name:
Date:

Goal: to define and work with function notation

Toolkit:

Main Ideas:

Functions

Ways to think about functions:
-rules
-formulas
-input/output machines

Input/output

INput - domain - independent → $\boxed{\text{function machine}}$ → OUTput range - dependent

A domain value goes IN, then the function machine changes it, and the (one and only) matching range value comes OUT.

Recall the "nickel drive" fund raiser. What does the machine do?
Account for: independent/dependent, domain/range, input/output, the variables

number of nickels → $\boxed{\times 0.05}$ → Amount of money

Function notation

Function notation shows us mathematically that the Amount of money raised (A) depends on (is a function of) the number of nickels (n) that come in.

$$A(n) = 0.05n$$

We say: 'A of n' is equal to 0.05n

A depends on n, OR A is a function of n

Ex1)

Ex1) Write the equation $y = 2x - 5$ in function notation.
label: independent/dependent, domain/range, input/output, the variables

dep, range, output → $y = 2x - 5$ ← ind, domain, input

y depends on x, so y is a function of x and we write
$f(x) = 2x - 5$

y and f(x) are the same thing!

Note: we can use letters other than f such as g, h, k

Note: we can work in the opposite direction by changing function notation back into the more familiar equations in 2 variables, e.g.
$g(x) = 3x + 4 \rightarrow y = 3x + 4$

Ex2)

"depends on"
equals
"is a function of"

Ex2) The equation $C = 23n + 550$ represents the cost (C) of a banquet where n people attend.

a) Describe the function The cost (C) depends on how many people attend (n). C is a function of n.

b) Write the function in function notation.

$C(n) = 23n + 550$

c) Find $C(100)$ = ___ and explain what this represents

"C of 100"

$C(n) = 23n + 550$

$C(100) = 23(100) + 550$

$C(100) = 2850$ $\quad$ ~~$C = 2850$~~

The cost for 100 people is \$2850.

d) Find n for $C(n) = 4000$ and explain what this represents

$C(n) = 23n + 550$

$4000 = 23n + 550$
$-550 \qquad -550$

$\frac{3450}{23} = \frac{23n}{23}$

$n = 150$

$C(150) = 4000$

For a cost of \$4000, 150 people can attend.

Ex3)

Ex3) For the function $f(x) = 3x - 4$

a) Write as a 2-variable equation

$y = 3x - 4$ same!

b) Determine the values of $f(6), f(4), f(-2)$

$f(x) = 3x - 4$	$f(4) = 3(4) - 4$	$f(-2) = 3(-2) - 4$
$f(6) = 3(6) - 4$	$f(4) = 8$	$f(-2) = -10$
$f(6) = 14$		
when x is 6, y is 14		

c) Determine the value of x for $f(x) = 2$ and for $f(x) = -1$

same as y

$f(x) = 3x - 4$

$2 = 3x - 4$
$+4 \qquad +4$

$\frac{6}{3} = \frac{3x}{3}$ $\quad 2 = x$

so $f(2) = 2$

$-1 = 3x - 4$

$3 = 3x$

$1 = x$

$f(1) = -1$

Reflection: For example 2 about the banquet, what values of n do not make sense as possible domain values? (Look back: what does n represent?)

5.3/5.4 – Interpreting and Drawing Graphs

Name:
Date:

Goal: to practice interpreting graphs and to practice drawing graphs (working back and forth between situations and their matching graphs)

Toolkit:

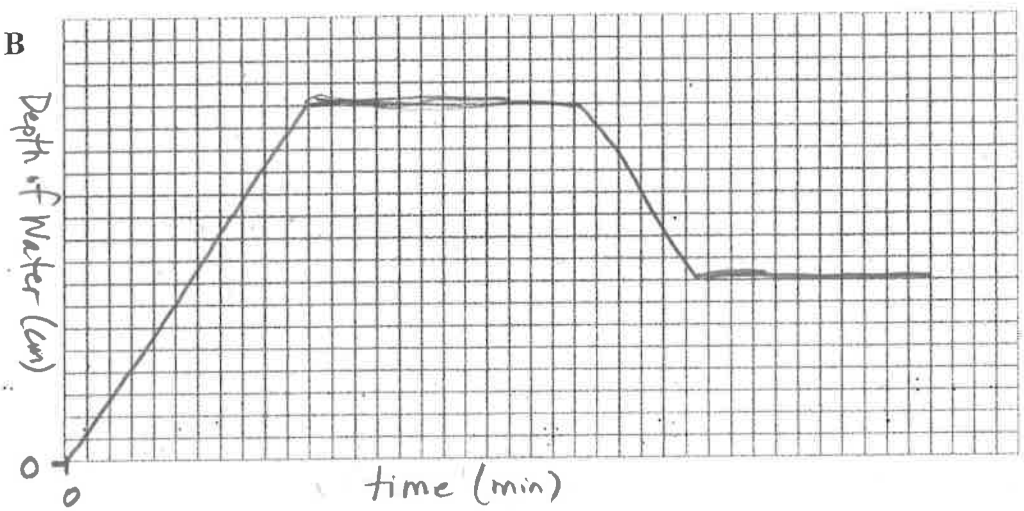
Main Ideas:

"Try This" p. 277

Work with a partner on the "Try This" on page 277

A

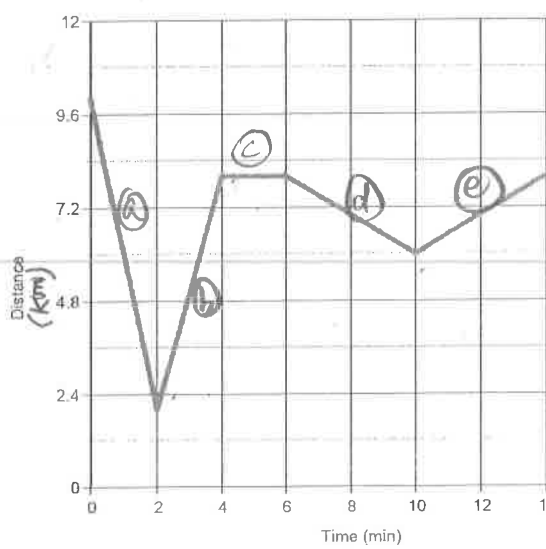
B



C

Ex1)

Ex1) Label key information on the following graph. When/how is it increasing? Decreasing?



distance = distance from home

a head toward home over 7km in 2 mins (driving?)

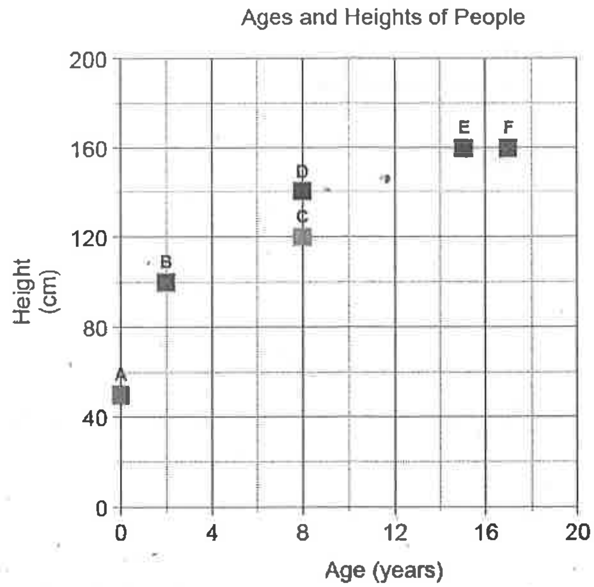
b head away from home ~5km in 2min

c stay still for 2mins

d head toward home slowly ~2km in 4 mins

e head away from home ~2km in 4 mins

Ex2) Interpret Graph	Ex2) Using the graph, EXPLAIN the answer to each question:



a) Who is the oldest? How old is s/he?

F, 17 yrs

b) Who is the youngest? How old is s/he?

A, new born

c) Who has the same height? What is that height?

E and F, 160 cm

d) Who has the same age? What is that age?

C and D, 8 yrs

e) Which person is taller for his/her age: person E or F?

E, younger

f) What are the coordinates (ordered pairs) for persons C and D?

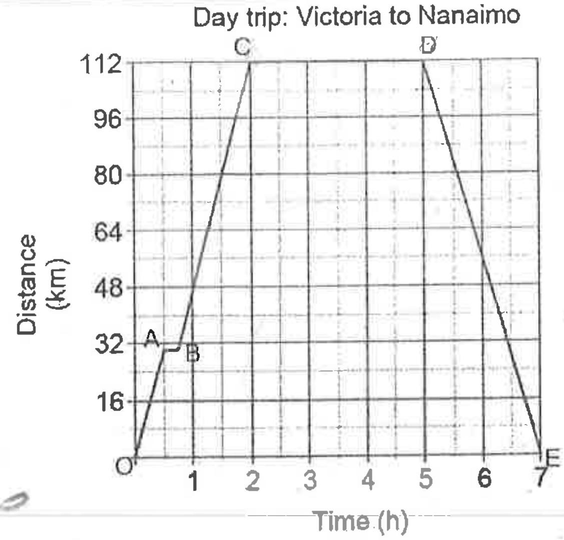
C(8,120) D(8,140)

g) Is this a function?

No, repeats in domain!

Ex3) Graph → Situation

Ex3) Use the graph to answer the following questions and to describe the journey for each segment of the graph.



a) How far is it from Victoria to Nanaimo? 112 km

b) Where do you start the day trip? End it?

start: Vic

End: Nanaimo

c) Which is the independent variable? Time

the dependent variable? distance

d) Fill in the following chart:

Segment	Graph	Journey
OA	The graph goes up to the right, so as time increases, the distance from Victoria increases.	
AB	The graph is ______________, so as time increases, the distance from Victoria ____________________.	
BC		The car travels approximately 80 km toward Nanaimo and ____________________.
CD		
DE	The graph goes down to the right, so as time increases, the distance ____________________.	The car takes 2 h to return to Victoria.

5.5A – Graphing Relations and Functions

Name:
Date:

Goal: to examine the properties of graphs of relations and graphs of functions

Toolkit:
- Discrete vs Continuous

Main Ideas:

Definitions

Function – a function has ordered pairs with different first coordinates (see VLT below)

Domain – the domain is the set of values of the independent variable (*x-axis*) [first element]

Range – the range is the set of values of the dependent variable (*y-axis*) [second element]

Discrete – (dots) The spaces between points on the graph have no literal meaning (e.g. you can't have 1.4 people)

Continuous – (connect the dots) The spaces between points have meaning (e.g. 1.4 seconds occurs between 1 second and 2 seconds, and something is happening then)

Warm-up

Warm-up: consider the relation that associates every natural number with its double

As a table of values:

domain range

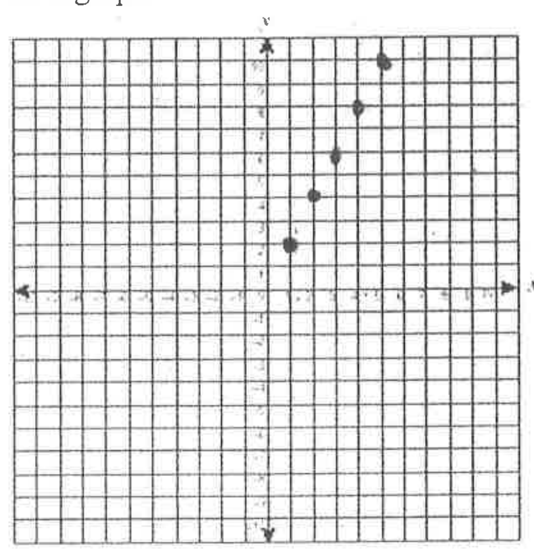
Natural number (x)	Double the number (y)
1	2
2	4
3	6
4	8
5	10

domain (x) range (y)

(1,2)
(2,4)
(3,6)
(4,8)
(5,10)

Discrete
so no line.

As a graph:



What is the domain value if the range value is 8?

4

As a formula:

$y = 2x$

Functions

Is the relation in the warm-up a FUNCTION? How can we tell?

Yes, no repeats in domain

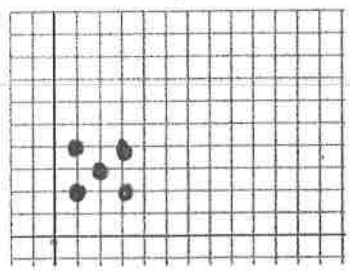
Vertical Line Test (VLT) - A graph represents a function when no two points on the graph lie on the same vertical line.

Non-functions

What if it is not a function? We can still call it a RELATION.

Graph the table of values

x	y
1	2
2	3
3	4
1	4
3	2

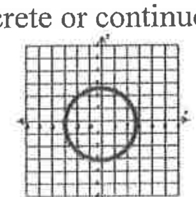


VLT: FAIL!

This is not a function!

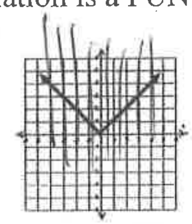
Ex1)

Ex1) State whether each relation is a FUNCTION (yes or no) and whether it is discrete or continuous.

a) 

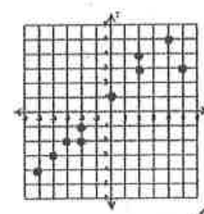
Function? Yes / No

Discrete / Continuous

b) 

Function? Yes / No

Discrete / Continuous

c) 

Function? Yes / No

Discrete / Continuous

Ex2)

Ex2) EXPLAIN whether the graph for each situation should be discrete or continuous.

a) The amount of money charged to your online music account is a function of the number of songs you download.

discrete; can't download a portion of a song

b) The amount of water in a bathtub is a function of time passing as it is filled, emptied, etc.

continuous; can find amount at any time you like.

Reflection: Return to your Frayer model from 5.2 and add anything you wish to. What are ALL the ways we have so far of recognizing a function?

5.5B – Domain and Range

Name:
Date:

Goal: to determine (and express mathematically) the domain and range of graphs and other relations

Toolkit: Inequality Signs

$>$ is greater than

$<$ is less than

$\geq$ is greater than or equal to

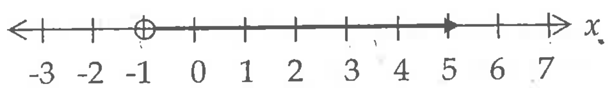
$\leq$ is less than or equal to

< is Like an L for Left/Less than/Lower than
> is the other one

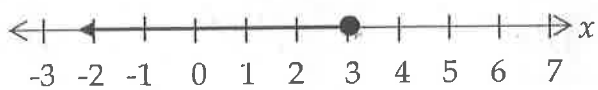
Main Ideas:

Review
Write an inequality that is represented by each graph.

o = point not included (> or <) • = point included (≥ or ≤)

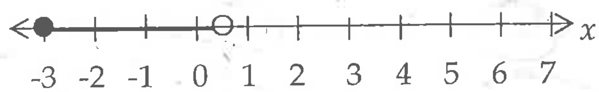


$x > -1$ $-1 < x$



$x \leq 3$

*New?



$-3 \leq x < 0.5$

$x \geq -3$
$x < 0.5$

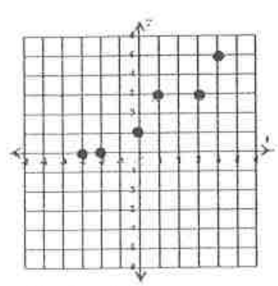
Domain and Range

The domain is the set of all x values (so we'll use the x-axis to help us)

The range is the set of all y values (so we'll use the y-axis to help us)

Ex1) State the domain and range for this relation.

Hint: For **discrete** graphs, list their coordinates (ordered pairs), then list all the first coordinates (x) for the domain, and second (y) for range, just like earlier in the chapter.



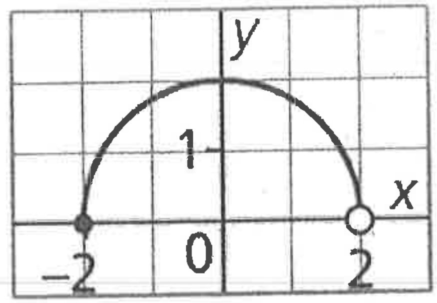
(-3, 0)
(-2, 0)
(0, 1)
(1, 3)
(3, 3)
(4, 5)

Domain: $\{-3, -2, 0, 1, 3, 4\}$

Range: $\{0, 1, 3, 5\}$

Ex2) For a **continuous** relation, we cannot describe every single x-value or y-value (there are infinitely many!).

Since we can't **list** ALL the domain values or ALL the range values, it helps to think about "minimum" and "maximum" values:



Writing domain and range as an **inequality**

Domain:
How far **left** does the graph go? (min) -2
How far **right**? (max) 2
Write the domain as an **inequality:**

$$-2 \le x < 2$$

Range:
How far **down** does the graph go? (min) 0
How far **up**? (max) 2
Write the range as an **inequality:**

$$0 \le y \le 2$$

There are 5 different ways to state domain and range:
We already did it one way above, as an **inequality**.

Writing domain and range other ways:
-in words
-on a number line
-interval notation
-set notation

Domain	Range
In words: All real numbers between -2 and 2, including -2 but not 2.	*In words:* All real numbers between 0 and 2, including 0 and 2.
Number Line: like the review on the last page 	*Number Line:* 
Interval Notation: included → $[-2, 2)$ ← not included; minimum, maximum	*Interval Notation:* $[0, 2]$
Set Notation: real numbers $\{x \mid -2 \le x < 2, x \in \mathbb{R}\}$	*Set Notation:* $\{y \mid 0 \le y \le 2, y \in \mathbb{R}\}$

such that

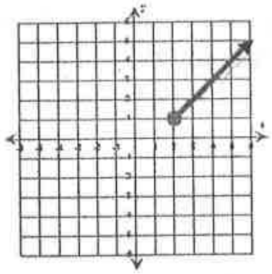
belongs to

~~$2 \le x \le \infty$~~

left?	right?	down?	up?
2	∞	1	∞

Ex3) State the domain and range for each relation

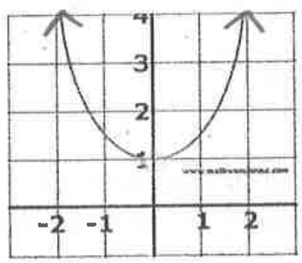
a)



	DOMAIN	RANGE
inequality:	$x \geq 2$	$y \geq 1$
interval:	$[2, \infty]$	$[1, \infty]$

line:

b)



left?	right?
$-\infty$	∞

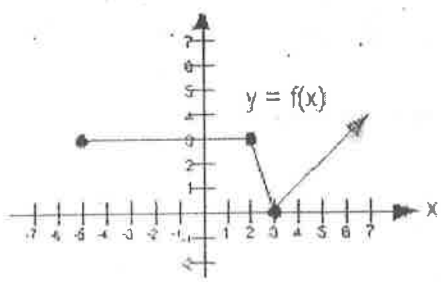
down?	up?
1	∞

inequality: $x \in \mathbb{R}$ (all values of x) $y \geq 1$

interval: $[-\infty, \infty]$ $[1, \infty]$

set: $\{x \mid x \in \mathbb{R}\}$ $\{y \mid y \geq 1, y \in \mathbb{R}\}$

c)



left?	right?
-5	∞

down?	up?
0	∞

	DOMAIN	RANGE
Words:	All real numbers greater than -5, including -5.	All real numbers greater than 0, including 0
Inequality:	$x \geq -5$	$y \geq 0$

Reflection: Describe in words how you would find the domain and range of a function using its graph. You may wish to use a specific graph to help explain.

5.6 – Properties of Linear Relations

Name:
Date:

Goal: to identify and represent linear relations in different ways

Toolkit:

- Independent Variable x, domain
- Dependent Variable y, range (depends on value of x)
- Constant = term that's just a number
- Reducing fractions
- Anything you remember about linear relations!

Main Ideas:

Warm-up

Warm-up: During a certain stretch of a road trip, you set your cruise control and start a timer (at zero) and reset your trip-meter to zero. Your friend watches to see how many kilometers you've gone after 30 minutes, one hour, an hour and a half, etc. and she keeps track of it in a table.

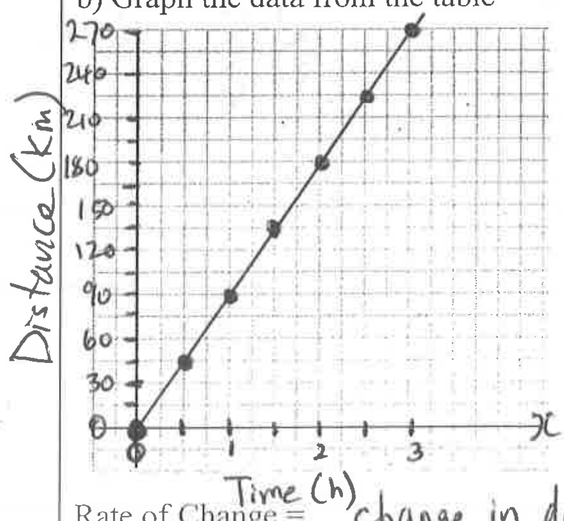
a) Identify the independent → time
and dependent → distance
variables

Time (h)	Distance (km)
0	0
0.5	45
1	90
1.5	135
2	180
2.5	225
3	270

+0.5 +0.5 +0.5

+45 +45 +45

b) Graph the data from the table



c) What do you notice about the pattern?
straight line

d) What is the rate of change? speed!
$90 \frac{km}{hr}$

Rate of Change = $\frac{\text{change in dependent variable } (y)}{\text{change in independent var } (x)} = \frac{90\text{km}}{1\text{ hr}}$
= speed!

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

(Hint: make sure to list independent variable (x) values in numerical order!)

Ex1) Recognizing a linear relation in **table** form

Ex1) Which tables of values represent linear relations? Identify the independent and dependent variables for each relation and IF LINEAR, find the rate of change.

a) Temperatures in Celsius (C) and Fahrenheit (F)

C (ind)	F (dep)
0	32
5	41
10	50
15	59

+5, +5, +5   +9, +9, +9

Yes, LINEAR

$R \text{ of } C = \frac{dep}{ind} = \frac{9}{5}$

b) Number of bacteria (n) growing on an old sandwich after t minutes

t (ind)	n (dep)
0	6
5	12
10	24
15	48
20	96

+5, +5, +5, +5   +6, +12, +24, +48

NOT LINEAR

c) The amount of HST (T for tax) charged on different purchases of Amount (A)

A (ind)	T (dep)
15	1.80
30	3.60
45	5.40
60	7.20
75	9.00
0	0

+15, +15, +15, +15   1.80, 1.80, 1.80, 1.80

Yes, LINEAR

$R \text{ of } C = \frac{dep}{ind} = \frac{1.80}{15} = \frac{0.12}{1}$

d) How else could we determine whether these tables of values represent linear relations?

graph it!

e) Below are the equations for each table of values. What do you notice about the equations of the *linear relations*? Relate the equation to what you know from the table of values.

a) $F = \frac{9}{5}C + 32$ b) $n = 6(2)^{\left(\frac{t}{5}\right)}$ c) $T = 0.12A + 0$

a) dep var; rate of change; ind var; constant (value when ind = 0)

b) exponent so non-linear

c) dep; RofC; ind var

$\text{dep var} = (\text{RofC})(\text{ind var}) + \text{constant}$

constant: it's the y value when $x = 0$

Ex2) Recognizing a linear relation in equation form

Ex2) Create a table of values for each equation, then graph it and decide whether it is a linear relation.

How many points do you NEED to tell whether a relation is linear? 3

a) $y = -2x + 8$

x	y
0	8
3	2
6	-4
9	-10

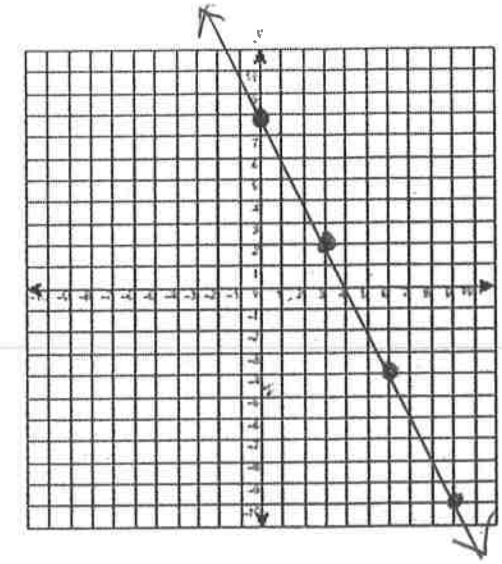
+3, +3; -6, -6

$y = -2(0) + 8$
$y = 8$

$y = -2(3) + 8$
$= -6 + 8$
$= 2$

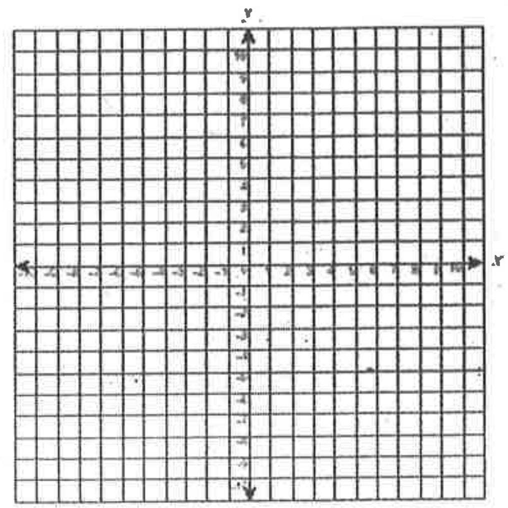
$y = -2(6) + 8$
$y = -12 + 8$
$= -4$

$R of C = \frac{-6}{3} = -2$



b) $y = 3x^2 - 3$

exponent; not linear

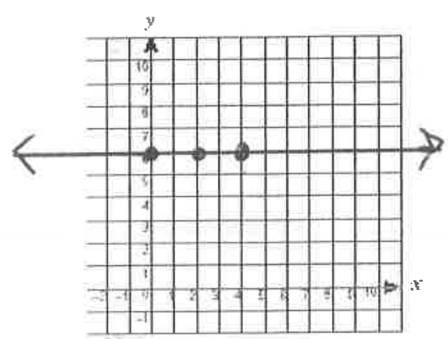


c) $y = 6$

x	y
0	6
2	6
4	6

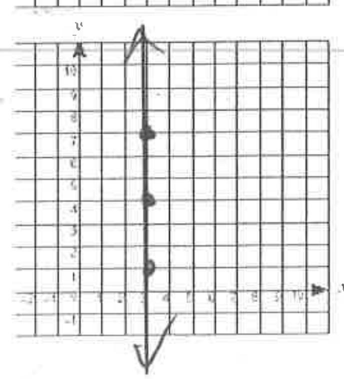
+2, +2; 0, 0

$R of C = \frac{0}{2} = 0$



d) $x = 3$

x	y
3	7
3	4
3	1



Ex3)

Ex3) Sort the equations we have seen so far by crossing out all NON-linear relations. How can we recognize linear relations **without** graphing?

$F = \frac{9}{5}C + 32$ $\qquad$ $\cancel{n = 6(2)^{\frac{t}{5}}}$ $\qquad$ $T = 0.12A + 0$

$y = -2x + 8$ $\qquad$ $\cancel{y = 3x^2 - 3}$ $\qquad$ $y = 6$ $\qquad$ $x = 3$

dep var = (R of C)(ind var) + constant

y value when x = 0

x = constant ← vert line

y = constant ← horiz line

Ex4)

Ex4) A banquet hall costs \$80 to rent, and it costs \$30 per person for catering. Write an equation to represent the total cost of the banquet (C) in relation to the number of people who attend (n).

$C = 30n + 80$

dep var $\quad$ r of c $\quad$ ind var $\quad$ constant $\Rightarrow$ linear!

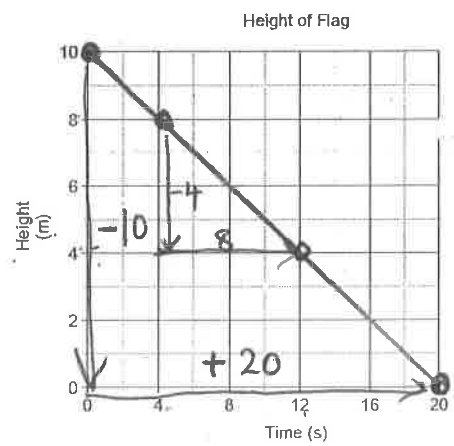
Ex5)

Ex5) Determine and explain the rate of change using the graph of the linear relation.

Step 1) Find the dependent (height) and independent variables (time)

Step 2) Find two EASY TO READ points

Step 3)
Find the change in height (y, dep. var.)
and the change in left/right (x, indep. var.)



R of C = $\frac{\Delta dep\ var}{\Delta indep\ var} = \frac{-10\ m}{20\ s}$

delta "change"

Step 4) Reduce the fraction and pay attention to units to help see what the rate represents.

R of C = $\frac{-1\,m}{2\,s}$ $\qquad$ $\frac{-4\,m}{8\,s} = \frac{-1\,m}{2\,s}$

Reflection: Compare (similarities and differences) how you find the rate of change for a table of values versus a graph.

5.7 – Interpreting Graphs of Linear Functions

Name:
Date:

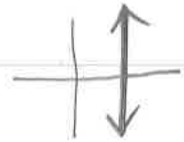
Goal: to use intercepts, rate of change, domain, and range to describe the graph of a linear function.

Toolkit:	**Main Ideas:**

Warm-up

Warm-up: True or False? A linear relation is always a linear function.

$x = \text{constant}$

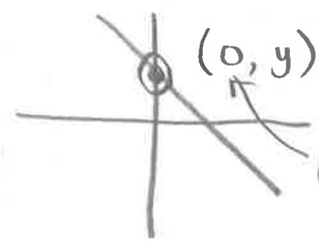


vertical lines are not functions

What is a horizontal intercept? (x-intercept)
it's where your graph crosses the x-axis

$(x, 0)$
always $y=0$

A vertical intercept? (y-intercept)
it's where your graph crosses the y-axis.

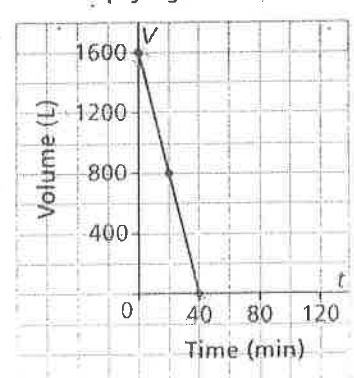


$(0, y)$
always $x=0$

Ex1) Determining features of a linear function's graph

Ex1) What are some of the key features of this graph? Intercepts

Emptying a Hot Tub



a) Write the coordinates of the points where the graph intersects the axes.
$(40, 0)$ $(0, 1600)$

b) Determine the vertical and horizontal intercepts.
y-int: $(0, 1600)$
x-int: $(40, 0)$

c) Describe what the points of intersection represent.
x-int: represents that it took 40 mins to empty tub
y-int: represents that the tub started with 1600L

d) What are the domain and range of this function?
D: $0 \leq t \leq 40$
R: $0 \leq V \leq 1600$

left right
0 40

down up
0 1600

e) What is the rate of change for this function?

$$R\text{ of }C = \frac{\Delta \text{dep}}{\Delta \text{ind}} = \frac{-1600\text{ L}}{40\text{ min}} = -40\text{ L/min}$$

Ex2) Sketching a graph using function notation and intercepts

Ex2) Sketch a graph of the linear function $f(x) = 2x - 4$

$y = 2x - 4$

Step 1: Determine the y-intercept

set $x = 0$

$f(0) = 2(0) - 4$

$f(0) = -4$

$(0, -4)$

Step 2: Determine the x-intercept

set $y = 0$

$0 = 2x - 4$

$+4 \quad\quad +4$

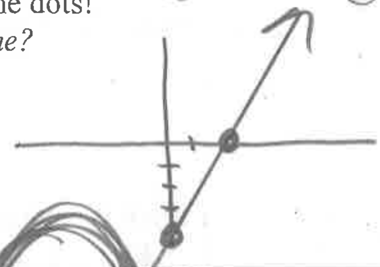
$4 = 2x$

$\frac{4}{2} = \frac{2x}{2}$

$2 = x$ $(2, 0)$

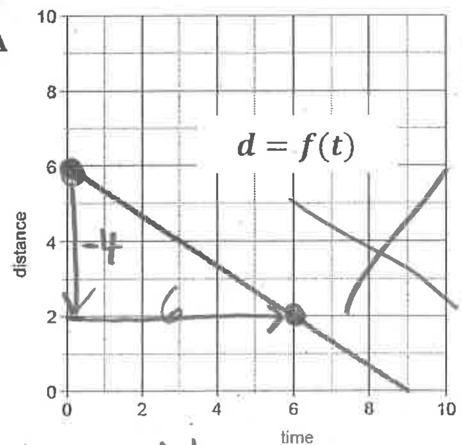
Step 3: Plot the intercepts and connect the dots!

How else could we have graphed this line?

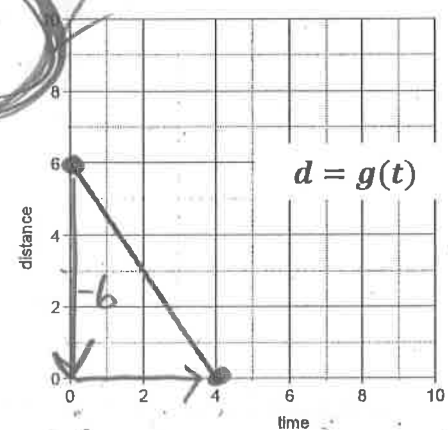


Ex 3) Which graph has a R of C of $\frac{-3}{2}$ and a y-intercept of 6?

A



B



r of C $= \frac{\Delta dep}{\Delta ind} = \frac{-4}{6} = \frac{-2}{3}$

$\frac{\Delta dep}{\Delta ind} = \frac{-6}{4} = \frac{-3}{2}$ ✓

a) Using the correct graph, what is the distance when time is 2?

3

b) Using the correct graph, what is the time when the distance is 1?

template: dep var = (r of c)(ind var) + constant

y value when $x = 0$ = y-intercept!

$d = \frac{-3}{2}t + 6$

$1 = \frac{-3}{2}t + \cancel{6}$

$-6 \quad\quad -6$

$-5 = \frac{-3t}{2}$

$-10 = -3t$

$t = \frac{10}{3} = 3.\overline{3}$

Reflection: Describe how you can tell from a graph whether a linear function has a positive or negative rate of change.