

Name: NOTES KEY  
Date: \_\_\_\_\_

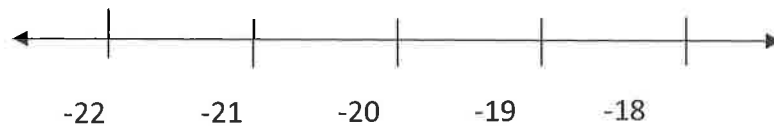
## CHAPTER 3 NOTES – Rational Numbers

*Calendar of Chapter: See the 'Homework' link on the webpage*

### What You'll Learn:

- 3.1 – Compare and order rational numbers
- 3.2/3.3 – Solve problems by adding & subtracting rational numbers
- 3.4A – Solve problems by multiplying & dividing rational numbers
- 3.4B – Explain and apply the order of operations with rational numbers

The label on a package of frozen cranberries says that it must be stored at a temperature between  $-18^{\circ}\text{C}$  and  $-22^{\circ}\text{C}$ . Name some possible temperatures and put them on the number line.



When in the 'real world' might negative fractions or decimals be needed?

### 3.1 – What is a Rational Number?

Focus: Compare and order rational numbers.

**INTERNET SEARCH:**

What is a rational number?

Are integers rational?

Which decimals are rational?

What is an irrational number, and give two examples.

**Comparing & Ordering Rational Numbers**

How do you order decimals?

Ex1 – Order from least to greatest:  
0.3, 0.31, 0.03, 0.29  
0.39, 0.039,  $0.\bar{3}$

Ex2 – Order from Least to greatest:  
-0.05, 0.55,  $-0.\bar{5}$ , 0.5,  
0.05, -0.55, -0.56

A number that can be written as a fraction.  
(any terminating (ending) or repeating decimal)

Yes (ex)  $2 = \frac{2}{1}$  ← written as a fraction

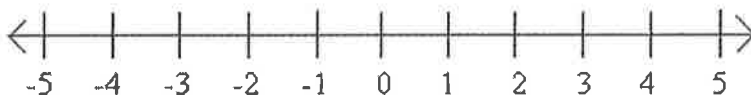
Write down 5 random fractions (pick at least one with a denominator of 9 or 11), and then divide each one.

What do you notice?

$\frac{3}{5} = 0.6$ ,  $\frac{2}{9} = 0.\bar{2}$ ,  $\frac{5}{11} = 0.\overline{45}$ ,  $\frac{37}{100} = 0.37$ ,  $\frac{7}{12} = 0.58\bar{3}$

Decimals that terminate (and) or repeat

A decimal that neither terminates or repeats  
(ex)  $\pi$ ,  $\sqrt{2}$ ,  $\sqrt{3}$



By place value

0.03, 0.039, 0.29, 0.3, 0.31,  
 $0.\bar{3}$ , 0.39

negs: -0.05,  $-0.\bar{5}$ , -0.55, -0.56

pos: 0.55, 0.5, 0.05

-0.56,  $-0.\bar{5}$ , -0.55, -0.05, 0.05,  
↑  
0.5, 0.55

↑  
furthest left on number line

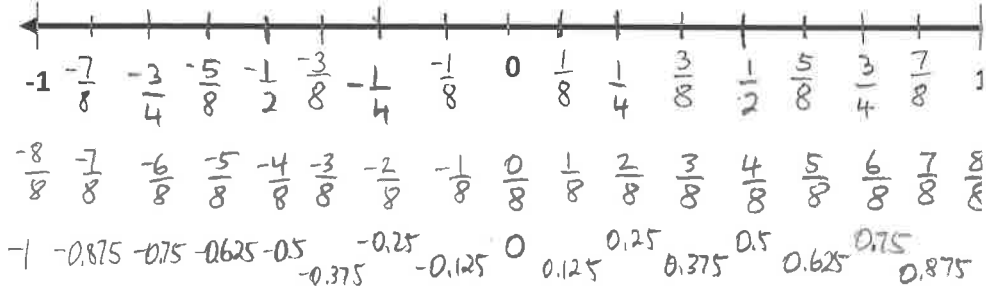
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	units	decimal	tenths	hundredths	thousandths	ten thousandths
								0	.	3			
								0	.	3	1		
								0	.	0	3		
								0	.	2	9		
										0	3	9	
										0	3	3	3

Ex3- On the # line:

a) Label:  $\frac{1}{2}, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{4}, \frac{3}{4}, -\frac{3}{4}, \frac{1}{8}, -\frac{1}{8}, \frac{3}{8}, -\frac{3}{8}, \frac{5}{8}, -\frac{5}{8}, \frac{7}{8}, -\frac{7}{8}$

b) Rewrite each fraction with a denominator of 8

c) underneath each fraction, change to decimal



Ex4

Order these numbers from least to greatest

$-\frac{2}{3}, -0.5, -1.31, 1\frac{1}{8}, \frac{3}{2}, -0.7, -1\frac{3}{10}, -0.5$

$-\frac{2}{3}, -0.5, -1.31, 1\frac{1}{8}, \frac{3}{2}, -0.7, -1\frac{3}{10}, -0.5$   
 $-0.6, -0.5, -1.31, 1.125, 1.5, -0.7, -1.3, -0.5$

neg:  $-0.6, -0.5, -1.31, -0.7, -1.3, -0.5$   
 pos:  $1.125, 1.5$

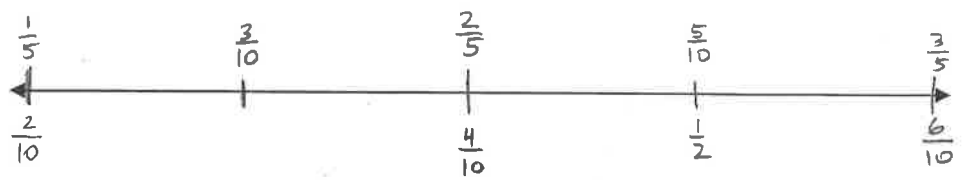
← furthest left on number line

$-1.31, -1\frac{3}{10}, -0.7, -\frac{2}{3}, -0.5, -0.5, 1\frac{1}{8}, \frac{3}{2}$

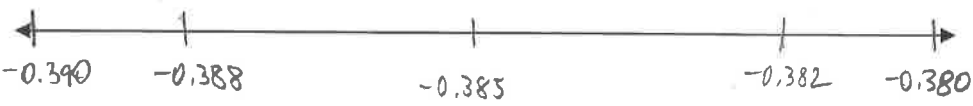
Ex5

Write 3 rational #s (fraction or decimal) between each pair of numbers on the number line.

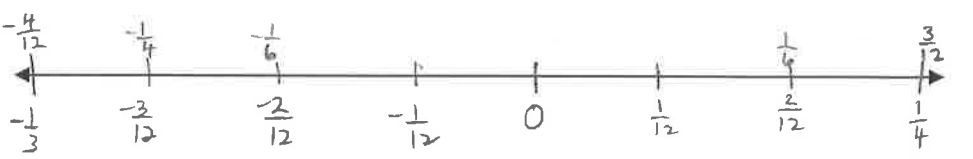
a)  $\frac{1}{5}$  and  $\frac{3}{5}$



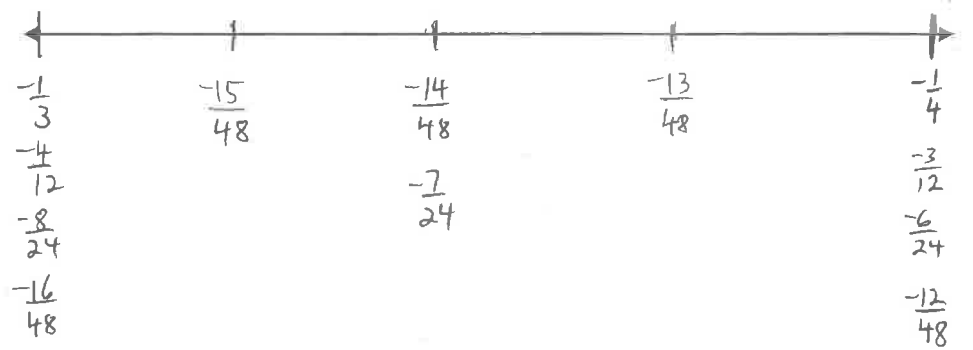
b) -0.38 and -0.39



c)  $-\frac{1}{3}$  and  $\frac{1}{4}$



d)  $-\frac{1}{3}$  and  $-\frac{1}{4}$



### 3.2/3.3 – Adding & Subtracting Rational Numbers

Focus: Solve problems that require adding & subtracting rational numbers.

#### Warmup:

Find each answer using strategies you've learned in the past.

- a)  $-3 + 5$
- b)  $4 + -3$
- c)  $-1 + -5$
- d)  $3 - 7$
- e)  $-2 - 6$
- f)  $3 - (-6)$

Not down your strategies for adding & subtracting integers.

#### Ex1

Let's add using fraction pies:

$$\frac{1}{2} + \frac{1}{3}$$

What is the problem with trying to combine the pie pieces?

What could we do to the pie pieces in order to be able to add them together?

Now, let's compare it to the method you've learned for adding fractions in middle school.

#### Ex2 – Subtract

$$\frac{3}{8} - \frac{1}{4}$$

- a) 2   b) 1   c) -6   d) -4
- e) -8   f) 9

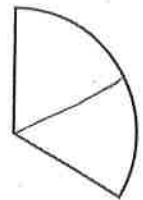
money strategy: negative is money owed  
positive is money gained

if you see  $4 - (-7)$   
turns into +

ex)  $3 - 7$   
 $\begin{matrix} \uparrow & \downarrow \\ \text{gain } 3 & \text{lose } 7 \end{matrix} = \text{lost } 4 = -4$

Be sure that you can add and subtract integers and get the correct answer consistently. If this is something you struggle with, work through section 3.2 and/or seek help from the teacher to learn any helpful strategies!

don't have same size pieces



cut them up to make all the pieces the same size

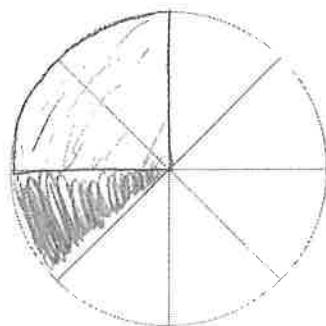
$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{1}{3} = \frac{2}{6}$$

$$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

getting common denominators = getting same size pieces!

$$\overset{3 \times}{\frac{1}{2}} + \overset{1 \times 2}{\frac{1}{3 \times 2}} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$



$$\begin{aligned} \frac{3}{8} - \frac{1}{4} \\ = \frac{3}{8} - \frac{2}{8} \\ = \frac{1}{8} \end{aligned}$$

Watch 'Act 1' of 'The Recipe'

Find your solution to 'The Recipe'

Watch 'Act 3'

How do you deal with negative fractions?

What are the steps involved with adding & subtracting fractions?

Ex3 - Simplify:

a)  $-\frac{2}{3} + \frac{1}{6}$

b)  $-\frac{3}{4} - \frac{5}{6}$

c)  $-\frac{1}{3} + \left(-\frac{7}{8}\right)$

d)  $-3\frac{1}{3} + 2\frac{5}{6}$

e)  $\frac{5}{-4} - \left(-3\frac{1}{5}\right)$

What questions come to mind? What do you need to know?

what types of quantities must be mixed in the measuring cup? Will it exceed 1 cup?

$$\frac{1^{\times 6}}{2^{\times 6}} + \frac{1^{\times 4}}{3^{\times 4}} + \frac{1^{\times 3}}{4^{\times 3}} = \frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{13}{12}$$

always put the negative on top  $\frac{1}{-3}$  becomes  $-\frac{1}{3}$   $-\frac{1}{3}$  becomes  $-\frac{1}{3}$

- ① Change any mixed to improper
- ② Put all negatives on top
- ③ Get common denominators
- ④ Add or subtract numerators
- ⑤ Reduce if necessary.

a)  $-\frac{2^{\times 2}}{3^{\times 2}} + \frac{1}{6}$     b)  $-\frac{3^{\times 3}}{4^{\times 3}} - \frac{5^{\times 2}}{6^{\times 2}}$     c)  $-\frac{1^{\times 8}}{3^{\times 8}} + \left(-\frac{7^{\times 3}}{8^{\times 3}}\right)$

$= -\frac{4}{6} + \frac{1}{6}$      $= -\frac{9}{12} - \frac{10}{12}$      $= -\frac{8}{24} + \frac{-21}{24}$

$= -\frac{3}{6} = \left(-\frac{1}{2}\right)$      $= \left(-\frac{19}{12}\right)$      $= \left(-\frac{29}{24}\right)$

d)  $-\left(3\frac{1}{3}\right) + 2\frac{5}{6}$     e)  $-\frac{5}{4} + 3\frac{1}{5}$

$= -\frac{10^{\times 2}}{3^{\times 2}} + \frac{17}{6}$

$= -\frac{20}{6} + \frac{17}{6}$

$= -\frac{3}{6} = \left(-\frac{1}{2}\right)$

$= -\frac{5^{\times 5}}{4^{\times 4}} + \frac{16^{\times 4}}{5^{\times 4}}$

$= -\frac{25}{20} + \frac{64}{20}$

$= \left(\frac{39}{20}\right)$

### 3.4A - Multiplying & Dividing Rational Numbers

Focus: Solve problems that require multiplying and dividing rational numbers

**Warmup:**

Multiply:

- a)  $-9 \times 4$
- b)  $3 \times -3$
- c)  $-24 \div -3$
- d)  $-3 \times 2 \div -1$
- e)  $-1 \times -1 \times -1$

Explain how you know whether the answer is positive or negative:

Ex1 - When multiplying fractions such as

$$\frac{1}{4} \times \frac{1}{2}$$

you may want to think: 'What's half of a quarter?' Use the fraction pie.

What are the **steps** You learned in middle school to multiply fractions?

Ex2 - Use fraction pies to find:

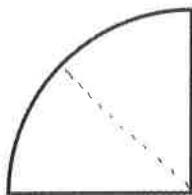
$$\frac{3}{4} \times \frac{1}{3}$$

Ex3 - Multiply

$$a) -\frac{3}{2} \times \frac{1}{-5}$$

a) -36 (b) -9 (c) 8 (d) 6 (e) -1

when multiplying and dividing integers, if there are an even number of integers the answer is positive. If there are an odd number of integers the answer is negative.



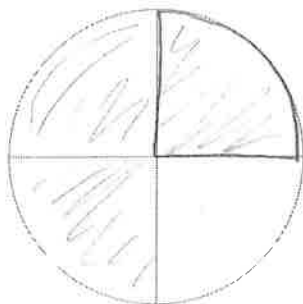
← here is  $\frac{1}{4}$ . What is half of that?

$$\frac{1}{8}$$

Same

- change mixed to improper
- multiply tops
- multiply bottoms
- reduce

$$\frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$$



What's  $\frac{1}{3}$  of  $\frac{3}{4}$ ?

$$\frac{1}{4}$$

Same

$$\frac{3}{4} \times \frac{1}{3} = \frac{3 \times 1}{4 \times 3} = \frac{3^{\cancel{3}}}{12^{\cancel{3}}} = \frac{1}{4}$$

**\*Remember:** Always put negatives on top!

$$-\frac{3}{2} \times \frac{-1}{5} = \frac{3}{10}$$

b)  $\frac{4}{7}(-3)$

Ex4 – Multiply

a)  $\left(\frac{-11}{7}\right)\left(-\frac{21}{44}\right)$

b)  $\left(2\frac{2}{3}\right)\left(-1\frac{3}{4}\right)$

**An Investigation:**

a)  $\frac{1}{4} \times \frac{1}{2}$

think: what's half of a quarter?

$$\frac{1}{4} \div 2$$

think: what's a quarter divided by 2?

What can you conclude about  $\times \frac{1}{2}$  and  $\div 2$ ?

If they are the same, then their opposites must also be the same.

Ex5 - Divide

$$\frac{1}{4} \div \frac{1}{2}$$

What are the steps to dividing fractions that you learned in middle school?

**\*Remember:** Always turn a whole number into a fraction first!

$$\frac{4}{7} \times \frac{-3}{1} = \frac{-12}{7}$$

**\*You can REDUCE fractions before multiplying them by 'cross reducing'. (can only do this process when MULTIPLYING fractions)**

$$\frac{-11}{7} \left( \frac{-21}{44} \right) = \frac{-1}{1} \left( \frac{-3}{4} \right) = \left( \frac{3}{4} \right)$$

b)  $\frac{8}{3} \left( -1\frac{3}{4} \right) = \frac{2}{3} \left( -\frac{7}{1} \right) = \left( -\frac{14}{3} \right)$

a)  $\frac{1}{8}$

$$\frac{1}{8}$$

they are the same!

What's the opposite of  $\times \frac{1}{2}$ ?  $\div \frac{1}{2}$  What's the opposite of  $\div 2$ ?  $\times 2$

Therefore,  $\div \frac{1}{2}$  &  $\times 2$  are the same things!  
 $\div \frac{1}{2}$  same as  $\times 2$

$$\frac{1}{4} \div \left( \frac{1}{2} \right) = \frac{1}{4} \times \left( \frac{2}{1} \right) = \frac{2}{4} = \left( \frac{1}{2} \right)$$

Notice how this is exactly what we figured out above!

- ① change mixed to improper
- ② Flip 2<sup>nd</sup> fraction and change  $\div$  to  $\times$
- ③ Put negatives on top
- ④ Multiply numerators (tops)
- ⑤ Multiply denominators (bottoms)
- ⑥ Reduce if necessary

What is a fraction that has been flipped called?  
 reciprocal

Ex6 - Divide

$$a) \left(-\frac{4}{3}\right) \div \left(\frac{-8}{3}\right)$$

$$b) -4\frac{2}{3} \div 1\frac{4}{5}$$

Ex7 - Simplify

$$\left(-\frac{1}{3}\right) \times \frac{4}{3} \div \left(\frac{5}{-6}\right)$$

$$a) \frac{-4}{3} \div \frac{-8}{3} = \frac{-4^{\cancel{+4}}}{3^{\cancel{+3}}} \times \frac{-3^{\cancel{+3}}}{8^{\cancel{+4}}} = \frac{-1}{1} \times \frac{-1}{2} = \left(\frac{1}{2}\right)$$

\*cross  
reduce

$$b) -\left(4\frac{2}{3}\right) \div 1\frac{4}{5}$$
$$= -\frac{14}{3} \div \frac{9}{5} = -\frac{14}{3} \times \frac{5}{9} = \left(\frac{-70}{27}\right)$$

$$\left(-\frac{1}{3}\right) \times \frac{4}{3} \div \frac{-5}{6}$$

$$= -\frac{4}{9} \div \frac{-5}{6}$$

$$= -\frac{4}{9} \times \frac{-6^{\cancel{+3}}}{5}$$

$$= -\frac{4}{3} \times \frac{-2}{5}$$

$$= \left(\frac{8}{15}\right)$$



### 3.4B - Order of Operations with Rational Numbers (DISCOVERY LESSON)

Focus: Explain and apply the order of operations with rational numbers.

**Warmup:**

- Find (circle) and explain the error that each student made
- Simplify the expression correctly.

Two students were asked to evaluate:  $-8 - 2(24 \div (-8))^2$

**Student 1:**

$$\begin{aligned} & -8 - 2(24 \div (-8))^2 \\ & -10(24 \div (-8))^2 \quad \text{subtracted first} \\ & -10(-3)^2 \\ & -10(9) \\ & -90 \end{aligned}$$

**Student 2:**

$$\begin{aligned} & -8 - 2(24 \div (-8))^2 \\ & -8 - 2(-3)^2 \\ & -8 - (-6)^2 \\ & -8 - 36 \quad \text{multiplied before exponents} \\ & -44 \\ & -8 - 2(24 \div (-8))^2 \\ & -8 - 2(-3)^2 \\ & -8 - 2(9) \\ & -8 - 18 \\ & -26 \end{aligned}$$

**B E D M A S**

P  
r  
a  
c  
k  
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E  
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d  
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i  
o  
n

S  
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r  
a  
c  
t

- \* for divide/multiply, do whichever comes first
- \* for add/subtract, do whichever comes first

What is the key word for order of operations?

Ex1 - Simplify

$$\begin{aligned} & 3\frac{2}{3} - \left(-1\frac{1}{3}\right) \div \frac{5}{3} + \left(-2\frac{1}{4}\right) \\ & \frac{11}{3} - \left(-\frac{4}{3}\right) \div \frac{5}{3} + \left(-\frac{9}{4}\right) \\ & \frac{11}{3} - \left(-\frac{4}{3}\right) \times \frac{3}{5} + \left(-\frac{9}{4}\right) \\ & \frac{11}{3} - \left(-\frac{4}{5}\right) + \left(-\frac{9}{4}\right) \\ & \frac{55}{15} + \frac{12}{15} - \frac{9}{4} \\ & \frac{4 \times 11}{4 \times 15} + \frac{-9 \times 15}{4 \times 15} = \frac{268}{60} + \frac{-135}{60} = \frac{133}{60} \end{aligned}$$

Ex2 - Simplify

$$\begin{aligned} & \left(\frac{8}{3}\right) \left(\frac{-1}{2}\right)^3 - 1\frac{1}{2} \div \frac{4}{-3} \\ & \frac{8}{3} \left(\frac{-1}{2^3}\right) - \frac{3}{2} \div \frac{-4}{3} \\ & \frac{8}{3} \left(\frac{-1}{8}\right) - \frac{3}{2} \div \frac{-4}{3} \\ & -\frac{1}{3} - \left(\frac{3}{2} \div \frac{-4}{3}\right) \\ & -\frac{1}{3} - \left(\frac{3}{2} \times \frac{-3}{4}\right) \\ & -\frac{1}{3} + \frac{9 \times 3}{8 \times 3} = \frac{-8}{24} + \frac{27}{24} = \frac{19}{24} \end{aligned}$$

Ex3 - Evaluate

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) - \left(-\frac{2}{3}\right) \div \left[\frac{1 \times 4}{3 \times 4} + \left(-\frac{3}{12}\right)\right]$$

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \frac{2}{3} \div \left[\frac{4}{12} + \frac{-3}{12}\right]$$

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) + \frac{2}{3} \div \frac{1}{12}$$

$$\frac{1}{4} + \frac{2}{3} \div \frac{1}{12}$$

$$\frac{1}{4} + \frac{2 \times 12}{3 \times 1}$$

$$\frac{1}{4} + \frac{24}{3}$$

$$\frac{1}{4} + 8$$

$$8\frac{1}{4} \text{ or } \frac{33}{4}$$