3.1 – Factors and Multiples

Name:
Date:

Goal: to determine prime factors, greatest common factors, and least common multiples of whole numbers

Toolkit:

- Division
- Multiplication
- Writing repeated multiplication using powers, e.g. 2×2×2×2×2×2= 2^6

Main Ideas:

Definitions

Factor – a term which divides evenly into another term

Prime number – when a number has only 2 distinct factors (1 and itself). **Examples:** 2, 3, 7 etc..

Composite number – when a number has more than 2 factors. **Examples:** 4, 6, 9 etc..

Prime factorization – a term written as a product of prime factors

every composite number can be expressed as a product of prime factors

Greatest common factor (GCF) – the largest term which will divide evenly into a series of separate terms

Least (or Lowest) common multiple (LCM) – the smallest multiple which is common to series of separate terms

Prime Factorization

Ex1) Write the prime factorization for each of the **composite** numbers:

a) 3

b) 6
2 3
$2 \cdot 3$

c) 45
5 9
3 3
$3^2 \cdot 5$

d) 47

e) 3300
33 100
11 3 10 10
2 5 2 5
$2^2 \cdot 3 \cdot 5^2 \cdot 11$

Finding the GCF by listing all the factors of each number (the rainbow method)

extra ex

24, 50
3 8 5 10
4 2 2 5
2 2

Ex2) Determine the greatest common factor of 126 and 144

Method 1 – list all the factors and find the largest one in common *(write small!)*

126: 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126

144: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

GCF = 18

*Backup method

126
9 14
3 3 2 7

144
12 12
3 4 3 4
2 2 2 2

Finding the GCF by writing the prime factorization of each number

24: $2^3 \cdot 3$

50: $2 \cdot 5^2$

GCF = 2

Method 2

1) write the prime factorization for each number
2) highlight the factors that they have in common
3) multiply all the common factors together go get the GCF

126: $2 \cdot 3^2 \cdot 7$

144: $2^4 \cdot 3^2$

$GCF = 2 \cdot 3^2 = 18$

* take the least of each prime factor from the lists.

Finding the LCM by listing the first multiples of each number

extra ex

$24: 2^3 \cdot 3$

$50: 2 \cdot 5^2$

$LCM = 2^3 \cdot 3 \cdot 5^2 = 600$

Ex3) Find the least common multiple of 28, 42, and 63

Method 1 – list the first few multiples of each number until you find (the first, lowest) one in common

28: 28, 56, 84, 112, 140, 168, 196, 224, 252

42: 42, 84, 126, 168, 210, 252

63: 63, 126, 189, 252

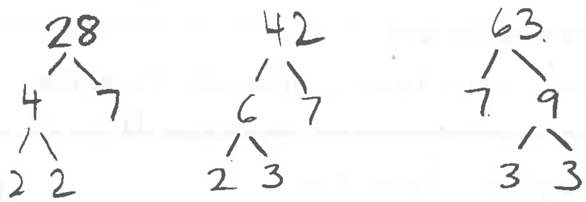
LCM = 252

*Backup method

Finding the LCM by writing the prime factorization of each number

Method 2

1) write the prime factors of each number
2) highlight the greatest power of each prime in ANY of the lists
3) multiply the greatest powers of each prime together to get the LCM



$28: 2^2 \cdot 7$

$42: 2 \cdot 3 \cdot 7$

$63: 3^2 \cdot 7$

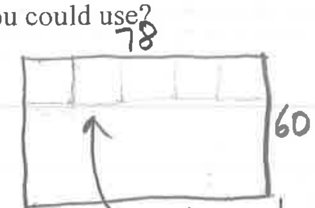
$LCM = 2^2 \cdot 3^2 \cdot 7 = 252$

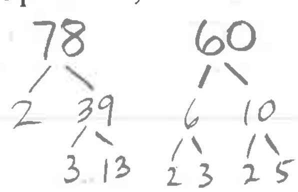
* Take the most of each prime factor from the lists!

What types of real-world problems involve GCFs and LCMs?

Ex4) Beside each problem, write whether you would need the GCF or the LCM, then answer the question!

a) A bathroom wall (the part above the bathtub) is a rectangle that measures 78" by 60". If you wanted to cover it exactly with square tiles, what is the largest possible square tile you could use?





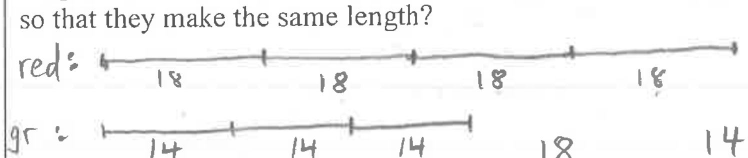
$78: 2 \cdot 3 \cdot 13$

$60: 2^2 \cdot 3 \cdot 5$

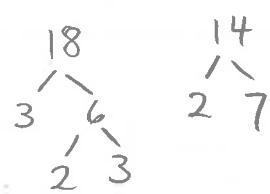
$GCF = 2 \cdot 3 = 6"$

need a value smaller than 78 and 60, so FACTOR, thus a GCF problem!

b) You have red bungee cords that are 18cm long and green bungee cords that are 14cm long. What is the shortest length of connected bungees you can make with each colour so that they make the same length?



LCM!



$18: 2 \cdot 3^2$

$14: 2 \cdot 7$

$LCM = 2 \cdot 3^2 \cdot 7 = 126cm$

Reflection: How can you remember the difference between a factor and a multiple? Write (or make) a memory trick to help you.

3.2 – Perfect Squares, Perfect Cubes, and their Roots

Name:
Date:

Goal: to identify perfect squares and perfect cubes, and to find square roots and cube roots

Toolkit:

- Prime factorization – no calculator!
- The opposite operation of squaring is the square root:

$$5^2 = 25 \text{ and } \sqrt{25} = 5$$

- The opposite operation of cubing is the cube root:

$$2^3 = 2 \times 2 \times 2 = 8 \text{ and } \sqrt[3]{8} = 2$$

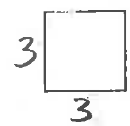
Main Ideas:

What is a Perfect Square?

A **perfect square** is a number that can be written as the product of 2 equal factors.

This means you can represent it as the **AREA OF A SQUARE!** $A = b \times b = b^2$

Picture an actual square!



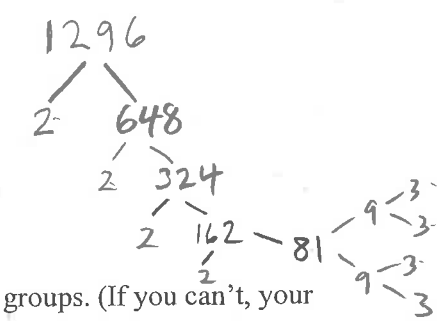
$3^2 = 3 \times 3 = 9$

The **square root** is the side length of the square

$\sqrt{9} = 3$

Determining a Square Root

Ex1) Determine the square root of 1296.
Step 1: Write 1296 as a product of its prime factors



$1296 = 2^4 \cdot 3^4$

$1296 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Step 2: Re-order the prime factors into TWO identical groups. (If you can't, your number is NOT a perfect square).

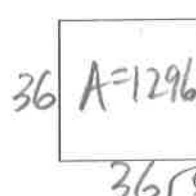
$1296 = (2 \cdot 2 \cdot 3 \cdot 3)(2 \cdot 2 \cdot 3 \cdot 3)$

Step 3: Multiply out each "group" again to see what number it represents

$1296 = 36 \cdot 36$

Since 1296 can be written as the product of TWO equal factors: 36 × 36, it can be represented as the area of a square.

The square root of 1296 is 36.



We write $\sqrt{1296} = 36$

Terminology: radical, radicand, index:

$\sqrt[2]{1296}$

index

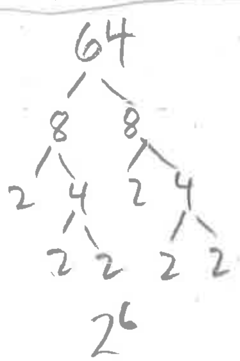
radical

radicand

extra examples

$2 \cdot 2 \cdot 7$

So 28 not a perfect square

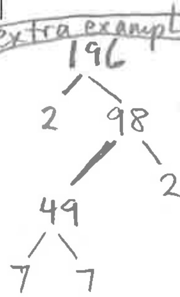


2^6

$2^3 \cdot 2^3$

So 64 is a perfect square

extra example



$196 = 2 \cdot 2 \cdot 7 \cdot 7$

$196 = (2 \cdot 7)(2 \cdot 7)$

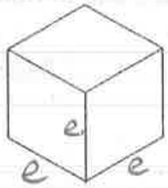
$\sqrt{196} = 14$

What is a Perfect Cube?

A **perfect cube** is a number that can be written as the product of 3 equal factors.

This means you can represent it as the **VOLUME OF A CUBE**! $V = e \times e \times e = e^3$.

Picture an actual cube!



The **cube root** is the edge length of the cube.

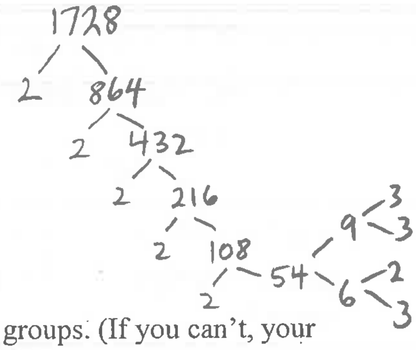
$$\sqrt[3]{V} = e$$

Determining a Cube Root

500
350
10
4

Ex2) Determine the cube root of 1728.

Step 1: Write 1728 as a product of its prime factors



$1728 : 2^6 \cdot 3^3$

Step 2: Re-order the prime factors into THREE identical groups. (If you can't, your number is NOT a perfect cube).

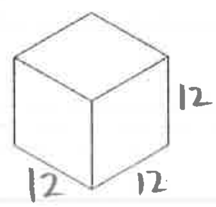
$$1728 : (2^2 \cdot 3)(2^2 \cdot 3)(2^2 \cdot 3)$$

Step 3: Multiply out each "group" again to see what number it represents

$$1728 = (12)(12)(12)$$

Since 1728 can be written as the product of THREE equal factors: 12 × 12 × 12, it can be represented as the volume of a cube.

The cube root of 1728 is 12.



We write $\sqrt[3]{1728} = 12$

radical, radicand, index?

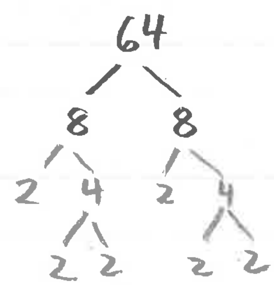
entire thing

Extend your thinking:

Ex3) Determine the edge length of a cube with volume $64x^6$.

$V_{cube} = 64x^6$

$e = \sqrt[3]{64x^6}$



$64 : 2^6$

$2^6 x^6$

$2^6 x^6 = (2^2 \cdot x^2)(2^2 \cdot x^2)(2^2 \cdot x^2)$

$e = 2^2 \cdot x^2$

$e = 4x^2$

Reflection: How could you ESTIMATE the square root or cube root of a number? (Think back to math 9?)

3.7 – Multiplying Polynomials

Name:
Date:

Goal: to expand monomial and binomial products (multiply out!)

Toolkit:

- Adding, subtracting, multiplying polynomials
- Multiplying powers with the same base: add the exponents

  Ex: $(x^3)(x^4) = x^{3+4} = x^7$

- Collecting like terms: same variable(s) with same exponents

  Ex: $2x^2 + 3x - 1x^2 + 2x + 1 = x^2+5x+1$

The sign in front of

Main Ideas:

The sign in front of a term is part of that term!

Definitions

Polynomial – poly many Nomial terms (terms separated by add and subtract)

Monomial – 1 term

Binómial – 2 terms

Trinomial – 3 terms

$-2x(x^2+4)$

$-2x^3-8x$

F irsts **O** utsides **I** nsides **L** asts

$(y-2)(y-5)$

$y^2-5y-2y+10$

$y^2-7y+10$

Ex1) Expand and simplify → translates to: distributive property (multiply) and then like terms (add/subtract)

a) $3x^2(x+3)$

$3x^3+9x^2$

b) $(x+2)(x+3)$

$x^2+3x+2x+6$

x^2+5x+6

c) $(2y+z)(3y-2z)$

$6y^2-4yz+3yz-2z^2$

$6y^2-yz-2z^2$

d) $(2a-1)(2a+3)+(a-1)(a-2)$

$4a^2+6a-2a-3+a^2-2a-a+2$

$4a^2+4a-3+a^2-3a+2$

$5a^2+a-1$

$(x-4)(x+5)$

$x^2+5x-4x-20$

x^2+x-20

Ex2) Expand and simplify:

a) $(x+3y)(x+y-3)$

$x^2 + xy - 3x + 3xy + 3y^2 - 9y$

$x^2 + 3y^2 + 4xy - 3x - 9y$

b) $(x+2)^3$

$(x+2)(x+2)(x+2)$

$x^2 + 2x + 2x + 4$

$(x^2+4x+4)(x+2)$

$(x+2)(x^2+4x+4)$

$x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$

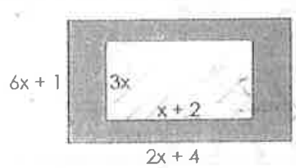
$x^3 + 6x^2 + 12x + 8$

c) $(r^2+3r-1)(2r^2-r+2)$

$2r^4 - r^3 + 2r^2 + 6r^3 - 3r^2 + 6r - 2r^2 + r - 2$

$2r^4 + 5r^3 - 3r^2 + 7r - 2$

Ex3) Find the area of the shaded region (simplified!):



$A_{shaded} = A_{big\ rec} - A_{small\ rec}$

$A_{shaded} = \underbrace{(2x+4)(6x+1)} - 3x(x+2)$

$12x^2 + 2x + 24x + 4$

$= 12x^2 + 26x + 4 - 3x^2 - 6x$

$A_{shaded} = 9x^2 + 20x + 4$