4.1 – Estimating Roots

Name:
Date:

Goal: to explore decimal representations of different roots of numbers

Toolkit:
- Finding a square root
- Finding a cube root
- Multiplication
- Estimating

Main Ideas:

Definitions:

Radical: an expression consisting of a radical sign, a radicand, and an index.

$\sqrt[n]{x}$ index, radical sign, the radicand

Perfect squares and cubes to memorize: $\sqrt{4}=2$, $\sqrt{9}=3$, $\sqrt{16}=4$, $\sqrt{25}=5$, $\sqrt{36}=6$, $\sqrt{49}=7$, $\sqrt{64}=8$, $\sqrt{81}=9$, $\sqrt[3]{8}=2$, $\sqrt[3]{27}=3$, $\sqrt[3]{64}=4$, $\sqrt[3]{125}=5$

$\sqrt{100}=10$

$\sqrt{121}=11$

$\sqrt{144}=12$

Ex 1) Evaluate the following radicals, identify the radicand and index for each:

a) $\sqrt{16}=4$

Radicand: 16
Index: 2

b) $\sqrt[3]{64}=4$

Radicand: 64
Index: 3

Estimating square roots

Estimate $\sqrt{79}$

$\sqrt{64}$ $\sqrt{79}$ $\sqrt{81}$

8 9

8.8 or 8.9

$\sqrt{79}\cong 8.89$

Ex.2) Estimate the value of $\sqrt{20}$ to one decimal place.

Step 1: Find the two perfect squares that are closest to the radicand you are looking for (one that is lower and one that is higher).

$\sqrt{16}$ $\sqrt{20}$ $\sqrt{25}$

4 4.4 or 4.5 5

Step 2: Find which of the two perfect squares is closest to your radicand; this will determine the decimal point of your root.

Evaluate $\sqrt{20}$, how close was your estimate? $\sqrt{20}\cong 4.47$

Estimating cube roots

$\sqrt[3]{47}$

$\sqrt[3]{27}$ $\sqrt[3]{47}$ $\sqrt[3]{64}$

3 4

3.6

$\sqrt[3]{47} \cong 3.61$

Why can you take the cube root of a negative number but not the square root of a negative number?

$\sqrt{0.25}$

0.5

Ex 3) Estimate the value of $\sqrt[3]{16}$

Step 1: Find the two perfect cubes that are closest to the radicand you are looking for. $\sqrt[3]{8}$ $\sqrt[3]{16}$ $\sqrt[3]{27}$

2 2.4 or 2.5 3

Step 2: Find which of the two perfect cubes is closest to your radicand.

Evaluate $\sqrt[3]{16}$, how close was your estimate? $\sqrt[3]{16} \cong 2.5$

Ex 4) Estimate the value of $\sqrt[3]{-32}$

$\sqrt[3]{-64}$ $\sqrt[3]{32}$ $\sqrt[3]{-27}$

-4 -3.1 or -3.2 -3

- If index is odd, radicand can be negative
- If index is even, radicand cannot be negative.

Ex 5) Evaluate $\sqrt{0.64}$

$= 0.8$

$0.8 \times 0.8 = 0.64$ $0.08 \times 0.08 = .0064$

Ex 6) Evaluate $\sqrt{0.0196}$

$= .14$

Estimate $\sqrt{0.12}$

$\sqrt{0.09}$ $\sqrt{0.16}$

.3 .34 .4

Ex 7) Write an equivalent form of 0.3 as a cube root.

$\sqrt[3]{.027} = 0.3$ $0.3 \times 0.3 \times 0.3 = .027$

Reflection: How would you write 5 as a square root? A cube root? A fourth root?

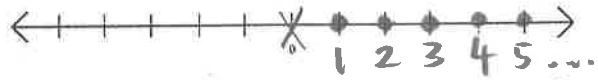
4.2 – Irrational Numbers

Name:
Date:

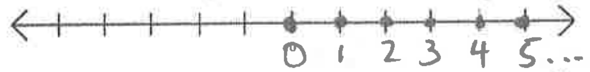
Goal: to classify real numbers, and to identify & order irrational numbers

Toolkit:	**Looking back:**
• Estimating roots • Placing numbers on number lines • Anything you remember about classifying Real Numbers	

Natural Numbers ($\mathbb{N}$) : Counting Numbers



Whole Numbers ($\mathbb{W}$) : Natural AND zero



Integers ($\mathbb{Z}$) : neg and pos non-decimals, and zero

Rational Numbers ($\mathbb{Q}$) : numbers that terminate (end) or repeat and thus can be written as a fraction.

