

5.1 – Representing Relations

Name: Notes Key  
Date:

**Goal:** to discuss the concept of a relation and to represent relations in different ways

<p><b>Toolkit:</b></p> <ul style="list-style-type: none"> <li>- anything you know about "relations"</li> <li>- " " " " graphs</li> <li>- word math</li> </ul>	<p><b>Main Ideas:</b></p>
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**Definitions:**

**Set** - a set is a collection of distinct objects

**Element** - an element of a set is one object in the set

**Relation** - a relation associates the elements of one set with the elements of another set

There are many ways to represent a relationship between two sets. Be prepared to recognize these terms and match them to the different representations:

Words, Table, Diagram, Arrow Diagram, Bar Graph, Ordered Pairs, <sup>(Line Graph)</sup> Later

Ex1)

When we talk about a Gulf Islands community, we may want to know on which island it is located.

Community	Gulf Island
Fulford Harbour	Salt Spring Island
Gillies Bay	Texada Island
Sturdies Bay	Galiano Island
Long Harbour	Salt Spring Island
Blubber Bay	Texada Island
Vesuvius	Salt Spring Island

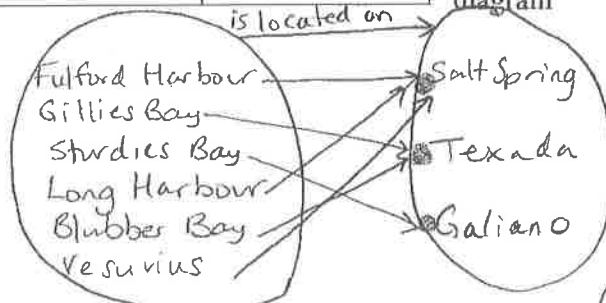
a) What type of relation is presented?

a table

b) Describe the relation in words

\_\_\_\_\_ "is located on"  
community \_\_\_\_\_ island

c) Represent the relation as an arrow diagram



d) Represent the relation as a set of ordered pairs

(community, island)  
(first element, 2nd element)

(Fulf. H., S Spring), (Gillies B., Tex.), (Sturdies B., Gal.), (Long H., S Spring),  
(Blubber B., Tex.), (Vesuvius, S Spring).

Note: could this be made into a bar graph? No - need numbers somewhere.

Ex2)

Ex2) This bar graph shows the relationship between different breeds and their mean (average) heights.

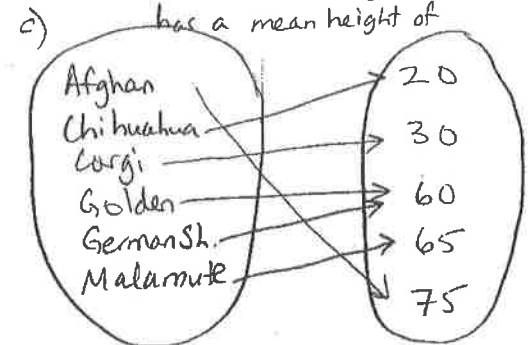
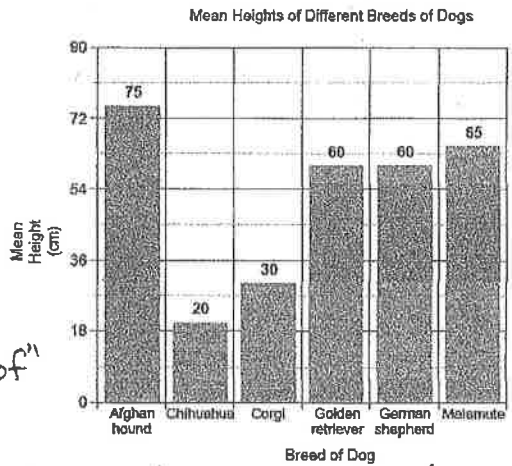
Represent this relation

- a) In words
- b) As a table
- c) As an arrow diagram

a) dog "has a mean height of"

height (cm)

breed	Mean height
Afghan	75
Chihuahua	20
Corgi	30
Golden Ret.	60
German Sh.	60
Malamute	65

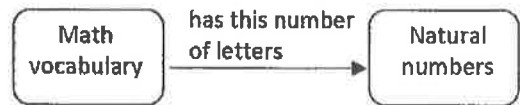


\* Note: arrow diagram - #s are smallest to largest; no repeats

Ex3)

Ex3) In the diagram,

- a) Describe the relation in words
- b) List 2 ordered pairs that belong to the relation



a) this vocab word "has this number of letters in it": #

b) (denominator, 11), (polynomial, 10)  
etc.

**Reflection:** Which method of representing a relation makes the most sense to you? Why? List its advantages and disadvantages.

5.2A – Properties of Functions & 5.3/5.4 – Interpreting Graphs

Name:

Date:

**Goal:** To develop the concept of a function and to be able to recognize functions. To practice interpreting graphs and drawing graphs (working between situations and their matching graphs)

**Toolkit:**

- understanding relations (5.1)
- arrow diagrams
- variables

**Main Ideas:**

**Definitions**

Domain – The set of first elements of a relation is called the **domain**

Range – The set of second elements of a relation is called the **range**

Function – A **function** is a special type of relation where each element in the domain is associated with exactly one element in the range (OR a set of ordered pairs in which no two ordered pairs have the same first co-ordinate)

Independent Variable – An **independent variable** is a variable whose value is not determined by the value of another variable

Dependent Variable – A **dependent variable** is a variable whose value is determined by the value of another (the independent) variable

Ex1)

Ex1) State the domain and range for each relation:

a) 1<sup>st</sup> set      2<sup>nd</sup> set

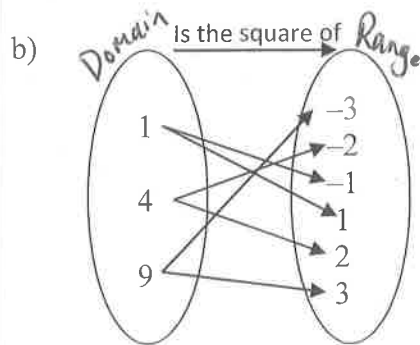
Domain      Range

Animal	# of legs
Chicken	2
Dog	4
Cat	4
Spider	8
Ladybug	6
Eagle	2

Domain: {chicken, dog, cat, spider, ladybug, eagle}

Range: {2, 4, 6, 8}

for numbers, smallest to largest, no repeats.



Domain: {1, 4, 9}

Range: {-3, -2, -1, 1, 2, 3}

Domain: {-2, -1, 1, 2, 3}

c)  $\{(-2, 4), (-1, 1), (1, 1), (2, 4), (3, 9)\}$  Range: {1, 4, 9}

Functions

How do we determine whether a relation is also a **function**?

For a table of values or ordered pairs: **NO REPEATS** in the domain

D	R
Animal	# of legs
Chicken	2
Dog	4
Cat	4
Spider	8
Ladybug	6
Eagle	2

{ (-2, 4), (-1,1), (1, 1), (2,4), (3,9) }

No repeats ⇒ **FUNCTION**

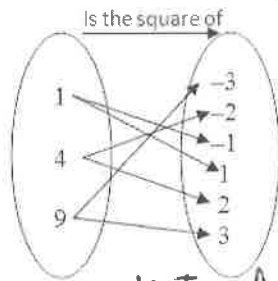
{ (1, 1), (1,2), (3, 3), (3,4) }

↑ ↑  
repeat

NOT a function

↑  
no repeats so it is a **FUNCTION**

For an arrow diagram:



Only one arrow can each entry on the left (each domain element)

NOT a function

Ex2)

Ex2) Students are doing a “nickel drive” fund raiser. The amount of money they raise will **depend on** the number of nickels turned in.

- label the domain/range, independent/dependent variables
- is this relation a function, or not a function?

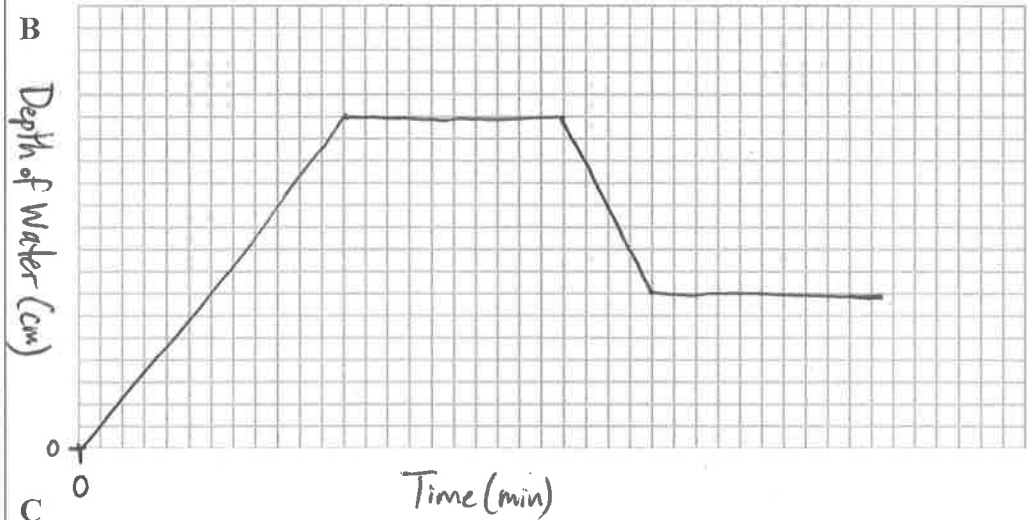
a) Independent Domain      Dependent Range

Number of nickels, $n$	Amount raised, $A$ (\$)
0	0
50	2.50
100	5.00
150	7.50
200	10.00
Would this pattern continue?	

b) It's a function as there are no repeats in the domain. A certain number of nickels can only give one specific \$ value.

A

B



C

Ex4)

Ex4) Use the graph to answer the following questions and to describe the journey for each segment of the graph.

a) How far is it from Victoria to Nanaimo?

112 km

b) Where do you start the day trip? End it?

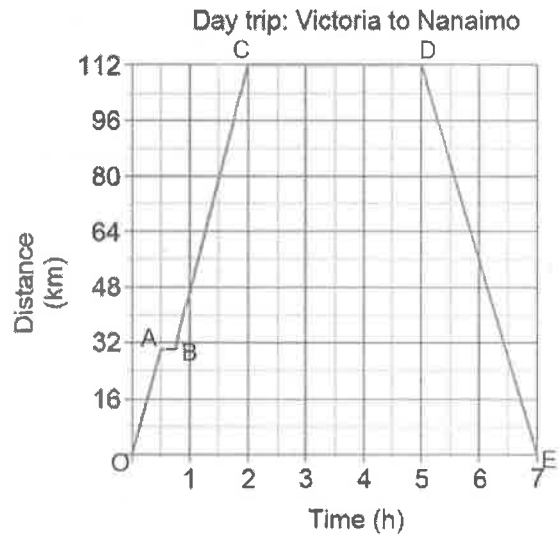
Start: Victoria

End: Victoria

c) Which is the independent variable?  
the dependent variable?

