3.3 – Common Factors of a Polynomial

Name:
Date:

Goal: to determine the factors of a polynomial by identifying the GCF

Toolkit:	**Main Ideas:**
• Finding the GCF • Distributive Property	

Factor a binomial using the GCF	Ex 1) Factor the binomial: $3g + 6$ Ex 2) Factor the binomial: $- 8y + 16y^2$
Factor a trinomial using the GCF	Ex 3) Factor the trinomial: $3x^2 + 12x - 6$ Ex 4) Factor the trinomial: $6 - 12z + 18z^2$
Factor polynomials in more than one variable	Ex 5) Factor the trinomial: $-20c^4d - 30c^3d^2 - 25cd$

Reflection: How are the processes of factoring and expanding related?

3.5 – Factoring Trinomials of the form $x^2 + bx + c$, where a=1

Name:
Date:

Goal: to use models and algebraic strategies to multiply binomials and to factor trinomials.

Toolkit:	**Main Ideas:**
• Factoring	

Definitions:
Descending order: the terms are written in order from the term with the greatest exponent to the term with the least exponent
Ascending order: the terms are written in order from the term with the least exponent to the term with the greatest exponent

Steps for Factoring a Trinomial in the form: x^2+bx+c, where a=1

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable *(biggest to smallest)*
Step 2: Find two numbers that multiply to equal the *c* term and add to equal the *b* term (add to the middle, multiply to the end)
Step 3: Factor into two binomials using the numbers from step 2, with the variable from the question placed first in each bracket

Multiplying two binomials

Ex 1) Expand and Simplify: $(x-1)(x-7)$ use FOIL

Remember: expanding and factoring are opposite operations....they UNDO each other!

Factoring a trinomial in the form $x^2 + bx + c$

Ex 2) Factor the trinomial: $x^2 - 8x + 7$ ....we should end up with $(x-1)(x-7)$!

Notice that a (the number in front of the x^2) will always end up being 1 in these questions!

Ex 3) Factor: $a^2 - 2a - 8$

Factoring a trinomial written in ascending order

Ex 4a) Factor: $-30 + 7m + m^2$

b) $x^2 - 4xy + 21y^2$

Ex 5) Factor: $-5h^2 - 20h + 60$

Always check to see if there is a GCF you can factor out first! IF there is a negative number in front of the x^2, factor out the negative as well.

Ex 6) Factor: $-12 - 9g + 3g^2$

Ex 7) Factor: $2x^2 - 6x - 80$

Ex 8) Factor: $x^2 + x - 2$

Reflection: Does the order in which the binomial factors are written affect the solution? Explain.

3.6 – Polynomials of the Form $ax^2 + bx + c$, $a \neq 1$

Name:
Date:

Goal: to extend the strategies for multiplying binomials and factoring trinomials

Toolkit:	**Main Ideas:**
• Multiplying binomials • Factoring	

Factoring by Decomposition: (needed when the $a \neq 1$ in $ax^2 + bx + c$)

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable *(biggest to smallest)*
Step 2: Find two numbers that multiply to equal *ac* and add to equal *b* *(add to the middle, multiply to **product** of first and last)*
Step 3: Re-write the expression but split or *decompose* the *b* term using the two numbers from step 2.
Step 4: Now the expression has FOUR terms, so we can factor by grouping the first two terms and the last two terms.
Step 5: When fully factored, the remaining two brackets need to be identical! These are now a common factor, and can be factored out, and what is left becomes the components of the second bracket.

Factor by Grouping

Ex. 1) Factor the following by grouping:

a) $3x^2 - 3x - 2x + 2$

b) $2x^2 - 4x + x - 2$

Factoring a trinomial of the form $ax^2 + bx + c$

Ex 2) Factor the trinomial: $4g^2 + 11g + 6$ by decomposition

notice that a (the number in front of x^2) is not = 1 in any of these questions!

Ex 3) Factor the trinomial: $-7m - 10 + 6m^2$

Ex 3) Factor: $8p^2 - 18pq - 5q^2$

Ex 4) Factor: $6x^2 + 14x - 12$

If you can make a trinomial have a=1 by removing a G.C.F., then you can use "the simple way"!

Ex 5) Factor: $3x^2 + 6x - 9$

Ex 6) Find an integer to replace □ so that the trinomial can be factored. How many integers can you find?

$4x^2 + \square x + 9$

Reflection: Will decomposition work if the *a* value of a trinomial is 1? Do an example to prove this.

3.8 – Factoring Special Polynomials

Name:
Date:

Goal: to investigate perfect square trinomials and difference of squares

Toolkit:	**Main Ideas:**
• Finding a square root • Finding GCF • Multiplying Polynomials	

Definitions:

Perfect Square Trinomial: a trinomial of the form $m^2 + 2mn + n^2$; it can be factored as $(m + n)^2$
or of the form $m^2 - 2mn + n^2$; it can be factored as $(m - n)^2$

Difference of Squares: a binomial of the form $m^2 - n^2$; it can factored as $(m - n)(m + n)$

Warmup: Factor the trinomial $4x^2 - 4x + 1$ using decomposition.

Factoring a perfect square trinomial

Decomposition works, but it is time consuming. Test to see if the trinomial is a perfect square! If so, it will be quicker to factor. $4x^2 - 4x + 1$

Step 1: Is the trinomial in order? Can you factor out a GCF?

Step 2: Are the first and last terms perfect squares?

Step 3: Make two brackets, and write the square roots into each. Then, figure out if the brackets should have a '+' or '-' in between the terms.

Step 4: Now test that the middle terms (the 'O' and 'I' of FOIL) add to the middle term of the original polynomial. If so, the trinomial is a perfect square.

Ex 1) Factor the trinomial: $36x^2 + 12x + 1$

Ex 2) Factor the trinomial: $18x^2 - 48xy + 32y^2$

Ex 3) Factor the trinomial: $25c^2 - 21cd + 4d^2$

Factoring a Difference of Squares	Difference of Squares is only possible if you have a binomial. The binomial must have a SUBTRACT (difference) in between two PERFECT SQUARES (of squares). Ex 4) Factor the binomial: $81m^2 - 49$ Step 1: Is there a subtract in the middle? Step 2: Is each term a perfect square? Step 3: If not, is there a GCF to factor out? Step 4: Make two brackets, one with a '+' and one with a '-'. Step 5 Square root each term and put into the appropriate position in each bracket. CHECK: Ex 5) Factor: $m^2 - 36$ Why is one bracket '+' and one '-' ? Ex 6) Factor: $32v^2 - 2w^2$ Ex 7) Factor: $\frac{x^2}{25} - \frac{y^2}{4}$ Ex 8) Factor: $x^2 + 9$ Ex 9) Factor: $2x^4 - 162$ *If you have a 4^{th} power variable, there is a good chance there will be TWO LAYERS of factoring to complete.

Reflection: Does a sum of squares factor? Explain.

3.9 – Factoring Synthesis

Name:
Date:

FACTORING FLOW CHART

STEP 1 Take out COMMON FACTORS (**GCF**)

STEP 2 Ask: How many terms are there?

TWO

Test for **difference of squares:**

*You need **subtraction** ("difference") and each term must be a **perfect square**

If you don't have perfect squares, check to see if you can factor out a GCF.

$a^2 - b^2 = (a + b)(a - b)$

Example:
$4x^2 - 9$
$(2x + 3)(2x - 3)$

Example:
$2m^2 - 32n^2$
$2(m^2 - 16n^2)$
$2(m + 4n)(m - 4n)$

Example:
$4w^2 + 9y^2$
*cannot factor
As it is a SUM
of squares*

THREE

Factoring **trinomials:** $ax^2 + bx + c$
Is the trinomial in order?
Can you factor out a GCF?

Type 1: a = 1
Example: $x^2 - 3x + 2$
Ask: what ADDS to "b" (here –3)
& MULTIPLIES to "c" (here +2)
Answer: –1, –2
Write factors: $(x - 1)(x - 2)$

Type 2: $a \neq 1$
Is it a perfect square trinomial?
Are first and last terms perfect squares?
Is the middle term correct?
Example: $4x^2 - 12x + 9$
Factor using square roots:
$(2x - 3)(2x - 3)$
Middle term: $-6x - 6x = -12x$

If it isn't a perfect square trinomial, factor using DECOMPOSITION.
Example: $2x^2 - x - 1$
Ask: what ADDS to "b" (here –1)
& MULTIPLIES to "ac" (here $2(-1) = -2$)
Answer: –2, 1
Use these to split (decompose) the middle term into two separate terms:

$$2x^2 - \mathbf{x} - 1$$
$$\underline{2x^2 - \mathbf{2x}} + \underline{\mathbf{1x} - 1}$$

Factor using grouping:
$2x(x - 1) + 1(x - 1)$
See if two brackets are the same.
Factor the bracket out front as a GCF, & the 'leftovers' make up the 2nd bracket.
$(x - 1)(2x + 1)$

STEP 3 Ask: **FF?** Look inside each factor (bracket) and see if you can **FACTOR FURTHER.**
*If the original question has an x^4 term, there is a good chance there will be 2 layers of factoring!

Practice factoring expressions using the flowchart for assistance.	Ex 1) Factor: $2x^2 - 22x + 60$ Ex 2) Factor: $p^2 - 25q^2$ Ex 3) Factor: $3y^2 - 7y - 6$ Ex 4) Factor: $4m^2 + 12m - 56$ Ex 5) Factor: $9x^2 - 42xy + 49y^2$ Ex 6) Factor: $8b^2 + 2c^2$ Ex 7) Factor: $8x^2 + 40x + 18$ Ex 8) Factor: $32x^2 - 50y^2$ Ex 9) Factor: $3n^4 - 48$

Reflection: