3.3 – Common Factors of a Polynomial

Name: Key
Date:

Goal: to determine the factors of a polynomial by identifying the GCF

Toolkit:	**Main Ideas:**
• Finding the GCF • Distributive Property	

Factor a binomial using the GCF

Ex 1) Factor the binomial: $3g + 6$ ⟹ 2 terms : $3g$, 6

What's the GCF of 3 and 6? 3

Can the variable be part of the GCF? No, because the 2nd term is a constant

$3(g+2)$

Check: $3(g+2)$

$3g+6$ ✓

Ex 2) Factor the binomial: $- 8y + 16y^2$

Reorder from highest to lowest degree (exponent on variable)

$\frac{16y^2}{8y} - \frac{8y}{8y}$

$8y(2y-1)$

Factor a trinomial using the GCF

Ex 3) Factor the trinomial: $\frac{3x^2}{3} + \frac{12x}{3} - \frac{6}{3}$

$3(x^2+4x-2)$

Ex 4) Factor the trinomial: $6 – 12z + 18z^2$

Reorder: $\frac{18z^2}{6} - \frac{12z}{6} + \frac{6}{6}$

$6(3z^2-2z+1)$

Factor polynomials in more than one variable

Ex 5) Factor the trinomial: $\frac{-20c^4d}{-5cd} - \frac{30c^3d^2}{-5cd} - \frac{25cd}{-5cd}$

$-5cd(4c^3+6c^2d+5)$

* If 1ST term is negative, the GCF should be negative!

Reflection: How are the processes of factoring and expanding related?

3.5 – Factoring Trinomials of the form $x^2 + bx + c$, where a=1

Name:
Date:

Goal: to use models and algebraic strategies to multiply binomials and to factor trinomials.

Toolkit:	**Main Ideas:**
• Factoring $ax^2 + bx + c$ (coefficient for x^2, coefficient for x, constant)	

Definitions:
Descending order: the terms are written in order from the term with the greatest exponent to the term with the least exponent
Ascending order: the terms are written in order from the term with the least exponent to the term with the greatest exponent

Steps for Factoring a Trinomial in the form: x^2+bx+c, where a=1

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable *(biggest to smallest)*
Step 2: Find two numbers that multiply to equal the *c* term and add to equal the *b* term (add to the middle, multiply to the end)
Step 3: Factor into two binomials using the numbers from step 2, with the variable from the question placed first in each bracket

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Multiplying two binomials

Ex 1) Expand and Simplify: $(x-1)(x-7)$ use FOIL

$x^2 - 7x - 1x + 7$
$x^2 - 8x + 7$

b=-8 came from adding -1 and -7
c=7 came from multiplying (-1)(-7)

Remember: expanding and factoring are opposite operations….they UNDO each other!

Factoring a trinomial in the form $x^2 + bx + c$

> Notice that a (the number in front of the x^2) will always end up being 1 in these questions!

Ex 2) Factor the trinomial: $x^2 - 8x + 7$ ….we should end up with $(x-1)(x-7)$!

a=1 b=-8 c=7

× 7, + -8

① re-order (not necessary)
② two numbers that multiply to c and add to b

7, 1
-7, -1

③ $(x-7)(x-1)$ or $(x-1)(x-7)$

Ex 3) Factor: $a^2 - 2a - 8$

a=1 b=-2 c=-8

① in order ✓
② × c, + b
× -8, + -2

-8, 1 4, -2
8, -1 -4, 2

$(a-4)(a+2)$

FOIL To check: $(a-4)(a+2)$
$a^2 + 2a - 4a - 8$
$a^2 - 2a - 8$

Factoring a trinomial written in ascending order

Ex 4a) Factor: $-30 + 7m + m^2$

① $m^2 + 7m - 30$
② × -30 + 7
10, -3
③ $(m+10)(m-3)$

b) $x^2 - 4xy + 21y^2$

① in order
② × 21, + -4
-7, 3
③ $(x-7y)(x+3y)$

Ex 5) Factor: $-5h^2 - 20h + 60$

Always check to see if there is a GCF you can factor out first! If there is a negative number in front of the x^2, factor out the negative as well.

a≠1!

gcf? (-5) $\quad -5(h^2+4h-12)$

copy (-5) and proceed! $\quad -5(h-2)(h+6)$

① reorder? ✓

② × to -12 + to +4

$-3 \cdot 4 \to +1$
$3 \cdot -4 \to -1$
$-2 \cdot +6 \to +4$
$2 \cdot -6 \to -4$

Check: FOIL first!