Independent is on x axis : Time

Dependent on y axis : Distance



Ex2) Interpret Graph

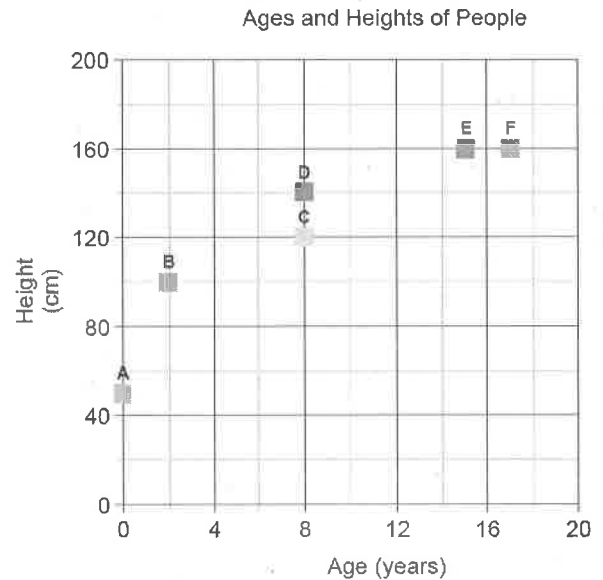
Ex2) Using the graph, EXPLAIN the answer to each question:

a) Who is the oldest? How old is s/he?

F ; 17

b) Who is the youngest? How old is s/he?

A ; newborn



c) Who has the same height? What is that height?

E & F ; 160cm

d) Who has the same age? What is that age?

D & C ; 8 yrs.

e) Which person is taller for his/her age: person E or F?

E is taller for his/her age  
(same height but younger)

f) What are the coordinates (ordered pairs) for persons C and D?

C(8, 120)      D(8, 140)

g) Is this a function?

No, C and D have a repeat in the domain

5.2B – Function Notation

Name: Notes Key  
Date:

Goal: to define and work with function notation

**Toolkit:**

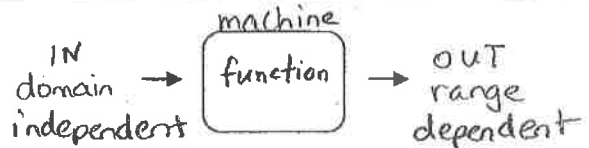
- functions
- substitution
- solving for a variable (rearranging)

**Main Ideas:**

Input/output

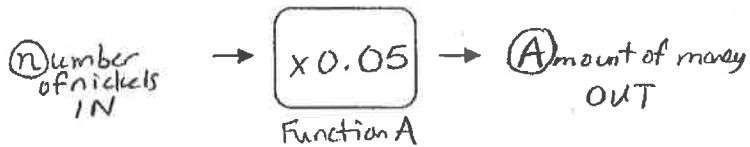
Ways to think about functions:

- rules
- formulas
- input/output machines



A domain value goes IN, then the function machine changes it, and the (one and only) matching range value comes OUT.

Recall the “nickel drive” fund raiser. What does the machine do?  $\times 0.05$   
Account for: independent/dependent, domain/range, input/output, the variables



Function notation

Function notation shows us mathematically that the Amount of money raised (A) depends on (is a function of) the number of nickels (n) that come in.

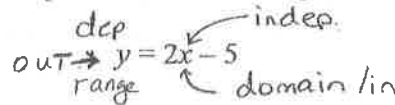
$$A(n) = 0.05n$$

We say: “A of n is equal to  $0.05n$ ”

Ex1)

Ex1) Write the equation  $y = 2x - 5$  in function notation.

label: independent/dependent, domain/range, input/output, the variables



y depends on x, so y is a function of x and we write

$$f(x) = 2x - 5$$

$f(x)$  is like  $y$

Note: we can use letters other than  $f$  such as  $g, h, k$   $g(x) = 2x - 5$ , etc.

Note: we can work in the opposite direction by changing function notation back into the more familiar equations in 2 variables, e.g.

$$g(x) = 3x + 4 \rightarrow y = 3x + 4$$

Ex2)

Ex2) The equation  $C = 23n + 550$  represents the cost ( $C$ ) of a banquet where  $n$  people attend.

- a) Describe the function The cost ( $C$ ) depends on the number of people who come ( $n$ )  $C$  is a function of  $n$ .
- b) Write the function in function notation.

$$C(n) = 23n + 550$$

- c) Find  $C(100) = \underline{\quad}$  and explain what this represents Sub in  $n=100$ .

$$C(n) = 23n + 550$$

$$C(100) = 23(100) + 550$$

$$C(100) = 2300 + 550$$

$$C(100) = 2850$$

"When  $n=100$  (100 people)  $C = 2850$  (cost is \$2850)."

- d) Find  $n$  for  $C(n) = 4000$  and explain what this represents replace  $C(n)$  with 4000

$$C(n) = 23n + 550$$

$$4000 = 23n + 550$$

$$\begin{array}{r} 4000 \\ -550 \\ \hline 3450 \end{array} = \frac{23n}{23}$$

Solve for  $n$

$$n = 150$$

"When the cost is 4000 ( $C=4000$ ) there were 150 people ( $n=150$ )"

Ex3)