Irrational Numbers ($\bar{\mathbb{Q}}$) : numbers that neither terminate nor repeat (cannot be written as a fraction)

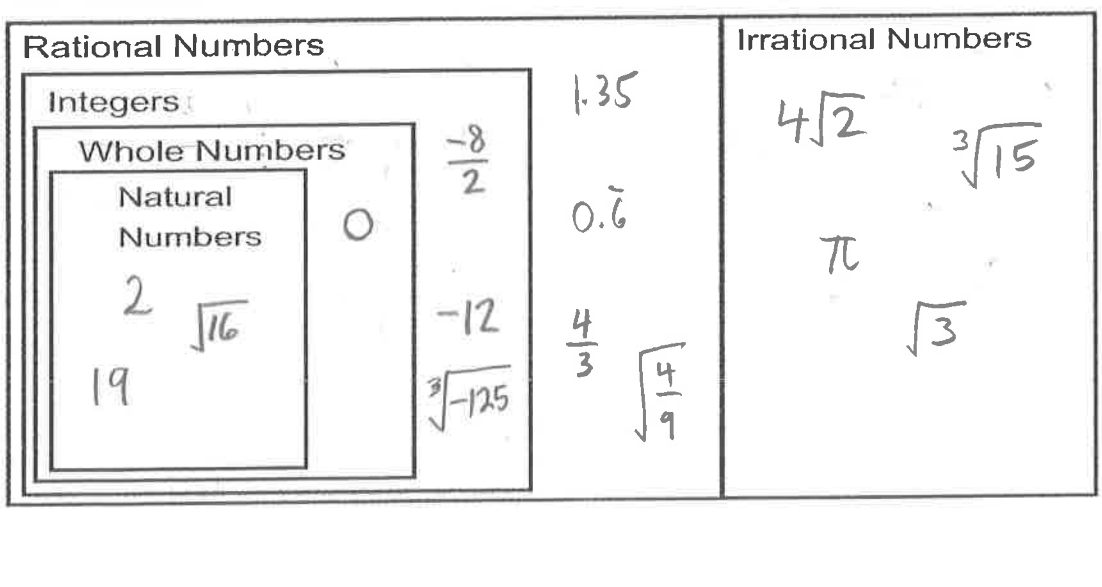
Classifying Real Numbers

Ex1) Where do these numbers belong in the diagram of Real numbers?

2 $\quad 0.\bar{6}$ $\quad 4\sqrt{2}$ $\quad \frac{4}{3}$ $\quad \frac{-8}{2}$ $\quad -12$ $\quad \pi$ $\quad 0$ $\quad \sqrt{16}$

1.35 $\quad \sqrt[3]{-125}$ $\quad \sqrt{3}$ $\quad \sqrt[3]{15}$ $\quad 19$ $\quad \sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

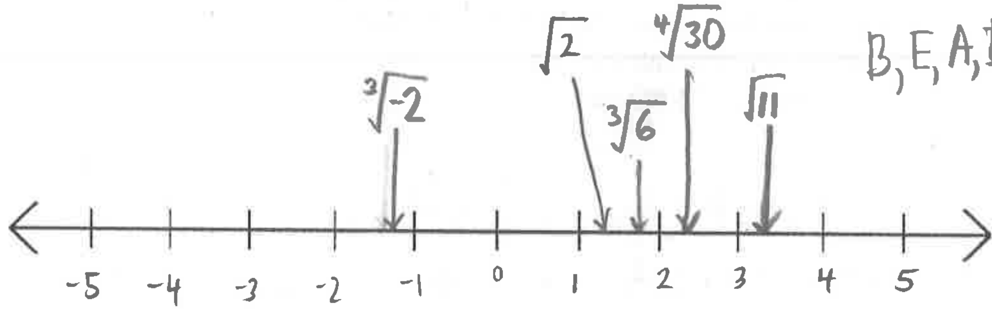
Real Numbers:



Ordering numbers on a number line	Ex2) Use a number line to order these numbers from least to greatest.
Connect:	Ex3) Is the tangent ratio for θ in each right triangle rational or irrational?

Ex2) Use a number line to order these numbers from least to greatest.