$-5(h^2+6h-2h-12)$
$-5(h^2+4h-12)$
$-5h^2-20h+60$ ✓

Circle answer!

Ex 6) Factor: $-12 - 9g + 3g^2$

reorder/gcf: $\quad 3g^2-9g-12$

$3(g^2-3g-4)$

$3(g+1)(g-4)$

× to -4 + to -3

$-1 \cdot 4 \to 3$
$1 \cdot -4 \to -3$
$2 \cdot -2 \to 0$

Check: FOIL first!

$3(g^2-4g+1g-4)$
$3(g^2-3g-4)$
$3g^2-9g-12$ ✓ Circle answer

Ex 7) Factor: $2x^2 - 6x - 80$

reorder? gcf?

$2(x^2-3x-40)$

need × to -40 + to -3

$2(x-8)(x+5)$

Is -1 × 40 smart to check? not really!

$-1 \cdot 40 = 39$

try: $-8 \times 5 \to -3$
$8 \times -5 \to +3$

Check: FOIL first!

$2(x^2+5x-8x-40)$
$2(x^2-3x-40)$
$2x^2-6x-80$ ✓

Ex 8) Factor: $x^2 + 1x - 2$

× to -2 + to +1

$1 \times -2 \to -1$
$-1 \times 2 \to (+1)$ ✓

$((x-1)(x+2))$

Reflection: Does the order in which the binomial factors are written affect the solution? Explain.

$(x-1)(x+2)$
$= x^2+2x-1x-2$
$= x^2+x-2$

$(x+2)(x-1)$
$= x^2-1x+2x-2$
$= x^2+x-2$

No! You get same product regardless of the order you multiply in.

3.6 – Polynomials of the Form $ax^2 + bx + c$, $a \neq 1$

Name: Notes key
Date:

Goal: to extend the strategies for multiplying binomials and factoring trinomials

Toolkit:

- Multiplying binomials
- Factoring

Main Ideas:

<u>Factoring by Decomposition:</u> (needed when the $a \neq 1$ in $ax^2 + bx + c$)

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable. *(biggest to smallest)*

Step 2: Find two numbers that multiply to equal ac and add to equal b *(add to the middle, multiply to product of first and last)*

Step 3: Re-write the expression but split or *decompose* the b term using the two numbers from step 2.

Step 4: Now the expression has FOUR terms, so we can factor by grouping the first two terms and the last two terms.

Step 5: When fully factored, the remaining two brackets <u>need to be identical</u>! These are now a common factor, and can be factored out, and what is left becomes the components of the second bracket.

Factor by Grouping

take out common bracket

Ex. 1) Factor the following by grouping: no gcf for all 4, but <u>each pair</u> has a gcf!

a) $3x^2 - 3x - 2x + 2$

gcf: $3x$ gcf: -2 so match

$3x(x-1) - 2(x-1)$

$(x-1)(3x-2)$

Can check!

b) $2x^2 - 4x + x - 2$

$2x(x-2) + 1(x-2)$

$(x-2)(2x+1)$

Factoring a trinomial of the form $ax^2 + bx + c$

notice that a (the number in front of x^2) is not = 1 in any of these questions!