Ex3) For the function  $f(x) = 3x - 4$

- a) Write as a 2-variable equation

$$y = 3x - 4$$

- b) Determine the values of  $f(6)$ ,  $f(4)$ ,  $f(-2)$  (sub in  $x = \dots$ )

$$f(x) = 3x - 4$$

$$f(6) = 3(6) - 4$$

$$f(6) = 18 - 4$$

$$f(6) = 14$$

$$x=6, y=14$$

$$f(x) = 3x - 4$$

$$f(4) = 3(4) - 4$$

$$f(4) = 12 - 4$$

$$f(4) = 8$$

$$x=4, y=8$$

$$f(x) = 3x - 4$$

$$f(-2) = 3(-2) - 4$$

$$f(-2) = -6 - 4$$

$$f(-2) = -10$$

$$x=-2, y=-10$$

- c) Determine the value of  $x$  for  $f(x) = 2$  and for  $f(x) = -1$  (sub in  $y = \dots$ )

$$f(x) = 3x - 4$$

$$2 = 3x - 4$$

$$\begin{array}{r} 2 \\ +4 \\ \hline 6 \end{array} = \frac{3x}{3}$$

$$\frac{6}{3} = \frac{3x}{3}$$

$$2 = x$$

$$x=2, y=2$$

$$f(x) = 3x - 4$$

$$-1 = 3x - 4$$

$$\begin{array}{r} -1 \\ +4 \\ \hline 3 \end{array} = \frac{3x}{3}$$

$$\frac{3}{3} = \frac{3x}{3}$$

$$1 = x$$

$$x=1, y=-1$$

**Reflection:** For example 2 about the banquet, what values of  $n$  do not make sense as possible domain values? (Look back: what does  $n$  represent?)

You can't have negative  $n$  (negative # of people) or decimals/fractions of people.  $n$  must be a whole number (0, 1, 2, ...).



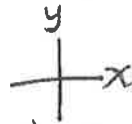
## 5.5A – Graphing Relations and Functions

Name: Notes key  
Date:

**Goal:** to examine the properties of graphs of relations and graphs of functions

### Toolkit:

- Discrete vs Continuous
- graphing  $(x, y)$  on the coordinate plane
- functions, domain, range



### Main Ideas:

### Definitions

**Function** – a function has ordered pairs with different first coordinates (see VLT below)

**Domain** – the domain is the set of values of the independent variable ( $x$ -axis) [first element]

**Range** – the range is the set of values of the dependent variable ( $y$ -axis) [second element]

**Discrete** – (dots) The spaces between points on the graph have no literal meaning (e.g. you can't have 1.4 people)

**Continuous** – (connect the dots) The spaces between points have meaning (e.g. 1.4 seconds occurs between 1 second and 2 seconds, and something is happening then)

### Warm-up

Warm-up: consider the relation that associates every natural number with its double

As a table of values:

Indep. Domain	dep. Range	
Natural number ( $x$ )	Double the number ( $y$ )	$(x, y)$
1	2	$(1, 2)$
2	4	$(2, 4)$
3	6	$(3, 6)$
4	8	$(4, 8)$
5	10	$(5, 10)$
etc.	etc.	

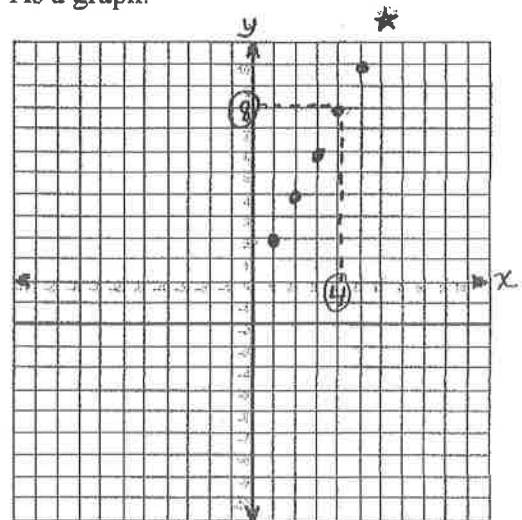
$(1, 2)$  start at  $(0, 0)$   
over 1 right, up 2  
domain value is 1, range value is 2

As a formula :

$$y = 2 \times x$$

$$y = 2x$$

As a graph:




What is the domain value if the range value is 8?

$$x = 4, y = 8$$

$$\text{check: } 4 \times 2 = 8 \quad \checkmark$$

Functions

Is the relation in the warm-up a FUNCTION? How can we tell?

Yes - no repeated  $x$ -values (domains) and 

Vertical Line Test - (VLT) - A graph represents a function when no two points on the graph lie on the same vertical line.

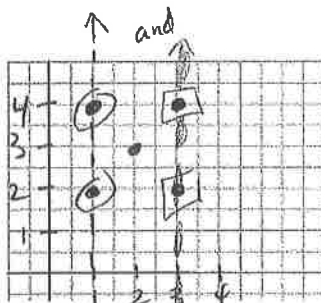
[Run a ruler vertically along graph: should only ever touch once!]

Non-functions

What if it is not a function? We can still call it a relation

Graph the table of values

$x$	$y$
1	2
2	3
3	4
1	4
3	2



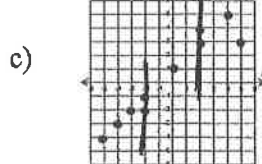
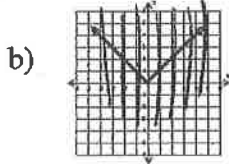
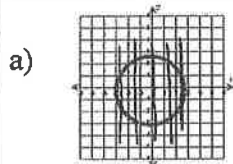
VLT: Fails VLT because the graph touches the same vertical line more than once!

repeated  $x$ 's (not a fn.)

Not a fn.

Ex1)

Ex1) State whether each relation is a FUNCTION (yes or no) and whether it is discrete or continuous.



Function? Yes No

Function? Yes No

Function? Yes No

Discrete / Continuous  
connected

Discrete / Continuous  
connected

Discrete / Continuous  
dots

Ex2)

Ex2) EXPLAIN whether the graph for each situation should be discrete or continuous.

a) The amount of money charged to your online music account is a function of the number of songs you download.

cannot buy 2.7 songs, or 1.45 songs.  
Discrete!

b) The amount of water in a bathtub is a function of time passing as it is filled, emptied, etc.

time continues (0.2 seconds, 0.3 seconds, 0.4, ...) with no breaks  
continuous!

**Reflection:** Return to your Frayer model from 5.2 and add anything you wish to. What are ALL the ways we have so far of recognizing a function?

- no repeated  $x$ 's (domain values / first elements)
- only one arrow leaving each element in first bubble of arrow diagram
- passes VLT (graph touches any vertical line only once)

5.5B – Domain and Range

Name: Key  
Date:

Goal: to determine (and express mathematically) the domain and range of graphs and other relations

**Toolkit: Inequality Signs**

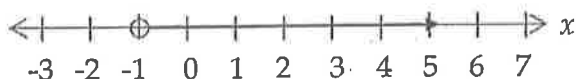
- > is greater than
- < is less than
- ≥ is greater than or equal to
- ≤ is less than or equal to
- < is Like an L for Left/Less than/Lower than
- > is the other one

**Main Ideas:**

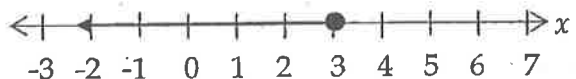
**Review**

Write an inequality that is represented by each graph.

○ = point not included (> or <)    ● = point included (≥ or ≤)



$x > -1$



$x \leq 3$

\*New?



$-3 \leq x < 0.5$

↑  
between two numbers  
always this pattern: smaller # ≤ x < # bigger #

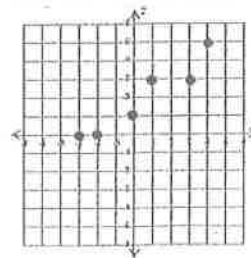
**Domain and Range**

The domain is the set of all x values (so we'll use the x-axis to help us)

The range is the set of all y values (so we'll use the y-axis to help us)

Ex1) State the domain and range for this relation.

**Hint:** For **discrete** graphs, list their coordinates (ordered pairs), then list all the first coordinates (x) for the domain, and second (y) for range, just like earlier in the chapter. (-3,0) (-2,0) (0,1) (1,3) (3,3) (4,5)



Domain:  $\{-3, -2, 0, 1, 3, 4\}$

Range:  $\{0, 1, 3, 5\}$

- \* - smallest to largest!
- \* - no repeats!

Ex2) For a **continuous** relation, we cannot describe every single x-value or y-value (there are infinitely many!).

Since we can't list ALL the domain values or ALL the range values, it helps to think about "minimum" and "maximum" values:

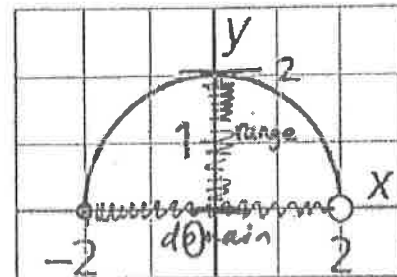
**Domain:**

How far **left** does the graph go? (min) -2

How far **right**? (max) 2

Write the domain as an **inequality**:

$$-2 \leq x < 2$$



**Range:**

How far **down** does the graph go? (min) 0

How far **up**? (max) 2

Write the range as an **inequality**: ← range uses 'y'

$$0 \leq y \leq 2$$

There are 5 different ways to state domain and range:

We already did it one way above, as an inequality.

Writing domain and range as an **inequality**

↑  
most common way to give domain & range for a continuous relation

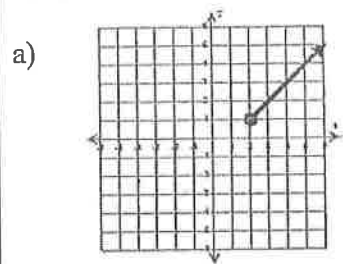
Writing domain and range other ways:

- in words
- on a number line
- interval notation
- set notation

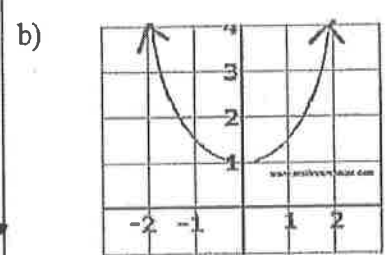
Domain	Range
<p><i>In words:</i> All real numbers between -2 and 2, including -2 but not 2.</p>	<p><i>In words:</i> All real numbers between 0 and 2, including 0 and 2.</p>
<p><i>Number Line:</i> like the review on the last page</p>	<p><i>Number Line:</i></p>
<p><i>Interval Notation:</i></p> <p><math>[-2, 2)</math> includes → x doesn't include ←</p>	<p><i>Interval Notation:</i></p> <p><math>[0, 2]</math></p>
<p><i>Set Notation:</i></p> <p><math>\{x \mid -2 \leq x &lt; 2, x \in \mathbb{R}\}</math></p> <p>↑ such that      ↑ belongs to      ↑ real numbers</p>	<p><i>Set Notation:</i></p> <p><math>\{y \mid 0 \leq y \leq 2, y \in \mathbb{R}\}</math></p> <p>↑ such that      ↑ belongs to      ↑ real numbers</p>

Ex3) State the domain and range for each relation

how far left: 2      how far down: 1  
 how far right:  $\infty$       how far up:  $\infty$

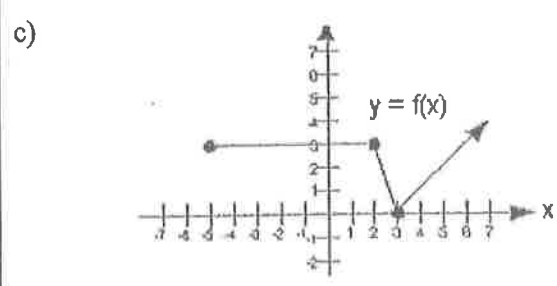


	<u>DOMAIN</u>	<u>RANGE</u>
inequality:	$x \geq 2$	$y \geq 1$
interval:	$[2, \infty]$	$[1, \infty]$
# line:		



inequality:	all real values of $x$	$y \geq 1$
interval:	$[-\infty, \infty]$	$[1, \infty]$
set:	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y \geq 1, y \in \mathbb{R}\}$

left?  $-\infty$       right?  $\infty$   
 down? 1      up?  $\infty$



left? -5      right?  $\infty$   
 down? 0      up?  $\infty$

	<u>DOMAIN</u>	<u>RANGE</u>
Words:	All real numbers greater than or equal to -5	all real numbers greater than or equal to 0
Inequality:	$x \geq -5$	$y \geq 0$

**Reflection:** Describe in words how you would find the domain and range of a function using its graph. You may wish to use a specific graph to help explain.

## 5.6 – Properties of Linear Relations

Name: *Notes Key*  
Date:

**Goal:** to identify and represent linear relations in different ways

### Toolkit:

- Independent Variable changes (x)
- Dependent Variable relies on indep. var. (y)
- Constant = an unchanging number (not a variable)
- Reducing fractions
- Anything you remember about linear relations!

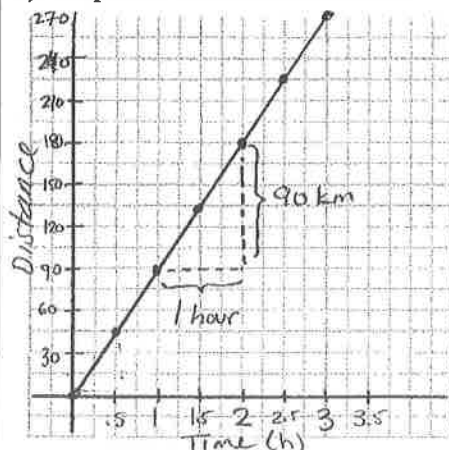
### Main Ideas:

### Warm-up

Warm-up: During a certain stretch of a road trip, you set your cruise control and start a timer (at zero) and reset your trip-meter to zero. Your friend watches to see how many kilometers you've gone after 30 minutes, one hour, an hour and a half, etc. and she keeps track of it in a table.

a) Identify the independent  $\rightarrow$  time and dependent  $\rightarrow$  distance variables

b) Graph the data from the table



Time (h)	Distance (km)
0	0
0.5	45
1	90
1.5	135
2	180
2.5	225
3	270

+0.5 ( )  
+0.5 ( )  
+0.5 ( )

) +45  
) +45  
) +45

c) What do you notice about the pattern?

It makes a line.

d) What is the rate of change?

$$\text{Rate of Change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{90 \text{ km}}{1 \text{ h}}$$

(You go 90 km in 1 hour, or 90 km per hour)

$\Rightarrow 90 \text{ km/h} \Rightarrow \text{speed!}$

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

(Hint: make sure to list independent variable (x) values in numerical order!)

Ex1) Recognizing a linear relation in table form

Ex1) Which tables of values represent linear relations? Identify the independent and dependent variables for each relation and IF LINEAR, find the rate of change.

$$\text{rate} = \frac{\text{change in DEP.}}{\text{change in INDEP.}}$$

a) Temperatures in Celsius (C) and Fahrenheit (F)

	indep.	dep.	
	C	F	
Constant change } +5	0	32	} +9 } constant change
	5	41	
	10	50	
	15	59	

rate of change =  $\frac{9}{5}$

✓ linear

b) Number of bacteria (n) growing on an old sandwich after t minutes

	indep.	dep.	
	t	n	
Constant change } +5	0	6	} +6 } dep. var. keeps changing
	5	12	
	10	24	
	15	48	
+5	20	96	+48

X nonlinear

c) The amount of HST (T for tax) charged on different purchases of Amount (A)

	A	T	
Constant change } +15	15	1.80	} +1.80 } constant change
	30	3.60	
	45	5.40	
	60	7.20	
+15	75	9.00	+1.80

rate of change =  $\frac{1.80}{15} \div = 0.12$

✓ linear

d) How else could we determine whether these tables of values represent linear relations?

Graphing! Plot the values: do they form a line?

e) Below are the equations for each table of values. What do you notice about the equations of the linear relations? Relate the equation to what you know from the table of values.

a)  $F = \frac{9}{5}C + 32$

rate of change!  
first dep. var. in table

b)  $n = 2^t$

Not linear exponents

c)  $T = 0.12A + 0$

rate of change!

$$\text{letter} = \# \times \text{letter} + \#$$

$$\text{dep. var} = \text{rate of change (indep. var)} + \text{constant (from table - dep. paired with 0)}$$

Ex2) Recognizing a linear relation in equation form

Ex2) Create a table of values for each equation, then graph it and decide whether it is a linear relation.

2 not enough (always makes a line)

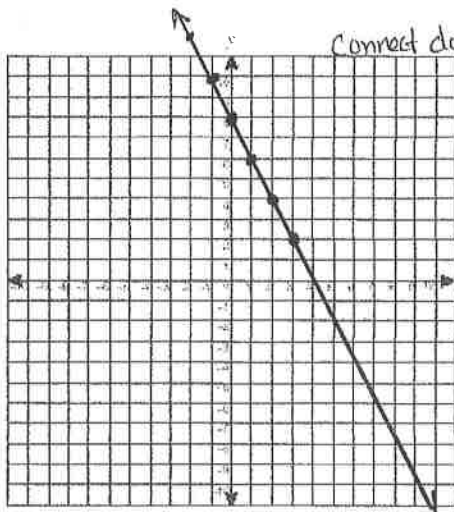
How many points do you NEED to tell whether a relation is linear? 3 (minimum)

a)  $y = -2x + 8$  Notice\*

x	y = f(x) = -2x + 8
-2	12
-1	10
0	8
1	6
2	4
3	2

Indep. →  
You choose  
(pick some\*)

✓ Linear



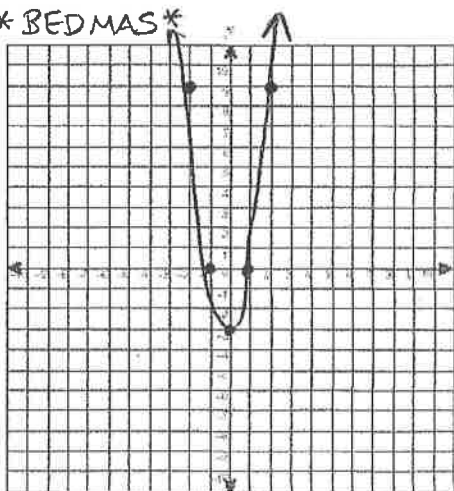
connect dots (why?)

we could've picked any x.  
x = 0.2  
x = 1.4  
etc.)

b)  $y = 3x^2 - 3$

x	y = f(x) = 3(x) <sup>2</sup> - 3
-2	9
-1	0
0	-3
1	0
2	9

\*BEDMAS\*



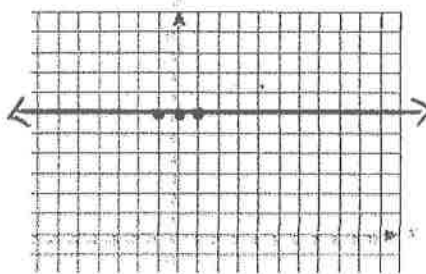
✗ non-linear

c)  $y = 6$

x	y
-1	6
0	6
1	6

horizontal line  
at a height of 6  
 $y = 6$

✓ Linear

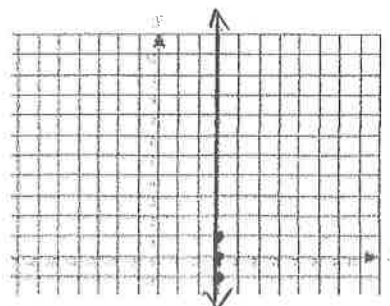


d)  $x = 3$

x	y
3	-1
3	0
3	1

vertical line  
along x-axis at 3  
 $x = 3$

✓ Linear





Ex3)

Ex3) Sort the equations we have seen so far by crossing out all NON-linear relations. How can we recognize linear relations **without** graphing?

$F = \frac{9}{5}C + 32$        $n = \cancel{6(2)^t}$        $T = 0.12A + 0$   
 $y = -2x + 8$        $y = 3x^2 - 3$        $y = 6$        $x = 3$

e.g.  $F = \frac{9}{5}C + 32$   
 Dep. Var.  $\uparrow$  rate of change  $\times$  Indep. Var.  $\uparrow$  initial value  $+ 32$   
 (no exponents, weird operations!)  
 $y = \text{constant}$        $x = \text{constant}$   
 hori z. line at height = constant      vertical line at left/right location of the constant

Ex4)

Ex4) A banquet hall costs \$80 to rent, and it costs \$30 per person for catering. Write an equation to represent the total cost of the banquet (C) in relation to the number of people who attend (n).  
initial value      rate  
indep. var      dep. var

$$C = 30n + 80$$

as n (# people) goes up by 1, cost goes up by 30.

80 is initial cost.

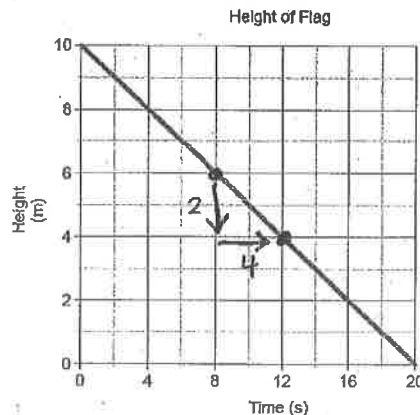
Ex5)

Ex5) Determine and explain the rate of change using the graph of the linear relation.  
 Step 1) Find the dependent (height) and independent variables (time)

Step 2) Find two EASY TO READ points

Step 3) Find the change in height (y, dep. var.) and the change in left/right (x, indep. var.)

$$\frac{\Delta \text{dep var}}{\Delta \text{indep var}} = \frac{\text{down } 2}{\text{right } 4} = -\frac{2}{4} \text{ m}$$



Step 4) Reduce the fraction and pay attention to units to help see what the rate represents.

$$-\frac{2}{4} \div 2 = -\frac{1}{2} \text{ m/s} \quad \text{the flag drops at } 0.5 \text{ metres per second. (speed!)}$$

**Reflection:** Compare (similarities and differences) how you find the rate of change for a table of values versus a graph.

rate is still  $\frac{\text{change in (dep)}}{\text{change in (indep)}}$ , but in table you might do  $\frac{\Delta \text{right column}}{\Delta \text{left column}}$  (count up/down)

on graph:  $\frac{\Delta \text{y-axis}}{\Delta \text{x-axis}}$  (count up/down and over)

# 5.7 – Interpreting Graphs of Linear Functions

Name: Notes Key

Date:

**Goal:** to use intercepts, rate of change, domain, and range to describe the graph of a linear function.

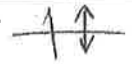
**Toolkit:**

- linear relations so far
- finding rates of change
- substitution & solving

**Main Ideas:**

**Warm-up**

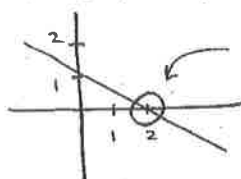
Warm-up: True or False? A linear relation is always a linear function.



False: vertical lines ( $x = \text{constant}$ ) are linear relations but fail the vertical line test ( $\therefore$  not functions)

What is a horizontal intercept?

Where graph crosses horizontal axis (often x-axis)

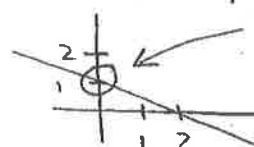


hor. int at 2  
 $(2, 0)$

or other dep. var.  
occurs when  $y = 0$

A vertical intercept? \* also "initial value"!

Where graph crosses vertical axis (often y-axis)



vert. int. at 1  
 $(0, 1)$

or other indep. var.  
occurs when  $x = 0$

Ex1) Determining features of a linear function's graph

Ex1) What are some of the key features of this graph?

a) Write the coordinates of the points where the graph intersects the axes.

$(0, 1600)$        $(40, 0)$

b) Determine the vertical and horizontal intercepts.

vert. int. at 1600

hor. int. at 40

c) Describe what the points of intersection represent. Here, ...  
vert. int. means the Volume of the tub when  $t = 0$  (at start)  
hor. int. is time when Volume is zero (tub empty).

d) What are the domain and range of this function?

D:  $0 \leq t \leq 40$

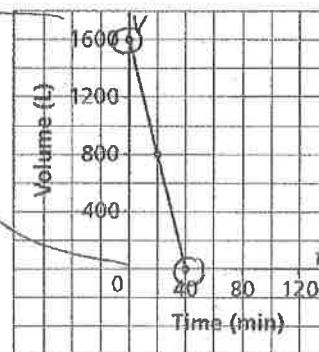
R:  $0 \leq V \leq 1600$

e) What is the rate of change for this function?

V drops 800 L  
in 40 min. time

$$-\frac{800 \text{ L}}{40 \text{ min}} = -20 \text{ L/min}$$

Emptying a Hot Tub



Ex2) Sketching a graph using function notation and intercepts

Ex2) Sketch a graph of the linear function  $f(x) = 2x - 4$

Step 1: Determine the y-intercept

y-int (set  $x=0$ )  
 $f(x) = 2x - 4$   
 $f(0) = 2(0) - 4$   
 $f(0) = -4$        $(0, -4)$   
 or  $y = -4$

When  $x=0$   $y=-4$

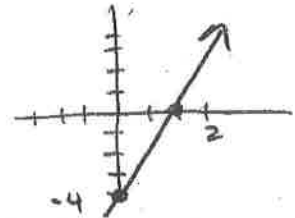
Step 2: Determine the x-intercept

x-int (set  $y=0$ )  
 ~~$f(x) = 2x - 4$~~   
 $0 = 2x - 4$   
 $+4$        $+4$   
 $\frac{4}{2} = \frac{2x}{2}$       When  $y=0$   
 $2 = x$        $x=2$   
 $(2, 0)$

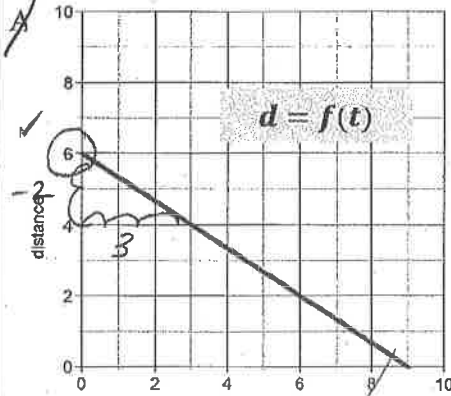
Step 3: Plot the intercepts and connect the dots!

How else could we have graphed this line?

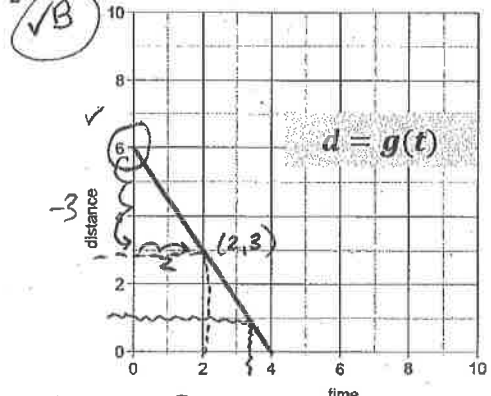
- Table of Values
- use one intercept as a starting point and then use rate of change



Ex3) Which graph has a rate of change of  $-\frac{3}{2}$  and a vertical intercept of 6?



rate:  $\frac{\text{down } 2}{\text{right } 3} = -\frac{2}{3}$



rate:  $\frac{\text{down } 3}{\text{right } 2} = -\frac{3}{2}$

a) Using the correct graph, what is the distance when time is 2?

When  $t=2$ , follow dotted line:  $d=3$

b) Using the correct graph, what is the time is it when the distance is 1?

When  $d=1$ , follow  $\sim$ ,  
 $t$  looks like 3.3 or 3.4.  
 But what is it really?

$d = -\frac{3}{2}t + 6$       sub  $ind=1$

$1 = -\frac{3}{2}t + 6$

$(\frac{2}{-3}) - 5 = -\frac{3}{2}t (\frac{2}{-3})$

$\frac{10}{3} = t$  or  $t = 3.\bar{3}$

\* Make and use an equation \*

rate:  $-\frac{3}{2}$       initial value: 6

$d$ : dep var       $t$ : indep var

**Reflection:** Describe how you can tell from a graph whether a linear function has a positive or negative rate of change.

**Up** from left to right:  $\oplus$  ↗

(we read left to right)

**down** from left to right:  $\ominus$  ↘