A	B	C	D	E
$\sqrt[3]{6}$	$\sqrt[3]{-2}$	$\sqrt{11}$	$\sqrt[4]{30}$	$\sqrt{2}$
$\cong 1.82$	$\cong -1.26$	$\cong 3.32$	$\cong 2.34$	$\cong 1.4$

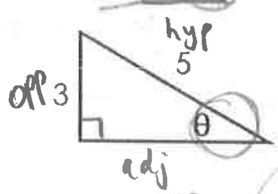


B, E, A, D, C

Connect:

Ex3) Is the tangent ratio for θ in each right triangle rational or irrational?

a)



$\tan\theta = \frac{opp}{adj} = \frac{3}{4}$

RATIONAL

$a^2+b^2=c^2$

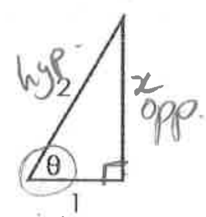
$a^2+3^2=5^2$

$a^2+9=25$

$a^2=16$

$a=\sqrt{16}=4$

b)



$\tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$

irrational

$2^2-1^2=x^2$

$4-1=x^2$

$3=x^2$

$x=\sqrt{3}$

Reflection: How could you order a set of irrational numbers if you do not have a calculator?

4.3A – From Entire to Mixed Radicals

Name:
Date:

Goal: to express an entire radical as a mixed radical

Toolkit:	**Main Ideas:**
• Understanding Radicals • Identifying Factors of a Number	

Perfect Squares - 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ...

Perfect Cubes - 1, 8, 27, 64, 125, 216, ...

What is an entire radical?

A radical sign with a number under it: ex $\sqrt{28}$, $\sqrt[3]{16}$

What is a mixed radical?

An expression that includes a radical with a coefficient
ex $2\sqrt{3}$

Equivalent Forms:
Ex 1)
a) $\sqrt{16 \cdot 9}$ is equivalent to $\sqrt{16} \cdot \sqrt{9}$ because:

$\sqrt{144}$ $\quad$ $4 \cdot 3$

12 $\quad$ 12

b) $\sqrt[3]{8 \cdot 27}$ is equivalent to $\sqrt[3]{8} \cdot \sqrt[3]{27}$ because:

$\sqrt[3]{216}$ $\quad$ $2 \cdot 3$

$= 6$ $\quad$ 6

What is the Multiplication Property of Radicals?

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, where n is a natural number, and a and b are real numbers

***We can use this property to simplify square roots and cube roots that are *not* perfect squares or perfect cubes, but have *factors* that are perfect squares or perfect cubes.**

Simplifying Square Roots

$\sqrt{10}$

We can simplify $\sqrt{24}$ because 24 has a perfect square factor of 4.
(hint:look at list of perfect squares!)

- Re-write $\sqrt{24}$ as a product of two factors, with the first one being the perfect square:

$\sqrt{4 \cdot 6}$

$= \sqrt{4} \cdot \sqrt{6}$

$= 2 \cdot \sqrt{6} = 2\sqrt{6}$

ex $\sqrt{45}$

$\sqrt{9 \cdot 5}$

$\sqrt{9} \cdot \sqrt{5}$

$3\sqrt{5}$

Simplifying Cube Roots

We can also simplify $\sqrt[3]{24}$ because 24 has a perfect cube factor of 8.
(hint:look at list of perfect cubes!)

- Re-write $\sqrt[3]{24}$ as a product of two factors, with the first one being the perfect cube:

$\sqrt[3]{8 \cdot 3}$

$\sqrt[3]{8} \cdot \sqrt[3]{3}$

$2 \cdot \sqrt[3]{3}$

$2\sqrt[3]{3}$

$\cancel{2\sqrt[3]{3}}$

Tip: If there is MORE than one perfect square or perfect cube factor, choose the LARGEST one!

Ex 2) Simplify each radical: (remember your list of perfect squares and perfect cubes!)

a) $\sqrt{80}$

$\sqrt{16\cdot 5}$

$\sqrt{16}\cdot\sqrt{5}$

$4\sqrt{5}$

b) $\sqrt{32}$

$\sqrt{16\cdot 2}$

$\sqrt{16}\cdot\sqrt{2}$

$4\sqrt{2}$

$\sqrt{4\cdot 8}$

$\sqrt{4}\cdot\sqrt{8}$

$2\sqrt{8}$

$2\cdot\sqrt{4\cdot 2}$

$2\cdot\sqrt{4}\cdot\sqrt{2}$

$2\cdot 2\cdot\sqrt{2}$

$4\sqrt{2}$

c) $\sqrt{98}$

$\sqrt{49\cdot 2}$

$\sqrt{49}\cdot\sqrt{2}$

$7\cdot\sqrt{2}$

$7\sqrt{2}$

d) $\sqrt[3]{162}$

$\sqrt[3]{27\cdot 6}$

$\sqrt[3]{27}\cdot\sqrt[3]{6}$

$3\sqrt[3]{6}$

e) $\sqrt[3]{108}$

$\sqrt[3]{27\cdot 4}$

$\sqrt[3]{27}\cdot\sqrt[3]{4}$

$3\sqrt[3]{4}$

How do you simplify something with an index of 4? (a fourth root?)

$\sqrt{80}$

$\sqrt{4\cdot 20}$

$\sqrt{4}\cdot\sqrt{20}$

$2\sqrt{20}$

$2\sqrt{4\cdot 5}$

$2\cdot\sqrt{4}\cdot\sqrt{5}$

$2\cdot 2\cdot\sqrt{5}$

$4\sqrt{5}$

Ex 3) Simplify $\sqrt[4]{162}$

Perfect Fourths: 16, 81, 256

-Rewrite radical with the prime factorization of 162

-Since $\sqrt[4]{162}$ is a fourth root, look for a factor that appears 4 times!

$\sqrt[4]{162}$

$\sqrt[4]{81\cdot 2}$

$\sqrt[4]{81}\cdot\sqrt[4]{2}$

$3\sqrt[4]{2}$

Ex 4) Simplify $\sqrt[4]{48}$

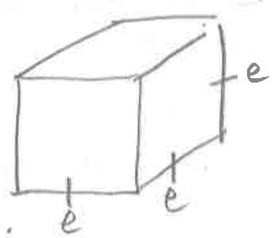
$\sqrt[4]{16\cdot 3}$

$\sqrt[4]{16}\cdot\sqrt[4]{3}$

$2\sqrt[4]{3}$

Word Problem

Ex 5) A cube has a volume of $128cm^3$. Write the edge length of the cube in simplest radical form.



$V = 128cm^3$

$128 = e^3$

$e = \sqrt[3]{128}$

$e = \sqrt[3]{64\cdot 2}$

$e = \sqrt[3]{64}\cdot\sqrt[3]{2}$

$e = 4\sqrt[3]{2}$ cm

Reflection: How do you use the index of a radical when you simplify a radical? Use an example.

4.3B – From Mixed to Entire Radicals

Name:
Date:

Goal: to express a mixed radical as an entire radical

Toolkit:
- List of Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100...
- List of Perfect Cubes: 1, 8, 27, 64, 125, 216, ...
- Multiplication Property of Radicals ($\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$)
- Mixed Radical....ex.
- Entire Radical......ex.

Main Ideas:

How do you write a mixed radical as an entire radical?

Write the mixed radical $4\sqrt{3}$ as an entire radical:

$4\sqrt{3}$
$= 4 \cdot \sqrt{3}$ — Use the Multiplication Property of Radicals (re-write 4 as a radical.....think .....4 = $\sqrt{?}$.....$\sqrt{16}$!)
$= \sqrt{16} \cdot \sqrt{3}$
$= \sqrt{16 \cdot 3}$ — Combine these under the same radical sign and multiply
$= \sqrt{48}$

(***NOTICE...these are the *opposite* steps to writing an entire radical as a mixed radical)

Ex. 1) Write each as an entire radical:

a) $5\sqrt{2}$
$5 \cdot \sqrt{2}$
$\sqrt{25} \cdot \sqrt{2}$
$\sqrt{25 \cdot 2}$
$\sqrt{50}$

b) $3\sqrt{3}$
$3 \cdot \sqrt{3}$
$\sqrt{9} \cdot \sqrt{3}$
$\sqrt{9 \cdot 3}$
$\sqrt{27}$

c) $3\sqrt[3]{2}$
$3 \cdot \sqrt[3]{2}$
$\sqrt[3]{27} \cdot \sqrt[3]{2}$
$\sqrt[3]{27 \cdot 2}$
$\sqrt[3]{54}$

d) $2\sqrt[3]{6}$
$2 \cdot \sqrt[3]{6}$
$\sqrt[3]{8} \cdot \sqrt[3]{6}$
$\sqrt[3]{8 \cdot 6}$
$\sqrt[3]{48}$

What do you do if the index is 4 or 5 (or higher?)

Write $3\sqrt[5]{2}$ as an entire radical:

First, re-write 3 as $\sqrt[5]{?}$ .... $3 = \sqrt[5]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \sqrt[5]{243}$

Perfect Fifths
32, 243, 1024

So now, $3\sqrt[5]{2}$
$= 3 \cdot \sqrt[5]{2}$
$= \sqrt[5]{243} \cdot \sqrt[5]{2}$ now, using the Multiplication Property of Radicals...
$= \sqrt[5]{243 \cdot 2}$
$= \sqrt[5]{486}$

Perfect Fourths
16, 81, 256

Ex. 2) Write each as an entire radical:

a) $2\sqrt[4]{5}$
$2 \cdot \sqrt[4]{5}$
$\sqrt[4]{16} \cdot \sqrt[4]{5}$
$\sqrt[4]{16 \cdot 5}$
$\sqrt[4]{80}$

b) $4\sqrt[5]{2}$
$4 \cdot \sqrt[5]{2}$
$\sqrt[5]{1024} \cdot \sqrt[5]{2}$
$\sqrt[5]{1024 \cdot 2}$
$\sqrt[5]{2048}$

How can entire radicals be used to help you order a set of mixed radicals with the same index?	Ex. 3) Arrange the following in order from greatest to least: $3\sqrt{5}$, $2\sqrt{13}$, $4\sqrt{3}$, 2, $9\sqrt{2}$

$3\sqrt{5}$ $2\sqrt{13}$ $4\sqrt{3}$ 2 $9\sqrt{2}$

$3\cdot\sqrt{5}$ $2\cdot\sqrt{13}$ $4\cdot\sqrt{3}$ $\sqrt{4}$ $9\cdot\sqrt{2}$

$\sqrt{9}\cdot\sqrt{5}$ $\sqrt{4}\cdot\sqrt{13}$ $\sqrt{16}\cdot\sqrt{3}$ $\sqrt{81}\cdot\sqrt{2}$

$\sqrt{9\cdot 5}$ $\sqrt{4\cdot 13}$ $\sqrt{16\cdot 3}$ $\sqrt{81\cdot 2}$

$\sqrt{45}$ $\sqrt{52}$ $\sqrt{48}$ $\sqrt{162}$

$9\sqrt{2}$, $2\sqrt{13}$, $4\sqrt{3}$, $3\sqrt{5}$, 2

Reflection: How do you use the index of a radical when you write a mixed radical as an entire radical? Use an example to help your explanation

4.4 – Fractional Exponents and Radicals

Name:
Date:

Goal: to relate rational exponents and radicals

Toolkit:
- Exponent Laws
- Taking square and cube roots
- Converting decimals to fractions
- Order of operations

Main Ideas:

Evaluating powers of the form $a^{\frac{1}{n}}$

Powers with Rational Exponents with Numerator 1

When n is as natural number and x is a rational number,

$x^{\frac{1}{n}} = \sqrt[n]{x}$ ... for example... $16^{\frac{1}{2}} = \sqrt[2]{16} = 4$

Ex 1) Write each power as a radical then evaluate without using a calculator.

index

a) $1000^{\frac{1}{3}}$

$= \sqrt[3]{1000}$

$= 10$

b) $0.25^{0.5}$

$0.25^{\frac{1}{2}}$

$\sqrt[2]{0.25}$

$= 0.5$

c) $(-8)^{\frac{1}{3}}$

$\sqrt[3]{-8}$

$= -2$

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

$\sqrt[4]{\frac{16}{81}} = \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$

Rewriting powers in radical and exponent form

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number,

$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$ ... ex) $25^{\frac{3}{2}} = \left(25^{\frac{1}{2}}\right)^3 = \left(\sqrt[2]{25}\right)^3 = (5)^3 = 125$

or

$x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$ ... ex) $25^{\frac{3}{2}} = (25^3)^{\frac{1}{2}} = \sqrt[2]{25^3} = \sqrt{15625} = 125$

Ex 2) Write $26^{\frac{2}{5}}$ in radical form in two different ways.

power

index

$26^{\frac{2}{5}}$

$26^{\frac{2}{5}}$

$= \left(\sqrt[5]{26}\right)^2$

$26^{\frac{2}{5}}$

$\sqrt[5]{26^2}$

$42^{\frac{2}{3}}$

$\left(\sqrt[3]{42}\right)^2$

OR

$\sqrt[3]{42^2}$

Ex 3) Write the following in exponent form.

a) $\sqrt[2]{6^5}$

$6^{\frac{5}{2}}$

b) $\left(\sqrt[4]{19}\right)^3$

$19^{\frac{3}{4}}$

$27^{\frac{4}{3}}$

$\left(\sqrt[3]{27}\right)^4$

3^4

81

Evaluating powers with rational exponents and rational bases	Ex 4) Evaluate the following:

Ex 4) Evaluate the following:

$\frac{75}{100}$ $\frac{3}{4}$

a) $0.01^{\frac{3}{2}}$

$\left(\sqrt{0.01}\right)^3$

$(.1)^3$

$.1 \times .1 \times .1$

0.001

b) $(-27)^{\frac{4}{3}}$

$\left(\sqrt[3]{-27}\right)^4$

$(-3)^4$

81

c) $32^{0.4}$

$32^{\frac{4}{10}}$

$32^{\frac{2}{5}}$

$\left(\sqrt[5]{32}\right)^2$

$(2)^2$

$= 4$

d) $16^{0.75}$

$16^{\frac{3}{4}}$

$\left(\sqrt[4]{16}\right)^3$

2^3

8

0.4 ← tenths

$= \frac{4}{10}$

$= \frac{2}{5}$

Applying rational exponents

_ . _ _ _

tenths, hundredths, thousandths

Ex 5) Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass, m kilograms. Use the formula to estimate the brain mass of each animal.

a) A moose with a body mass of 512kg

$b = 0.01m^{\frac{2}{3}}$

$b = 0.01(512)^{\frac{2}{3}}$

$b = 0.01\left(\sqrt[3]{512}\right)^2$

$b = 0.01(8)^2$

$b = 0.01(64) = 0.64\text{ kg}$

b) A cat with a body mass of 5kg

$b = 0.01m^{\frac{2}{3}}$

$b = 0.01(5)^{\frac{2}{3}}$

$b = 0.01\left(\sqrt[3]{5}\right)^2$

$b = 0.01(1.7)^2 = 0.01(2.924) = 0.029\text{ kg}$

Reflection: In the power $x^{\frac{m}{n}}$, m and n are natural numbers and x is a rational number. What does the numerator m represent? What does the denominator n represent? Use an example to explain your answer.

4.5 – Negative Exponents and Reciprocals

Name:
Date:

Goal: To relate negative exponents to reciprocals

Toolkit:

- Simplifying and evaluating with rational exponents
- Multiplying fractions

Main Ideas:

What is a reciprocal?

Two numbers with a product of **1** are reciprocals.

Ex. 1) Since $4 \cdot \frac{1}{4} = \mathbf{1}$, the numbers 4 and $\frac{1}{4}$ are reciprocals

$\frac{4}{1}$

Ex. 2) Since $\frac{2}{3} \cdot \frac{3}{2} = \mathbf{1}$, the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals

Powers with Negative Exponents

When x is any non-zero number and n is a rational number, x^{-n} is the reciprocal of x^n.

That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, $x \neq 0$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

Evaluate a power with a negative exponent

$\frac{(-3)^{-3}}{4^{-3}}$ $\frac{4^3}{(-3)^3}$

Evaluate each power:

Ex. 3) a) 3^{-2} b) $(-5)^{-3}$ c) $(-\frac{3}{4})^{-3}$ d) $(\frac{10}{3})^{-2}$

a) $= \frac{1}{3^2}$ $= \frac{1}{9}$

b) $= \frac{1}{(-5)^3}$ $= -\frac{1}{125}$

c) $\left(-\frac{4}{3}\right)^3$ $\frac{(-4)^3}{3^3} = \frac{-64}{27}$

d) $\left(\frac{3}{10}\right)^2$ $\frac{3^2}{10^2} = \frac{9}{100}$

Evaluate a power with a negative *rational* exponent

To evaluate a power with a negative *rational* (fraction) exponent:

Ex. 4) Evaluate $8^{-\frac{2}{3}}$ ← exponent ← index

$= \frac{1}{8^{\frac{2}{3}}}$ *write with a positive exponent*

$= \frac{1}{(\sqrt[3]{8})^2}$ *re-write into radical form, then work from inside out*

$= \frac{1}{(2)^2}$ *evaluate (write answer with NO exponents)*

$= \frac{1}{4}$

$$49^{-\frac{5}{2}} = \frac{1}{49^{\frac{5}{2}}} = \frac{1}{(\sqrt{49})^5} = \frac{1}{7^5} = \frac{1}{16807}$$

Ex. 5) Evaluate:

a) $(\frac{9}{16})^{-\frac{3}{2}}$

$$\left(\frac{16}{9}\right)^{\frac{3}{2}} \quad \frac{16^{\frac{3}{2}}}{9^{\frac{3}{2}}} \quad \frac{(\sqrt{16})^3}{(\sqrt{9})^3} = \frac{4^3}{3^3} = \frac{64}{27}$$

b) $(\frac{25}{36})^{-\frac{1}{2}}$

$$\left(\frac{36}{25}\right)^{\frac{1}{2}} \quad \frac{36^{\frac{1}{2}}}{25^{\frac{1}{2}}} \quad \frac{\sqrt{36}}{\sqrt{25}} = \frac{6}{5}$$

c) $16^{-\frac{5}{4}}$

$$\frac{1}{16^{\frac{5}{4}}} \quad \frac{1}{(\sqrt[4]{16})^5} \quad \frac{1}{2^5} = \frac{1}{32}$$

d) $-25^{-1.5}$

no brackets, not part of base!

(hint: change 1.5 to a fraction in lowest terms!)

$$\frac{15 \div 5}{10 \div 5} = \frac{3}{2}$$

$$-25^{-\frac{3}{2}} \quad -\frac{1}{25^{\frac{3}{2}}} \quad -\frac{1}{(\sqrt{25})^3} = -\frac{1}{5^3} = -\frac{1}{125}$$

Applying Negative Exponents (word problems)

Ex. 6) Use the formula $v = 0.155 s^{\frac{5}{3}} f^{-\frac{7}{6}}$ to estimate the speed of a dinosaur when $s = 1.5$ and $f = 0.3$ (answer is a speed in m/s)

Substitute values into the proper places in the formula

$$V = 0.155(1.5)^{\frac{5}{3}}(0.3)^{-\frac{7}{6}}$$

Evaluate, using your calculator

$$V = 0.155(1.5)^{\frac{5}{3}}\left(\frac{1}{0.3^{\frac{7}{6}}}\right)$$

$$V = \frac{0.155(1.5)^{\frac{5}{3}}}{0.3^{\frac{7}{6}}}$$

$$V = \frac{0.155(\sqrt[3]{1.5})^5}{(\sqrt[6]{0.3})^7} = \frac{(0.155)(1.965556)}{0.2455} = 1.24\,\text{m/s}$$

Reflection:

4.6A – Simplifying with Exponent Laws

Name:
Date:

Goal: to apply all of the exponent laws to simplify expressions

Toolkit:
- Exponent Laws
- Fractional and negative exponents
- Operations with fractions, integers

Main Ideas:

Exponent Laws

Product of powers: $x^m \cdot x^n = x^{m+n}$ ex $(5^3)(5^4) = 5^7$

Quotient of powers: $\frac{x^m}{x^n} = x^{m-n}$ ex $\frac{4^7}{4^4} = 4^3$

Power of a power: $(x^m)^n = x^{mn}$ ex $(2^3)^2 = 2^6$

Power of a product: $(xy)^m = x^m y^m$ ex $(2x)^3 = 2^3x^3 = 8x^3$

Power of a quotient: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$ ex $\left(\frac{y}{3}\right)^3 = \frac{y^3}{3^3} = \frac{y^3}{27}$

Power of zero: $x^0 = 1$

Fractional exponents: $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$

Negative exponents:

$a^{-n} = \frac{1}{a^n}$ | $a^n = \frac{1}{a^{-n}}$ | $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$ | $a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{(\sqrt[n]{a})^m}$

Note: write all powers with POSITIVE EXPONENTS.

Ex 1) Simplify by writing as a single power.

a) $0.6^2 \cdot 0.6^{-6}$

$0.6^{2+(-6)}$

$= 0.6^{-4}$

$\frac{1}{0.6^4}$

product of powers

b) $x^{-4} \cdot x^7$

x^3

product of powers!

c) $m^7 \div m^{-2}$

$m^{7-(-2)}$

m^9

quotient of powers

d) $\frac{0.4^3}{0.4^4}$

0.4^{-1}

$\frac{1}{0.4}$

$= \frac{10}{4}$

$= \frac{5}{2}$

e) $(n^2)^{-4}$

n^{-8}

$\frac{1}{n^8}$

Which law(s) did you use?

$\frac{5}{4} + \frac{-1}{4}$

$\frac{4}{4}$

Ex 2) Simplify by writing as a single power.

a) $\left[\left(-\frac{4}{7}\right)^2\right]^{-3} \div \left[\left(-\frac{4}{7}\right)^4\right]^{-5}$

$\left(\frac{-4}{7}\right)^{-6} \div \left(\frac{-4}{7}\right)^{-20}$

$\left(\frac{-4}{7}\right)^{14} = \frac{(-4)^{14}}{7^{14}}$

b) $\frac{(2.3^{-3})^{-5}}{2.3^5}$

$\frac{2.3^{15}}{2.3^5}$

2.3^{10}

c) $\frac{8^{\frac{5}{4}} \cdot 8^{-\frac{1}{4}}}{8^{\frac{3}{4}}}$

$\frac{8^{\frac{4}{4}}}{8^{\frac{3}{4}}}$

$\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$

$8^{\frac{1}{4}} = \sqrt[4]{8}$

Note: write all powers with POSITIVE EXPONENTS.

Ex 3) Simplify. multiply

a) $(x^4y^{-2})(x^2y^3)$

$x^4y^{-2}x^2y^3$

x^6y

b) $(27x^6y^9)^{\frac{1}{3}}$

$\frac{6}{1} \cdot \frac{1}{3} = \frac{6}{3} = 2$

$27^{\frac{1}{3}}x^2y^3$

$\sqrt[3]{27}\,x^2y^3$

$3x^2y^3$

c) $\left(\frac{6a^4b^{-3}}{14a^{-2}b^2}\right)^{-2}$

$\left(\frac{3}{7}a^6b^{-5}\right)^{-2}$

$\frac{3^{-2}}{7^{-2}}a^{-12}b^{10}$

$\frac{7^2b^{10}}{3^2a^{12}}$

$\frac{49b^{10}}{9a^{12}}$

$\frac{-2}{1} \cdot \frac{1}{2} = \frac{-2}{2} = -1$

d) $\left(\frac{50m^2n^4}{2m^4n^2}\right)^{\frac{1}{2}}$

$(25m^{-2}n^2)^{\frac{1}{2}}$

$25^{\frac{1}{2}}m^{-1}n$

$\sqrt{25}\,m^{-1}n$

$5m^{-1}n$

$\frac{5n}{m}$

$\frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$

e) $\frac{\left(2x^{\frac{3}{2}}y^2\right)\left(3x^{\frac{1}{2}}y^{-1}\right)}{(4x^3y^{-1})}$

$\frac{6x^2y^1}{4x^3y^{-1}}$

$1-(-1)$
$1+1$
2

$\frac{3}{2}x^{-1}y^2$

$\frac{3y^2}{2x}$

Reflection: How would you simplify the expression $\left(\frac{x^a}{x^3}\right)^2$ and how is it similar/different compared to the other problems we've done?

4.6B – Evaluating with Exponent Laws

Name:
Date:

Goal: to apply all of the exponent laws to evaluate expressions

Toolkit:	**Main Ideas:**
• Exponent Laws, incl. fractional /negative • Operations with fractions, integers • Substitution, BEDMAS	

What is the difference between "simplifying" and "evaluating"?

Simplify: Write answer as a single power with positive exp.

Ex 1) Simplify $x^{\frac{5}{3}} \cdot x^{\frac{1}{3}}$

$x^{\frac{6}{3}}$

x^2

Evaluate: write answer as a single number – no exponents or variables.

- integer
- decimal
or
- frac in lowest terms

Ex 2) Evaluate $1.5^{\frac{5}{3}} \cdot 1.5^{\frac{1}{3}}$

$1.5^{\frac{6}{3}}$

1.5^2

$= 2.25$

Ex 3) Evaluate each expression for $m = -1$ and $n = 2$

Step 1: Simplify the expression (use exponent laws)
Step 2: Substitute → replace letters with numeric values
Step 3: Evaluate

a) $(m^2n^3)(m^3n^2)$

m^5n^5

$= (-1)^5(2)^5$

$= (-1)(32)$

$= -32$

b) $\left(\dfrac{m^{-5}n^5}{m^{-2}n^6}\right)^{-3}$

$(m^{-3}n^{-1})^{-3}$

m^9n^3

$(-1)^9(2)^3$

$(-1)(8) = -8$

c) $\dfrac{(m^n)^2}{m^3}$

$\dfrac{m^{2n}}{m^3}$

m^{2n-3}

$(-1)^{2(2)-3}$

$(-1)^1 = -1$

OR $\dfrac{(m^n)^2}{m^3}$

$\dfrac{((-1)^2)^2}{(-1)^3}$

$\dfrac{(-1)^4}{(-1)^3} = (-1)^1$

$= -1$

Solving Problems using the Exponent Laws

Ex 4) A sphere has volume 600m^3.

a) Write an expression for the radius in exponent form
b) What is the radius of the sphere to the nearest tenth of a metre?

$V_{sphere} = \dfrac{4\pi r^3}{3}$

$(3)600 = \dfrac{4\pi r^3}{\cancel{3}}\cancel{(3)}$

$\dfrac{1800}{4\pi} = \dfrac{\cancel{4\pi} r^3}{\cancel{4\pi}}$

$\dfrac{450}{\pi} = r^3$

$\sqrt[3]{\dfrac{450}{\pi}} = \sqrt[3]{r^3}$

$\sqrt[3]{\dfrac{450}{\pi}} = r$

$r = \left(\dfrac{450}{\pi}\right)^{\frac{1}{3}}$

b) use calc

$r = 5.2\text{m}$

Reflection: Why is it important to simplify BEFORE evaluating?