Ex 2) Factor the trinomial: $4g^2 + 11g + 6$ by decomposition

$ac = 4 \times 6 = 24$; $a=4$ $b=11$ $c=6$

✓ gcf
✓ reorder

② Need to
× to 24 + to 11

$1 \times 24 \xrightarrow{+} 25$
$2 \times 12 \xrightarrow{+} 14$
$3 \times 8 \xrightarrow{+} 11$! ✓

③ split ⓑ into $3g$ and $8g$
④ grouping

① $4g^2 + 11g + 6$
②
③ $4g^2 + 3g + 8g + 6$
④ $g(4g+3) + 2(4g+3)$
⑤ $(4g+3)(g+2)$

check
$(4g+3)(g+2)$
$4g^2 + 8g + 3g + 6$
$4g^2 + 11g + 6$ ✓

Ex 3) Factor the trinomial $-7m - 10 + 6m^2$

reorder:

$ac = -60$

$6m^2 - 7m - 10$

× to -60 + to -7
$6 \times -10 \rightarrow -4$
$3 \times -20 \rightarrow -17$
$4 \times -15 \rightarrow -11$
$5 \times -12 \rightarrow -7$ ✓

$6m^2 + 5m - 12m - 10$

$m(6m+5) - 2(6m+5)$

$(6m+5)(m-2)$

Ex 3) Factor: $8p^2 - 18pq - 5q^2$

$a=8 \quad b=-18 \quad c=-5$

in order ✓

$\times ac, +b \Rightarrow \times -40, + -18$

$(-20, 2)$

$8p^2 - 18pq - 5q^2$

$8p^2 + 2pq - 20pq - 5q^2$

$2p(4p+q) - 5q(4p+q)$

$(4p+q)(2p-5q)$

Ex 4) Factor: $6x^2 + 14x - 12$

GCF $2(3x^2 + 7x - 6)$

$a=3, b=7, c=-6$

$\times -18, + 7$

$(9, -2)$

$2(3x^2 + 7x - 6)$

$3x^2 + 9x - 2x - 6$

$3x(x+3) - 2(x+3)$

$2(x+3)(3x-2)$

If you can make a trinomial have a=1 by removing a G.C.F., then you can use "the simple way"!

Ex 5) Factor: $3x^2 + 6x - 9$

GCF $3(x^2 + 2x - 3)$

$a=1 \quad b=2 \quad c=-3$

remember, if $a=1$, can factor easier!

$\times c \quad + b$

$\times -3 \quad + 2$

$(3, -1)$

$3(x+3)(x-1)$

Ex 6) Find an integer to replace □ so that the trinomial can be factored. How many integers can you find?

$4x^2 + \square x + 9$

$\times ac, + b$

$\times 36, + b$

factors of 36:

$36, 1 \Rightarrow 37$

$-36, -1 \Rightarrow -37$

$18, 2 \Rightarrow 20$

$-18, -2 \Rightarrow -20$

$12, 3 \Rightarrow 15$

$-12, -3 \Rightarrow -15$

$9, 4 \Rightarrow 13$

$-9, -4 \Rightarrow -13$

$6, 6 \Rightarrow 12$

$-6, -6 \Rightarrow -12$

any of these integers!

10 different integers in total!

Reflection: Will decomposition work if the *a* value of a trinomial is 1? Do an example to prove this.

Yes!

$x^2 + 2x - 3$

$x^2 - 1x + 3x - 3$

$x(x-1) + 3(x-1)$

$(x-1)(x+3)$ ✓

$\times ac \quad + b$

$\times -3 \quad + 2$

$3, -1$

3.8 – Factoring Special Polynomials

Name: Key
Date:

Goal: to investigate perfect square trinomials and difference of squares

Toolkit:	**Main Ideas:**
• Finding a square root • Finding GCF • Multiplying Polynomials	

Definitions:

Perfect Square Trinomial: a trinomial of the form $m^2 + 2mn + n^2$; it can be factored as $(m + n)^2$
or of the form $m^2 - 2mn + n^2$; it can be factored as $(m - n)^2$

Difference of Squares: a binomial of the form $m^2 - n^2$; it can factored as $(m - n)(m + n)$

Warmup: Factor the trinomial $4x^2 - 4x + 1$ using decomposition.

