

4.1 – Estimating Roots

Name: _____

Date: _____

Goal: to explore decimal representations of different roots of numbers

Toolkit:

- Finding a square root
- Finding a cube root
- Multiplication
- Estimating

Main Ideas:

Definitions:

Radical: an expression consisting of a radical sign, a radicand, and an index.

$$\sqrt[n]{x}$$

Perfect squares and cubes to memorize: $\sqrt{4} =$, $\sqrt{9} =$, $\sqrt{16} =$, $\sqrt{25} =$, $\sqrt{36} =$
 $\sqrt{49} =$, $\sqrt{64} =$, $\sqrt{81} =$, $\sqrt[3]{8} =$, $\sqrt[3]{27} =$, $\sqrt[3]{64} =$, $\sqrt[3]{125} =$

Estimating square roots

Ex 1) Evaluate the following radicals, identify the radicand and index for each:

a) $\sqrt{16}$

b) $\sqrt[3]{64}$

Radicand: _____

Radicand: _____

Index: _____

Index: _____

Ex 2) Estimate the value of $\sqrt{20}$ to one decimal place.

Step 1: Find the two perfect squares that are closest to the radicand you are looking for (one that is lower and one that is higher).

Step 2: Find which of the two perfect squares is closest to your radicand; this will determine the decimal point of your root.

Evaluate $\sqrt{20}$, how close was your estimate?

Estimating cube roots

Ex 3) Estimate the value of $\sqrt[3]{16}$

Step 1: Find the two perfect cubes that are closest to the radicand you are looking for.

Step 2: Find which of the two perfect cubes is closest to your radicand.

Evaluate $\sqrt[3]{16}$, how close was your estimate?

Why can you take the cube root of a negative number but not the square root of a negative number?

Ex 4) Estimate the value of $\sqrt[3]{-32}$

Ex 5) Evaluate $\sqrt{0.64}$

Ex 6) Evaluate $\sqrt{0.0196}$

Ex 7) Write an equivalent form of 0.3 as a cube root.

Reflection: How would you write 5 as a square root? A cube root? A fourth root?

4.2 – Irrational Numbers

Name:

Date:

Goal: to classify real numbers, and to identify & order irrational numbers

Toolkit:

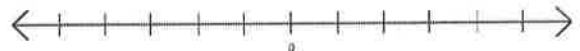
- Estimating roots
- Placing numbers on number lines
- Anything you remember about classifying Real Numbers

Looking back:

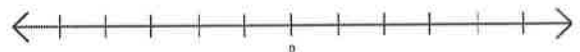
Natural Numbers ()



Whole Numbers ()



Integers ()



Rational Numbers ()

Irrational Numbers ()

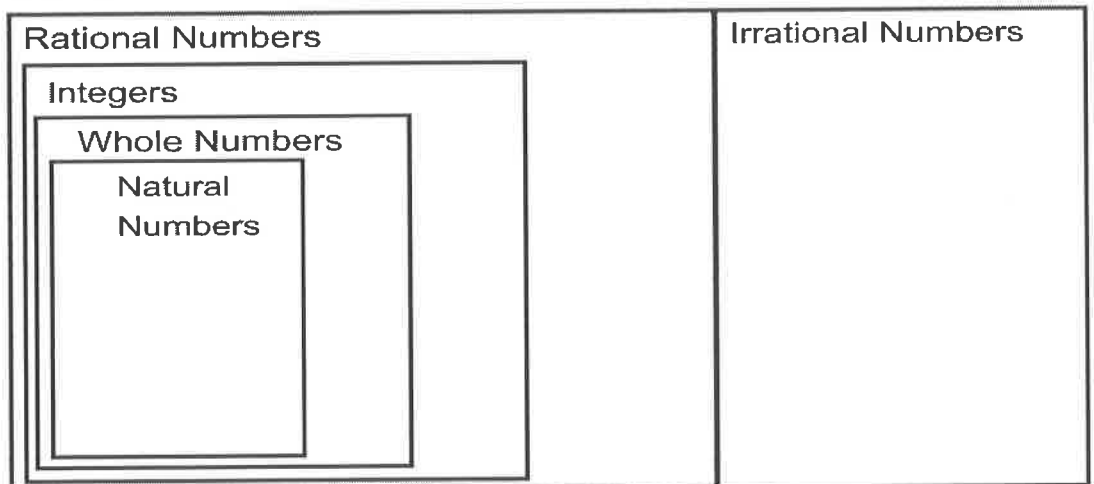
Classifying Real Numbers

Ex1) Where do these numbers belong in the diagram of Real numbers?

2 $0.\bar{6}$ $4\sqrt{2}$ $\frac{4}{3}$ $\frac{-8}{2}$ -12 π 0 $\sqrt{16}$

1.35 $\sqrt[3]{-125}$ $\sqrt{3}$ $\sqrt[3]{15}$ 19 $\sqrt{\frac{4}{9}}$

Real Numbers:



Ordering numbers on a number line

Ex2) Use a number line to order these numbers from least to greatest.

A $\sqrt[3]{6}$

B $\sqrt[3]{-2}$

C $\sqrt{11}$

D $\sqrt[4]{30}$

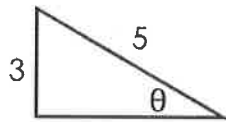
E $\sqrt{2}$



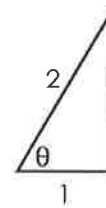
Connect:

Ex3) Is the tangent ratio for θ in each right triangle rational or irrational?

a)



b)



Reflection: How could you order a set of irrational numbers if you do not have a calculator?

4.3A – From Entire to Mixed Radicals

Name:

Date:

Goal: to express an entire radical as a mixed radical

Toolkit:

- Understanding Radicals
- Identifying Factors of a Number

Main Ideas:

Perfect Squares - 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144,

Perfect Cubes - 1, 8, 27, 64, 125, 216,

What is an entire radical?

What is a mixed radical?

Equivalent Forms:

Ex 1)

a) $\sqrt{16 \cdot 9}$ is equivalent to $\sqrt{16} \cdot \sqrt{9}$ because: b) $\sqrt[3]{8 \cdot 27}$ is equivalent to $\sqrt[3]{8} \cdot \sqrt[3]{27}$ because:

What is the Multiplication Property of Radicals?

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}, \quad \text{where } n \text{ is a natural number, and } a \text{ and } b \text{ are real numbers}$

***We can use this property to simplify square roots and cube roots that are *not* perfect squares or perfect cubes, but have *factors* that are perfect squares or perfect cubes.**

Simplifying Square Roots

We can simplify $\sqrt{24}$ because 24 has a perfect square factor of _____.

(hint: look at list of perfect squares!)

- Re-write $\sqrt{24}$ as a product of two factors, with the first one being the perfect square:

Simplifying Cube Roots

We can also simplify $\sqrt[3]{24}$ because 24 has a perfect cube factor of _____.

(hint: look at list of perfect cubes!)

- Re-write $\sqrt[3]{24}$ as a product of two factors, with the first one being the perfect cube:

Tip: If there is MORE than one perfect square or perfect cube factor, choose the LARGEST one!

How do you simplify something with an index of 4? (a fourth root?)

Word Problem

Ex 2) Simplify each radical: (remember your list of perfect squares and perfect cubes!)

a) $\sqrt{80}$ b) $\sqrt{32}$ c) $\sqrt{98}$ d) $\sqrt[3]{162}$ e) $\sqrt[3]{108}$

Ex 3) Simplify $\sqrt[4]{162}$

-Rewrite radical with the prime factorization of 162
-Since $\sqrt[4]{162}$ is a fourth root, look for a factor that appears 4 times!

Ex 4) Simplify $\sqrt[4]{48}$

Ex 5) A cube has a volume of 128cm^3 . Write the edge length of the cube in simplest radical form.

Reflection: How do you use the index of a radical when you simplify a radical? Use an example.

How can entire radicals be used to help you order a set of mixed radicals with the same index?

Ex. 3) Arrange the following in order from greatest to least: $3\sqrt{5}$, $2\sqrt{13}$, $4\sqrt{3}$, 2 , $9\sqrt{2}$

Reflection: How do you use the index of a radical when you write a mixed radical as an entire radical? Use an example to help your explanation

4.4 – Fractional Exponents and Radicals

Name:

Date:

Goal: to relate rational exponents and radicals

Toolkit:

- Exponent Laws
- Taking square and cube roots
- Converting decimals to fractions
- Order of operations

Main Ideas:

Evaluating powers of the form $a^{\frac{1}{n}}$

Powers with Rational Exponents with Numerator 1

When n is a natural number and x is a rational number,

$$x^{\frac{1}{n}} = \sqrt[n]{x} \dots \text{for example... } 16^{\frac{1}{2}} = \sqrt{16} = 4$$

Ex 1) Write each power as a radical then evaluate without using a calculator.

a) $1000^{\frac{1}{3}}$

b) $0.25^{0.5}$

c) $(-8)^{\frac{1}{3}}$

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

Rewriting powers in radical and exponent form

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number,

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m \dots \text{ex) } 25^{\frac{3}{2}} = \left(25^{\frac{1}{2}}\right)^3 = \left(\sqrt{25}\right)^3 = (5)^3 = 125$$

or

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m} \dots \text{ex) } 25^{\frac{3}{2}} = \left(25^3\right)^{\frac{1}{2}} = \sqrt{25^3} = \sqrt{15625} = 125$$

Ex 2) Write $26^{\frac{2}{5}}$ in radical form in two different ways.

Ex 3) Write the following in exponent form.

a) $\sqrt{6^5}$

b) $(\sqrt[4]{19})^3$

Evaluating powers with rational exponents and rational bases

Ex 4) Evaluate the following:

a) $0.01^{\frac{3}{2}}$

b) $(-27)^{\frac{4}{3}}$

c) $32^{0.4}$

d) $16^{0.75}$

Applying rational exponents

Ex 5) Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass, m kilograms. Use the formula to estimate the brain mass of each animal.

a) A moose with a body mass of 512kg

b) A cat with a body mass of 5kg

Reflection: In the power $x^{\frac{m}{n}}$, m and n are natural numbers and x is a rational number. What does the numerator m represent? What does the denominator n represent? Use an example to explain your answer.

4.5 – Negative Exponents and Reciprocals

Name:

Date:

Goal: To relate negative exponents to reciprocals

Toolkit:

- Simplifying and evaluating with rational exponents
- Multiplying fractions

Main Ideas:

What is a reciprocal?

Two numbers with a product of 1 are reciprocals.

Ex. 1) Since $4 \cdot \frac{1}{4} = 1$, the numbers 4 and $\frac{1}{4}$ are reciprocals

Ex. 2) Since $\frac{2}{3} \cdot \frac{3}{2} = 1$, the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals

Powers with Negative Exponents

When x is any non-zero number and n is a rational number, x^{-n} is the reciprocal of x^n .

That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, $x \neq 0$

Evaluate a power with a negative exponent

Evaluate each power:

Ex. 3) a) 3^{-2} b) $(-5)^{-3}$ c) $(-\frac{3}{4})^{-3}$ d) $(\frac{10}{3})^{-2}$

Evaluate a power with a negative rational exponent

To evaluate a power with a negative rational (fraction) exponent:

Ex. 4) Evaluate $8^{-\frac{2}{3}}$

$$= \frac{1}{8^{\frac{2}{3}}} \quad \text{write with a positive exponent}$$

$$= \frac{1}{(\sqrt[3]{8})^2} \quad \text{re-write into radical form, then work from inside out}$$

$$= \frac{1}{(2)^2} \quad \text{evaluate (write answer with NO exponents)}$$

$$= \frac{1}{4}$$

Ex. 5) Evaluate:

a) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

b) $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$

c) $16^{-\frac{5}{4}}$

d) $-25^{-1.5}$

(hint: change 1.5 to a fraction in lowest terms!)

Applying
Negative
Exponents
(word problems)

Ex. 6) Use the formula $v = 0.155s^{\frac{5}{3}}f^{-\frac{7}{6}}$ to estimate the speed of a dinosaur when $s = 1.5$ and $f = 0.3$ (answer is a speed in m/s)

Substitute values into the proper places in the formula

Evaluate, using your calculator

Reflection:

4.6A – Simplifying with Exponent Laws

Name:

Date:

Goal: to apply all of the exponent laws to simplify expressions

Toolkit:

- Exponent Laws
- Fractional and negative exponents
- Operations with fractions, integers

Main Ideas:

Exponent Laws

Product of powers:

Quotient of powers:

Power of a power:

Power of a product:

Power of a quotient:

Power of zero:

Fractional exponents:

Negative exponents:

Note: write all powers with **POSITIVE EXPONENTS.**

Ex 1) Simplify by writing as a single power.

a) $0.6^2 \cdot 0.6^{-6}$ b) $x^{-4} \cdot x^7$ c) $m^7 \div m^{-2}$ d) $\frac{0.4^3}{0.4^4}$ e) $(n^2)^{-4}$

Which law(s) did you use?

Ex 2) Simplify by writing as a single power.

a) $\left[\left(-\frac{4}{7} \right)^2 \right]^{-3} \div \left[\left(-\frac{4}{7} \right)^4 \right]^{-5}$

b) $\frac{(2 \cdot 3^{-3})^{-5}}{2 \cdot 3^5}$

c) $\frac{8^{\frac{5}{4}} \cdot 8^{-\frac{1}{4}}}{8^{\frac{3}{4}}}$

Note: write all powers with POSITIVE EXPONENTS.

Ex 3) Simplify.

a) $(x^4 y^{-2})(x^2 y^3)$

b) $(27x^6 y^9)^{\frac{1}{3}}$

c) $\left(\frac{6a^4 b^{-3}}{14a^{-2} b^2} \right)^{-2}$

d) $\left(\frac{50m^2 n^4}{2m^4 n^2} \right)^{\frac{1}{2}}$

e) $\frac{(2x^{\frac{3}{2}} y^2)(3x^{\frac{1}{2}} y^{-1})}{(4x^3 y^{-1})}$

Reflection: How would you simplify the expression $\left(\frac{x^a}{x^3} \right)^2$ and how is it similar/different compared to the other problems we've done?

4.6B – Evaluating with Exponent Laws

Name: _____

Date: _____

Goal: to apply all of the exponent laws to evaluate expressions

Toolkit:

- Exponent Laws, incl. fractional /negative
- Operations with fractions, integers
- Substitution, BEDMAS

Main Ideas:

What is the difference between “simplifying” and “evaluating”?

Simplify:

Ex 1) Simplify $x^{\frac{5}{3}} \cdot x^{\frac{1}{3}}$

Evaluate:

Ex 2) Evaluate $1.5^{\frac{5}{3}} \cdot 1.5^{\frac{1}{3}}$

Ex 3) Evaluate each expression for $m = -1$ and $n = 2$

Step 1: Simplify the expression

Step 2: Substitute → replace letters with numeric values

Step 3: Evaluate

a) $(m^2n^3)(m^3n^2)$ b) $\left(\frac{m^{-5}n^5}{m^{-2}n^6}\right)^{-3}$ c) $\frac{(m^n)^2}{m^3}$

Solving Problems using the Exponent Laws

Ex 4) A sphere has volume 600m^3 .

- Write an expression for the radius in exponent form
- What is the radius of the sphere to the nearest tenth of a metre?

Reflection: Why is it important to simplify BEFORE evaluating?