$ac = (4)(1) = 4$

$b = -4$

$-2, -2$

$4x^2 - 2x - 2x + 1$

$2x(2x-1) - 1(2x-1)$

$(2x-1)(2x-1) = (2x-1)^2$

Factoring a perfect square trinomial

Decomposition works, but it is time consuming. Test to see if the trinomial is a perfect square! If so, it will be quicker to factor. $4x^2 - 4x + 1$

Step 1: Is the trinomial in order? Yes   Can you factor out a GCF? No

Step 2: Are the first and last terms perfect squares? Yes

Step 3: Make two brackets, and write the square roots into each. Then, figure out if the brackets should have a '+' or '-' in between the terms.

$(2x - 1)(2x - 1) = (2x-1)^2$

Step 4: Now test that the middle terms (the 'O' and 'I' of FOIL) add to the middle term of the original polynomial. If so, the trinomial is a perfect square.

$-2x - 2x = -4x$ ✓ Yes.

Ex 1) Factor the trinomial: $36x^2 + 12x + 1$

In order? Yes
GCF? No

$(6x+1)(6x+1)$

Check middle term:
$6x + 6x = 12x$ ✓

$(6x+1)^2$

Ex 2) Factor the trinomial: $18x^2 - 48xy + 32y^2$

In order? Yes
GCF? Yes

$2(9x^2 - 24xy + 16y^2)$

$2(3x - 4y)(3x - 4y)$

Check middle term:
$-12xy - 12xy = -24xy$ ✓

$2(3x-4y)^2$

Ex 3) Factor the trinomial: $25c^2 - 29cd + 4d^2$

In order? Yes
GCF? No

$(5c - 2d)(5c - 2d)$

Check middle term:
$-10cd - 10cd = -20cd$ X

Therefore, not a perfect square trinomial. Must factor by decomp.

$25c^2 - 29cd + 4d^2$

$25c^2 - 25cd - 4cd + 4d$

$25c(c-d) - 4d(c-d)$

$(c-d)(25c-4d)$

Factoring a Difference of Squares

Difference of Squares is only possible if you have a binomial. The binomial must have a SUBTRACT (difference) in between two PERFECT SQUARES (of squares).

Ex 4) Factor the binomial: $81m^2 - 49$

Step 1: Is there a subtract in the middle? Yes

Step 2: Is each term a perfect square? Yes

Step 3: If not, is there a GCF to factor out? No

Step 4: Make two brackets, one with a '+' and one with a '-'.

Step 5 Square root each term and put into the appropriate position in each bracket.

$(9m+7)(9m-7)$

CHECK: $(9m+7)(9m-7)$

$= 81m^2 - 63m + 63m - 49$

$= 81m^2 - 49$ ✓

Ex 5) Factor: $m^2 - 36$

$(m+6)(m-6)$

Why is one bracket '+' and one '-' ?

This will cause the middle terms to be opposites, thereby adding to zero.

Ex 6) Factor: $32v^2 - 2w^2$

$2(16v^2 - w^2) = 2(4v+w)(4v-w)$

Ex 7) Factor: $\frac{x^2}{25} - \frac{y^2}{4}$

$\left(\frac{x}{5} + \frac{y}{2}\right)\left(\frac{x}{5} - \frac{y}{2}\right)$

Ex 8) Factor: $x^2 + 9$

not a difference of squares

A sum of squares CANNOT be factored.

Ex 9) Factor: $2x^4 - 162$

$2(x^4 - 81)$

$2(x^2-9)(x^2+9)$

sum of squares, cannot factor.

another difference of squares

$2(x+3)(x-3)(x^2+9)$

*If you have a 4th power variable, there is a good chance there will be TWO LAYERS of factoring to complete.

Reflection: Does a sum of squares factor? Explain.

3.9 – Factoring Synthesis

Name: Key
Date:

FACTORING FLOW CHART

STEP 1 Take out COMMON FACTORS (GCF)

STEP 2 Ask: How many terms are there?

TWO

Test for **difference of squares:**

*You need **subtraction** ("difference") and each term must be a **perfect square**

If you don't have perfect squares, check to see if you can factor out a GCF.

$a^2 - b^2 = (a + b)(a - b)$

Example:
$4x^2 - 9$
$(2x + 3)(2x - 3)$

Example:
$2m^2 - 32n^2$
$2(m^2 - 16n^2)$
$2(m + 4n)(m - 4n)$

Example:
$4w^2 + 9y^2$
cannot factor As it is a SUM of squares

THREE

Factoring **trinomials:** $ax^2 + bx + c$
Is the trinomial in order?
Can you factor out a GCF?

Type 1: a = 1
Example: $x^2 - 3x + 2$
Ask: what ADDS to "b" (here –3)
& MULTIPLIES to "c" (here +2)
Answer: –1, –2
Write factors: $(x - 1)(x - 2)$

Type 2: a ≠ 1
Is it a perfect square trinomial?
Are first and last terms perfect squares?
Is the middle term correct?
Example: $4x^2 - 12x + 9$
Factor using square roots:
$(2x - 3)(2x - 3)$
Middle term: $-6x - 6x = -12x$

If it isn't a perfect square trinomial, factor using DECOMPOSITION.
Example: $2x^2 - x - 1$
Ask: what ADDS to "b" (here –1)
& MULTIPLIES to "ac" (here $2(-1) = -2$)
Answer: –2, 1
Use these to split (decompose) the middle term into two separate terms:

$$2x^2 - x - 1$$
$$\underline{2x^2 - 2x} + \underline{1x - 1}$$

Factor using grouping:

$$2x(x - 1) + 1(x - 1)$$

See if two brackets are the same.
Factor the bracket out front as a GCF, & the 'leftovers' make up the 2nd bracket.

$$(x - 1)(2x + 1)$$

STEP 3 Ask: **FF?** Look inside each factor (bracket) and see if you can **FACTOR FURTHER.**
*If the original question has an x^4 term, there is a good chance there will be 2 layers of factoring!

Practice factoring expressions using the flowchart for assistance.	Ex 1) Factor: $2x^2 - 22x + 60$

$a = 1$ now so

GCF $2(x^2 - 11x + 30) \rightarrow$ two numbers that $\times c$ and $+ b$

$\times 30, + -11$

$2(x-6)(x-5)$

$-6, -5$

can go right to brackets.

Ex 2) Factor: $p^2 - 25q^2$ ← subtract

diff of squares $(p+5q)(p-5q)$

Ex 3) Factor: $3y^2 - 7y - 6$

No GCF

No perfect squares

thus decomposition

$\times ac, + b$

$\times -18, + -7$

$-9, 2$

$3y^2 - 7y - 6$

$3y^2 - 9y + 2y - 6$

$3y(y-3) + 2(y-3)$

$(y-3)(3y+2)$

Ex 4) Factor: $4m^2 + 12m - 56$

GCF $4(m^2 + 3m - 14)$ $a = 1$, so two numbers that $\times c, + b$

$\times -14, + 3$ NOT POSSIBLE

so can't factor further

Ex 5) Factor: $9x^2 - 42xy + 49y^2$

Perfect squares!

$(3x-7y)(3x-7y)$

test middle term: $-21xy - 21xy = -42xy$ ✓

$(3x-7y)^2$

Ex 6) Factor: $8b^2 + 2c^2$

GCF $2(4b^2 + c^2)$

← sum of squares so cannot factor further

Ex 7) Factor: $8x^2 + 40x + 18$

GCF $2(4x^2 + 20x + 9)$

perfect squares!

$2(2x+3)(2x+3)$

test middle term: $6x + 6x = 12x$ X

must factor by decomp:

$\times ac, + b$ so $\times 36, + 20 \Rightarrow 18, 2$

$2[4x^2 + 2x + 18x + 9]$

$2[2x(2x+1) + 9(2x+1)]$

$2(2x+1)(2x+9)$

Ex 8) Factor: $32x^2 - 50y^2$

GCF $2(16x^2 - 25y^2)$ Diff of squares!

$2(4x+5y)(4x-5y)$

Ex 9) Factor: $3n^4 - 48$

GCF $3(n^4 - 16)$ diff of sq!

$3(n^2+4)(n+2)(n-2)$

$3(n^2+4)(n^2-4)$

sum of squares ↗ ↖ diff of squares!

Reflection: