2.0 – Naming Triangles and Pythagoras

Name: Notes key
Date:

Goal: To learn how to correctly name triangles, their sides and their angles, and to use Pythagoras.

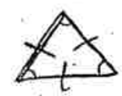
Toolkit:
- Labeling angles and sides of triangles
- All angles in a triangle add to ________
- Pythagoras: $a^2 + b^2 = c^2$ (c is hyp!)
- Labelling triangles from a target angle

Main Ideas:

Definitions

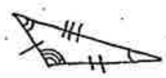
Right triangle – A triangle with a 90° angle in one corner. (note: other 2 angles must be acute – less than 90°).

Equilateral triangle – All 3 sides (and 3 angles!) are the same



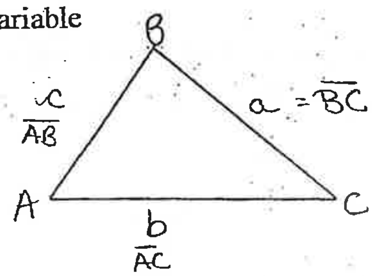
Isosceles triangle – 2 sides (and the 2 angles across from them) are equal.

Scalene triangle – no sides/angles are equal.



Labelling angles and sides of triangles

Ex 1) Draw a triangle, ΔABC, and label all angles and sides.
Label sides using both:
- One lower case variable
- Two endpoints (capital letters!)



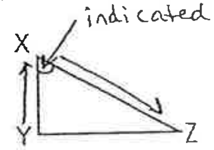
We label the side across from an angle (say, capital A) using the same letter, only in lower case.

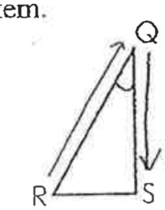
Note: Endpoints – capitals.

Three point system of naming angles – An angle is named using the two origins of the angle, and the vertex, with the vertex ALWAYS in the middle!

Ex2) Name each indicated angle using the three point system.

indicated angle





∠YXZ
X is in the middle!
or ∠ZXY

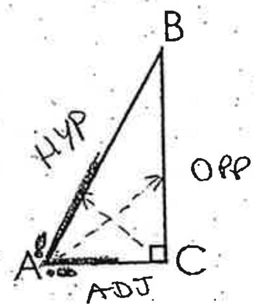
∠RQS
or ∠SQR

Labelling angles from a target angle

(Only for right triangles!)

In this chapter, we will also want to label the sides of a RIGHT triangle based their position in relation to a target angle which we use as a reference point.

Ex 3) In reference to angle A, label
-the hypotenuse (HYP)
-the side opposite to A (OPP)
- the side adjacent to A (ADJ)



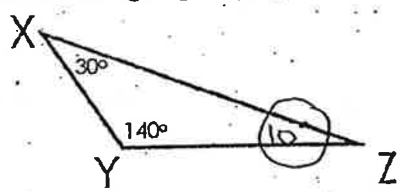
- HYP is always across from 90°
- A is the target angle. from there, ("stand at A" → draw footprints). label what side is opposite, adjacent (next to, but not the HYP).

Angles in a triangle

The sum of the angles in a triangle is 180°

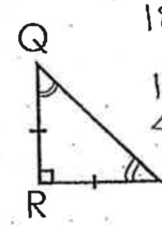
Ex4) Find the missing angle(s).

a)



$\angle Z = 180° - 140° - 30° = 10°$

b)



180° − 90° = 90° left for ∠Q, ∠S
∠Q = ∠S (isosceles!) so 90° ÷ 2 = 45° each

$\angle R = 90°$
$\angle Q = \angle S = 45°$

Pythagoras

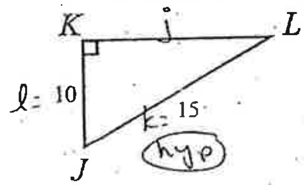
(Only for right triangles!)

Pythagoras – Remember, "c" MUST be the hypotenuse, or the side across from the right angle!

$$a^2 + b^2 = c^2$$

Ex 5) Name and find the missing side(s) (nearest tenth)

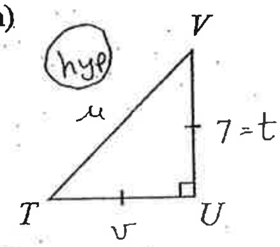
a)



$\boxed{j}$

$j^2 + l^2 = k^2$
$j^2 + 10^2 = 15^2$
$j^2 = 15^2 - 10^2$
$\sqrt{j^2} = \sqrt{125}$
$j = 11.2$

b)



$\boxed{v} = t = 7$

$\boxed{u}$ $v^2 + t^2 = u^2$
$7^2 + 7^2 = u^2$
$\sqrt{98} = \sqrt{u^2}$
$u = 9.9$

Reflection: Is it possible to have an equilateral triangle that is also a right triangle? Explain.

No! Equil. means all angles the same, and you can't have 90°-90°-90°!

2.1 – Angles from the Tangent Ratio

Name: Key
Date:

Goal: to develop the tangent ratio and relate it to the angle of inclination of a line

Toolkit:
- Similar Triangles
- Labeling sides and angles of a triangle

Main Ideas:

Terminology:

Hypotenuse: The longest side of a right triangle (and always opposite the right angle) (HYP)

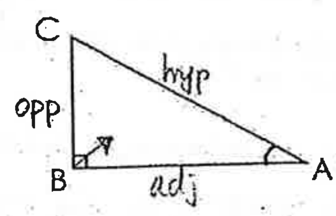
Opposite: The side that does NOT touch the angle (OPP)

Adjacent: The side that DOES touch the angle (and is not the hypotenuse) (ADJ)

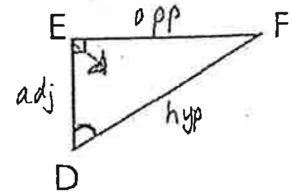
Naming Sides:

We name the sides of a right triangle (a triangle with a 90° angle) in relation to one of its acute angles (one of the angles that is NOT 90°)

Ex1)



TRY:



What is trigonometry?

the relationship between the angles and sides in a triangle

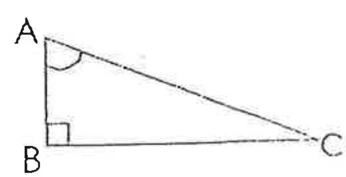
What is the TANGENT RATIO?

The relationship between an angle and its opposite and adjacent sides in a right triangle.

THE TANGENT RATIO

If $< A$ is an acute angle in a right triangle, then:

$$\text{Tan } A = \frac{\textit{length of side opposite} <A}{\textit{length of side adjacent to} <A}$$



You can use a scientific calculator to find an angle when you know its tangent. The $\boxed{\tan^{-1}}$ operation does this.

→ Shift Tan

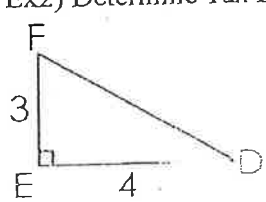
or

→ 2ndF Tan

Determining the Tangent Ratios for Angles:

* MAKE SURE CALCULATOR IS IN DEGREE MODE*

Ex2) Determine Tan D and Tan F. Then, determine <D and <F.



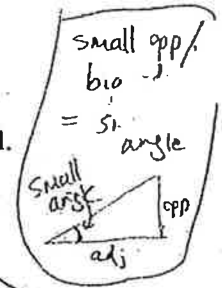
$\text{Tan } D = \frac{opp}{adj} = \frac{3}{4}$ $\quad \angle D = \text{Tan}^{-1}\frac{3}{4} = 36.9°$

$\text{Tan } F = \frac{4}{3}$ $\quad \angle F = \text{Tan}^{-1}\frac{4}{3} = 53.1°$

check: $36.9° + 53.1° + 90° = 180°$

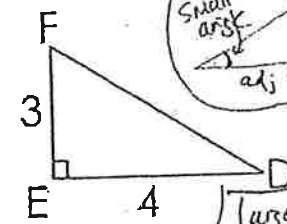
Write Tan D again. At the moment, the Tan D ratio is written as a fraction. Ratios can also be written as decimals. Write Tan D as a decimal:

Tan D = $\frac{3}{4} = 0.75$



Trigonometric **ratios** such as tangent can be written as a fraction or a decimal.

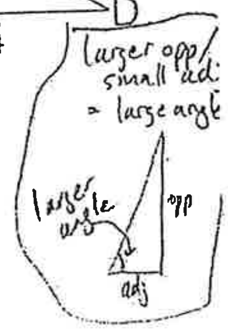
Why is the measure of <D = 36.9° if the ratio for Tan D = ¾ ?



Look at the triangle. If the opposite is less than the adjacent side, <D will be less than 45°. Thus, it is 36.9°.

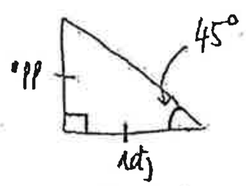
So, why is <F = 53.1° ?

Because the opposite side = 4 is greater than the adjacent side = 3, making a bigger angle



What would the opposite and adjacent sides have to be to have a 45° angle?

opp and adj sides would be equal

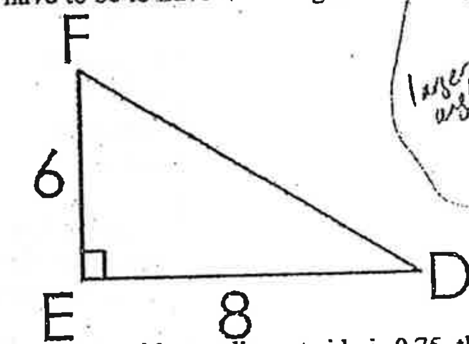


tan 45° = 1 check on calc!

top and bottom of fraction equal!!

What would the other acute angle measure be in this situation? 45°

b/c 45° + 45° + 90° = 180°



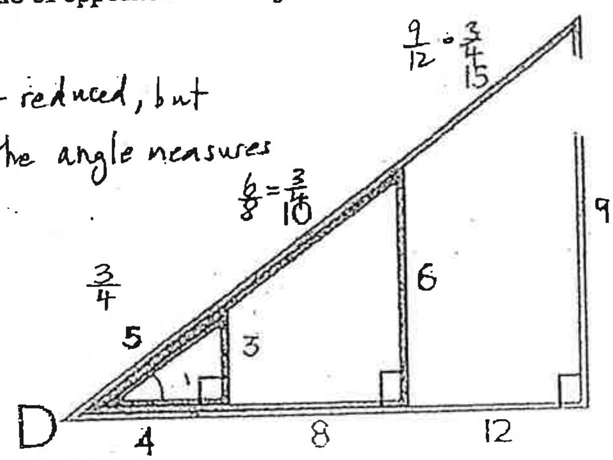
Back to Triangle DEF:

What would the measure of <D be for this triangle?

Tan D = $\frac{6}{8} = \frac{3}{4}$, <D = 36.9° ⇒ same!!

No matter how big the triangle, if the ratio of opposite side to adjacent side is 0.75, then the angle will measure 36.9°.

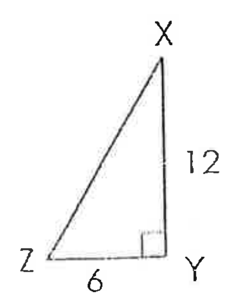
If the triangle is enlarged or reduced, but maintains its proportions, the angle measures will not change!



Ex3) Determine Tan X and Tan Z

Tan X = $\frac{6}{12} = \frac{1}{2} = 0.5$

Tan Z = $\frac{12}{6} = 2$

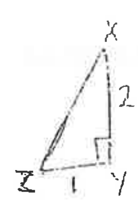


Now, determine <X and <Z:

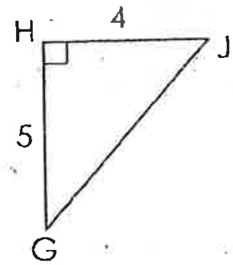
<X = tan⁻¹ 0.5 = 26.6°

<Z = tan⁻¹ 2 = 63.4°

Sketch another right triangle with the same angle measures:



Ex4) Determine the measures of <G and <J to the nearest tenth of a degree. Start by writing the tangent ratio properly.



$\text{Tan } G = \frac{4}{5} = 0.8$

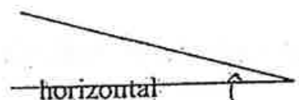
$\angle G = \tan^{-1} 0.8 = \underline{38.7°}$

$\text{Tan } J = \frac{5}{4} = 1.25$

$\angle J = \tan^{-1} 1.25 = \underline{51.3°}$

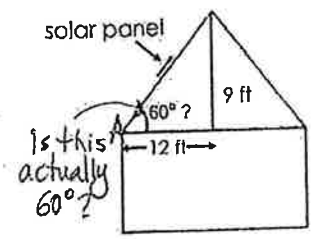
Definition:

Angle of Inclination – This is the ACUTE angle that a line makes with the horizontal



angle of inclination

Using the Tan Ratio to Determine the Angle of Inclination:



Ex6) The latitude of Fort Smith, NWT, is approximately 60°. Determine whether this design for a solar panel is best for Fort Smith.

$\tan A = \frac{9}{12} = 0.75$

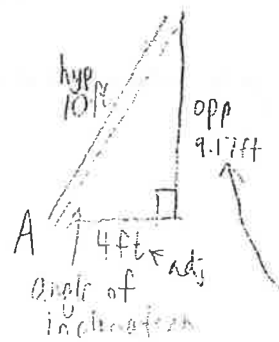
$\angle A = \tan^{-1} 0.75 = 36.9°$

$\angle A$ is an angle of inclination

No, this particular design would not maximize the amount of sun that hits the solar panel.

Ex7) A 10ft ladder leans against the side of a building with its base 4ft from the wall. What is the angle of inclination of the ladder?

DRAW A DIAGRAM!



can't use tan until we know opp...

PYTHAGORAS!

$a^2 + b^2 = c^2$

$a^2 + 4^2 = 10^2$

$a^2 + 16 = 100$

$a^2 = 84$

$a = \sqrt{84} = 9.17$

$\text{Tan } A = \frac{9.17}{4} = 2.29$

$\angle A = \tan^{-1} 2.29$

$= 66°$

The angle of inclination of the ladder is 66°.

Reflection: You have just studied the Tan ratio, which is the ratio of the opposite side to the adjacent side of a right triangle. What are the other two pairs of sides you could have in a right triangle? (think opp, adj, and hyp!)

2.2 – Sides from the Tangent Ratio

Name: Key
Date:

Goal: Apply the tangent ratio to calculate lengths of sides of triangles

Toolkit:
- Similar Triangles
- Labeling sides and angles of a triangle
- Tan Ratio (opposite and adjacent sides)

Main Ideas:

REMEMBER:

Calculator MUST be is DEGREE mode!

Terminology:

Direct Measurement: When we use a measuring instrument (eg. Ruler, protractor) to determine a length or an angle.

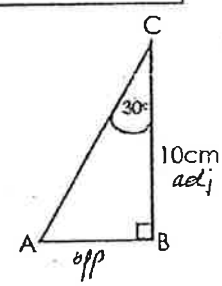
Indirect Measurement: When we use math concepts (eg. Trig, Pythagoras) to calculate a length or an angle

We can use the Tan ratio as a tool to calculate the length of a side of a right triangle *indirectly.*

Steps:
1) Use the Tan ratio ($\frac{opposite}{adjacent}$) to write an equation
2) When we know the measure of an angle (that is NOT the 90° angle!) and the length of one of the legs (not the hypotenuse), solve the equation to determine the length of the other leg.

Determining the Length of a Side **Opposite** a Given Angle:

Ex1) Determine the length of AB to the nearest tenth of a centimeter.
side c



$\tan 30° = \frac{c}{10}$

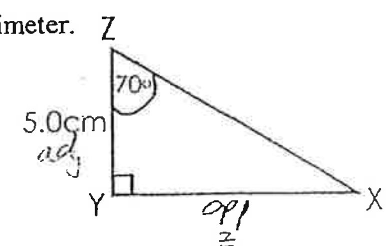
$(10)\tan 30° = \frac{c}{\cancel{10}}\cancel{(10)}$

$10 \tan 30° = c$

$c = 5.8$ cm

Does this seem reasonable? Yes!

Ex2) Determine the length of XY to the nearest tenth of a centimeter.
side z



$\tan 70° = \frac{z}{5}$

$5 \tan 70° = z$

$13.7\text{cm} = z$

reasonable
yes

Note: when solving a question where you have two equal fractions…..
"multiply the pair, divide by the spare!"

Ex 1. $\frac{3}{1} = \frac{8}{x}$

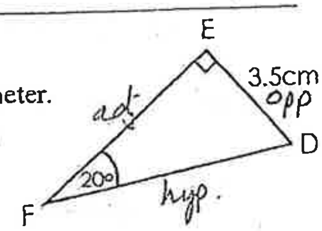
$x = (8 \times 1) \div 3$
$x = 2.67$

Ex 2. $\frac{2.4}{1} = \frac{y}{11}$

$x = (2.4 \times 11) \div 1$
$x = 26.4$

Determining the Length of a Side Adjacent a Given Angle:

Ex3) Determine the length of EF to the nearest tenth of a centimeter.
side d



$\tan 20° = \frac{3.5}{d}$

$\frac{\tan 20°}{1} = \frac{3.5}{d}$

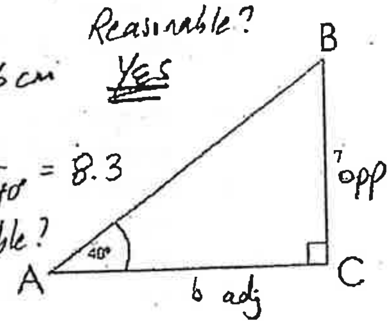
$d \tan 20° = (3.5)(1)$

$\frac{d \tan 20°}{\tan 20°} = \frac{3.5}{\tan 20°}$

$d = \frac{3.5}{\tan 20°} = 9.6\text{ cm}$

Reasonable? Yes

Ex 4) Find side AC to the nearest tenth.
Side b



$\tan 40° = \frac{7}{b}$

$\frac{\tan 40°}{1} = \frac{7}{b}$

$b \tan 40° = (7)(1)$

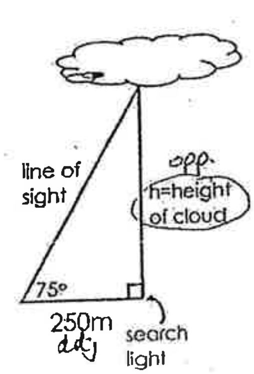
$\frac{b \tan 40°}{\tan 40°} = \frac{7}{\tan 40°}$

$b = \frac{7}{\tan 40°} = 8.3$

reasonable? Yes

Using the Tan Ratio to Solve an Indirect Measurement Problem:

Ex5) A searchlight beam shines vertically on a cloud. At a horizontal distance of 250m from the searchlight, the angle between the ground and the line of sight to the cloud is 75°. Determine the height of the cloud to the nearest metre.



$\tan 75° = \frac{opp}{250} = \frac{h}{250}$

$h = 250 \tan 75$

$h = 933\text{m}$

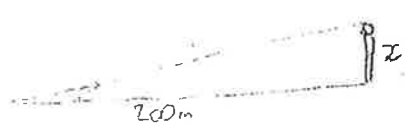
The cloud is 933m high.

Ex6) At a horizontal distance of 200m from the base of an observation tower, the angle between the ground and the line of sight to the top of the tower is 8°. How high is the observation tower, to the nearest metre?

Start by sketching and labeling a diagram to represent the information in the problem………

$\tan 8 = \frac{x}{200}$

$x = 200 \tan 8$



$x = 28\text{m}$

The tower is 28m high.

Reflection: Write, in your own words, how you can find the length of a side by using a known angle and a known side, and using the Tan Ratio.

2.4 – The Sine and Cosine Ratios

Name: Key.
Date:

Goal: to develop and apply the sine and cosine ratios to determine angle measures

Toolkit:

- Labeling sides and angles of a triangle
- What you have learned about the Tan ratio
- Angle of elevation vs depression

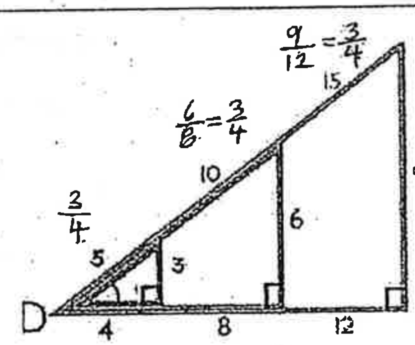
(HYP) – **Hypotenuse:** The longest side of a right triangle (and always opposite the right angle)
(OPP) – **Opposite:** The side that does NOT touch the angle
(ADJ) – **Adjacent:** The side that DOES touch the angle (and is not the hypotenuse)

Main Ideas:

Remember from yesterday that:

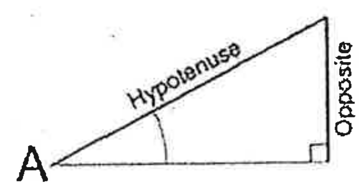
$$\text{Tan } D = \frac{\text{opposite side}}{\text{adjacent side}}$$

If the ratio of opposite to adjacent doesn't change, then angle D doesn't change. Look at angle D compared to the length of the opposite vs. the length of the adjacent sides.



You can also compare the ratio of other pairs of sides compared to the target angle. Tangent ratio is opposite side divided by adjacent side, but you can make a ratio with each of these sides and the hypotenuse, and these ratios can be related to the size of the target angle:

THE SINE RATIO



If < A is an acute angle in a right triangle, then

$$\text{Sin } A = \frac{\text{length of side opposite} \angle A}{\text{length of hypotenuse}}$$

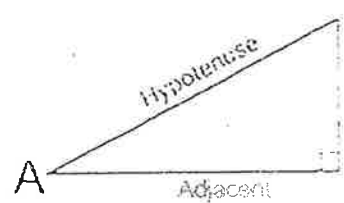
What values will the sine ratio always be between?

hypotenuse is always longest side, so sine ratio will always be a proper fraction, So between 0 and 1

If the opposite side is quite small compared to the hypotenuse, will the target angle be closer to 0^0 or 90^0 ? closer to 0°

THE COSINE RATIO

If < A is an acute angle in a right triangle, then

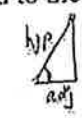


$$\text{Cos } A = \frac{\text{length of side adjacent} \angle A}{\text{length of hypotenuse}}$$

What values will the cosine ratio always be between?

0 and 1 for same reason as sine ratio

If the adjacent side is quite small compared to the hypotenuse, will the target angle be closer to 0^0 or 90^0? closer to $90°$. smaller the adj, bigger the target angle!!

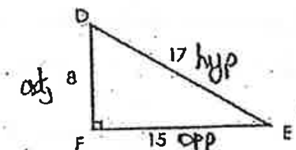


S O H C A H T O A

i n | o p p | h y p | c o s | a d j | h y p | t a n | o p p | a d j

Determining the Sine and Cosine of an Angle, and Determining the Measure of the Angle

Ex1) a) In triangle DEF, identify the side opposite <D, the side adjacent to <D, and the hypotenuse



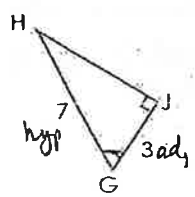
b) Determine the ratios Sin D and Cos D, and give the values as decimals (nearest hundredth)

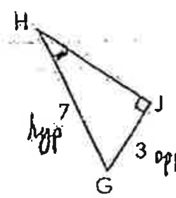
$\sin D = \frac{15}{17} = 0.88$ $\cos D = \frac{8}{17} = 0.47$

c) Determine angles D and E to the nearest tenth

$\sin^{-1} 0.88 = 61.9°$ $\angle E = 180° - 90° - 61.9° = 28.1°$

Ex2) Determine the measures of <G and <H to the nearest tenth of a degree.





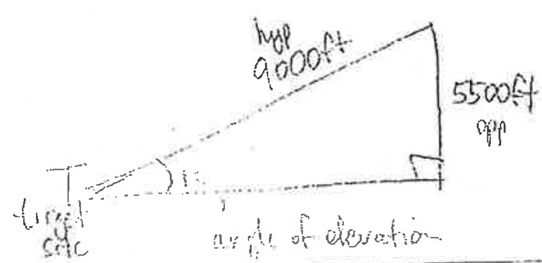
$\cos G = \frac{3}{7}$ $\sin H = \frac{3}{7}$

$<G = 64.6°$ $<H = 25.4°$

Using Sine or Cosine to Solve a Problem

Ex3) A water bomber is flying at an altitude of 5500 ft. The plane's radar shows that it is 9000 ft from the target site in a forest fire. What is the angle of elevation of the plane measured from the target site, to the nearest degree?

$\sin T = \frac{5500}{9000}$



$\angle T = \sin^{-1} \frac{5500}{9000}$

$\angle T = 38°$

The angle of elevation is $38°$

Reflection: What will you do to remember the calculator steps when finding an ANGLE (whether it's a sin, cos, or tan problem)?

2.5 – Missing Sides from Sine and Cosine

Name: Key
Date:

Goal: Use the sine and cosine ratios to determine lengths indirectly.

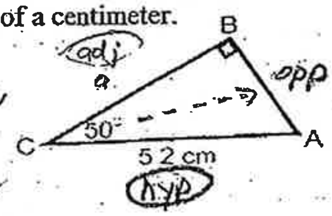
Toolkit:

- What you have learned about the Tan ratio
- Angle of elevation vs depression
- SOHCAHTOA

Main Ideas:

Using the sine or cosine ratio to determine the length of a leg.

Ex1) Determine the length of side a to the nearest tenth of a centimeter.



which ratio? we know $\angle C = 50°$, and the hypotenuse is 5.2cm, and side A is ADJACENT $\angle C$, so use ratio that is $\frac{adj}{hyp} \to \cos$

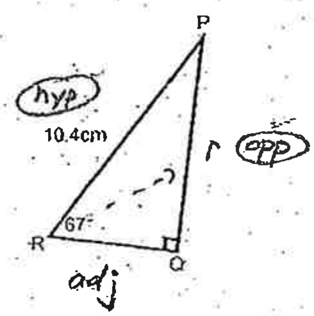
$\cos \angle C = \frac{adj}{hyp}$

$\frac{\cos 50°}{1} = \frac{a}{5.2} \longrightarrow$ $a = 5.2 \times (\cos 50°)$

$\boxed{a = 3.3\,cm}$

Ex2) Determine the length of side r to the nearest tenth of a centimeter.

which ratio is $\frac{opp}{hyp}$? $\to$ sin!



$\sin \angle R = \frac{opp}{hyp}$

$\sin 67° = \frac{r}{10.4}$

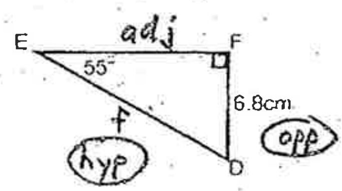
$r = 10.4 \times (\sin 67°)$

$\boxed{r = 9.6\,cm}$

Using the sine or cosine ratio to determine the length of the hypotenuse.

Ex. 3) Determine the length of side f to the nearest tenth of a centimeter.

which ratio is $\frac{opp}{hyp}$? $\to$ sin!



$\sin \angle E = \frac{opp}{hyp}$

$\frac{\sin 55°}{1} = \frac{6.8}{f}$

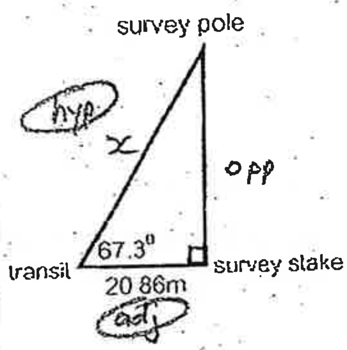
* multiply the pair, divide by the spare!

$f = \frac{6.8}{\sin 55°}$

$\boxed{f = 8.3\,cm}$

Solving an Indirect Measurement Problem

Ex. 4) A surveyor made the measurements shown in the diagram. How could the surveyor determine the distance from the transit to the survey pole to the nearest hundredth of a metre?



find x

which ratio?

we have $\frac{adj}{hyp} \rightarrow$ cos!

$\cos 67.3° = \frac{adj}{hyp}$

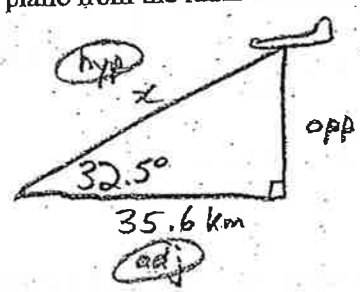
$\frac{\cos 67.3°}{1} = \frac{20.86}{x}$

$x = \frac{20.86}{\cos 67.3}$

$\boxed{x = 54.05m}$

The distance from the transit to the survey pole is 54.05m

Ex. 5) From a radar station, the angle of elevation of an approaching airplane is 32.5°. The horizontal distance between the plane and the radar station is 35.6km. How far is the plane from the radar station to the nearest tenth of a kilometer? (Draw a picture!)



which ratio is $\frac{adj}{hyp}$? $\rightarrow$ cos!

$\cos 32.5° = \frac{adj}{hyp}$

$\frac{\cos 32.5°}{1} = \frac{35.6}{x}$

$x = \frac{35.6}{\cos 32.5°}$

$\boxed{x = 42.2 km}$

The plane is 42.2km from the radar station.

Reflection: Explain when you might use the sine or cosine ratio instead of the tangent ratio to determine the length of a side in a right triangle.

Any time you need to involve the hypotenuse!

2.6 – Solving Triangles

Name:
Date:

Goal: Use a trigonometric ratio to solve a problem involving a right triangle

Toolkit:

- $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$ (SOHCAHTOA!)
- The sum of the angles in any triangle is 180°
- Pythagoras → $a^2 + b^2 = c^2$

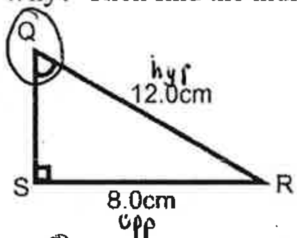
Main Ideas:

Which Trig Ratio should be used?

Find the missing angle or side using trig...

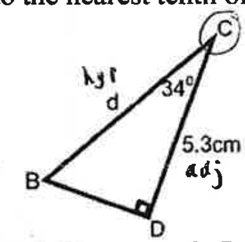
Ex1) To determine the measure of the indicated angle or side, which trig ratio would you use? Why? Then find the indicated angle or side, to the nearest tenth of a degree.

a)



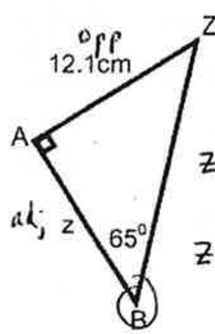
$\sin Q = \frac{8}{12}$; $\angle Q = \sin^{-1}\frac{8}{12} = 41.8°$

b)



$\cos 34° = \frac{5.3}{d}$; $d = \frac{5.3}{\cos 34°} = 6.4\text{cm}$

c)

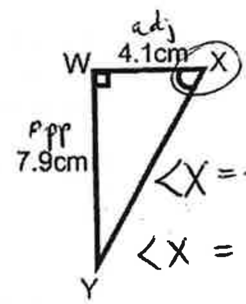


$\tan 65° = \frac{12.1}{z}$

$z = \frac{12.1}{\tan 65°}$

$z = 5.6\text{cm}$

d)



$\tan X = \frac{7.9}{4.}$

$\angle X = \tan^{-1}\frac{7.9}{4.1}$

$\angle X = 62.6°$

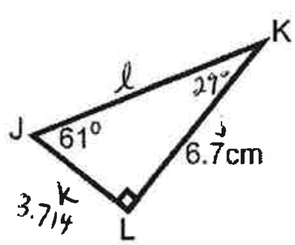
Solving a triangle means to determine the measures of all the angles and the lengths of all the sides in a triangle. We will need to use:

- $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$
- The sum of the angles in any triangle is 180°
- Pythagoras → $a^2 + b^2 = c^2$

How do you *SOLVE* a triangle?

Ex2) Solve ΔJKL. Give measures to the nearest tenth.

$\angle J = 61°$ $j = 6.7\text{cm}$
$\angle K = 29°$ $k = 3.7\text{cm}$
$\angle L = 90°$ $l = 7.7\text{cm}$



side l: $\sin 61° = \frac{6.7}{l}$

$l = \frac{6.7}{\sin 61°} = 7.7\text{cm}$

$\angle K$: $180 - 90 - 61° = 29°$

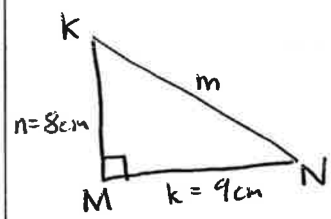
side k: $\tan 61° = \frac{6.7}{k}$

$k = \frac{6.7}{\tan 61°} = 3.714$

How do you solve a triangle without the picture of the triangle?

Ex3) In right triangle ΔKMN, $< M = 90°, KM = 8cm, and\ MN = 9cm$. Solve this triangle. Give measures to the nearest tenth.

(Draw and label the triangle, then solve!)



$\angle K = 48.4°$ $k = 9cm$

$\angle M = 90°$ $m = 12.0cm$

$\angle N = 41.6°$ $n = 8cm$

Side m:

$8^2 + 9^2 = m^2$

$64 + 81 = m^2$

$145 = m^2$

$m = \sqrt{145} = 12.0cm$

$\angle K$:

$\tan K = \frac{9}{8}$

$\angle k = 48.3664°$

$\angle N$:

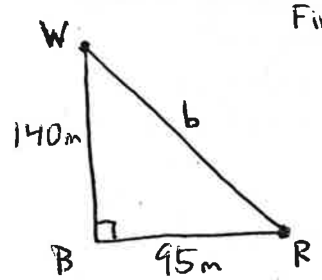
$180 - 90 - 48.3664$

$= 41.6336°$

Word Problems

Ex1) A whale-watching boat is stopped near a rock to look at some sea lions. Then it goes 95m due west to head towards a possible whale sighting. The captain points out a pod of whales, which the radar shows are 140m north of the boat. How far are the whales from the sea lions, and what is the angle at the rock (between the boat's path and the whales' direct line to the sea lions)? *Answer to the nearest tenth.*

Find side b and $\angle R$



side b:

$95^2 + 140^2 = b^2$

$b^2 = 28625$

$b = \sqrt{28625} = 169.2m$

$\angle R$:

$\tan R = \frac{140}{95}$

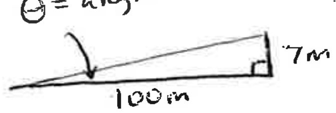
$\angle R = 55.8°$

The whales are 169.2m from the sea lions and the angle at the rock is 55.8°.

Ex2) As Sam is driving, she sees a sign telling her that the road has a 7% grade (i.e., a rise of 7 meters for a horizontal change of 100m).

a) What is the angle of inclination of the road? (nearest degree)
b) If she travels 500m along the road, how much has she risen vertically? (nearest meter)

θ = angle of inclination



a) $\tan \theta = \frac{7}{100}$

$\theta = 4°$

b)

$\sin 4° = \frac{x}{500}$

$x = 500 \sin 4° = 34.9 = 35m$

The road has risen 35m vertically

Reflection: What is the advantage of determining the unknown angle before the unknown sides?

2.7 – Applications, Two Triangles

Name: Notes Key
Date:

Goal: to apply trigonometry to solve problems with two right triangles

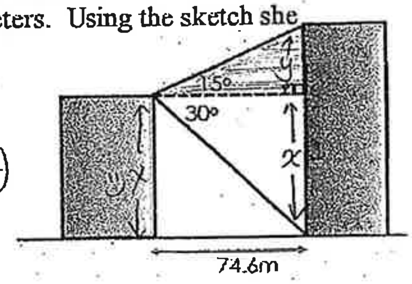
Toolkit:

- $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$ (SOHCAHTOA!)
- The sum of the angles in any triangle is 180°
- Pythagoras → $a^2 + b^2 = c^2$
- A PLAN: you'll need to come up with a PLAN to use more than one triangle to help you answer the question before you jump in.
- Try re-drawing the pieces.

Main Ideas:

2-D

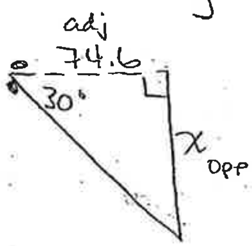
Ex1) From the top of one building, a surveyor measures the angle of elevation to the top of another (taller!) building, and the angle of depression to the base of the other building. The distance between the buildings is 74.6 meters. Using the sketch she made, find the height of the buildings (nearest tenth).

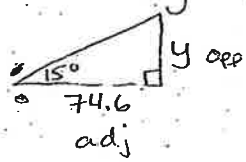


See the 2 triangles?

Plan? Find x (→ small bldg height)

Find y ($x+y$ → tall bldg height)





$\tan\theta = \frac{opp}{adj}$

$\tan 30° = \frac{x}{74.6}$

$x = 74.6 \tan 30°$

$x = 43.1\text{m}$

↑ short bldg

$\tan\theta = \frac{opp}{adj}$

$\tan 15° = \frac{y}{74.6}$

$y = 74.6 \tan 15°$

$y = 19.984$

$y = 20.0\text{m}$

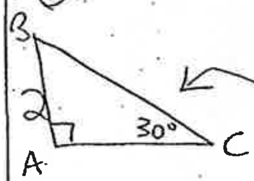
↑ tall bldg $= x+y$
$= 43.1 + 20.0$
$= 63.1\text{m}$

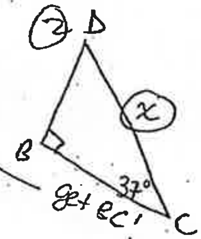
The short building is 43.1m high, and the tall building is 63.1m high.

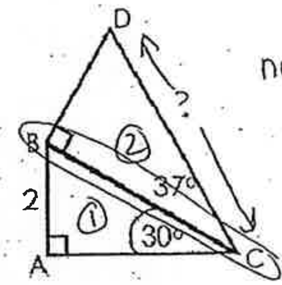
Ex2) For each questions, write out a PLAN to find the missing side CD.

a) Find the length CD. What's the PLAN?

See the 2 triangles?

① 

② 



not enough info yet. Need help from other triangle!

What do they share? BC.

PLAN:

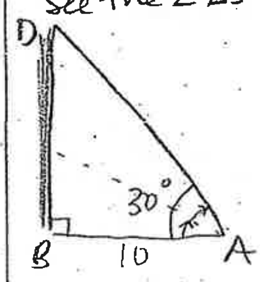
1) Use sin to get BC in △①.

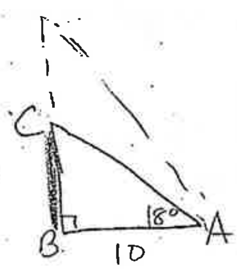
2) Use BC in △②, with cos, to get CD. ☺

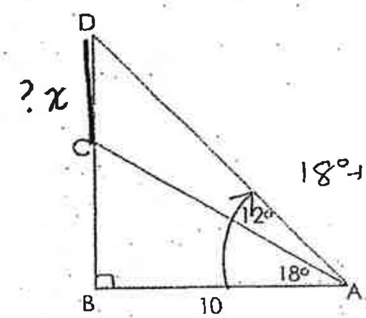
Wait to check later? $BC = 4$ $CD = 5.0$

b) Find the length CD. What's the PLAN?

See the 2 △s?







$18° + 12° = 30°$

PLAN: Find big height − little height = left over

① Use tan to get BD. (add 18° + 12° first!)

② Use tan to get BC

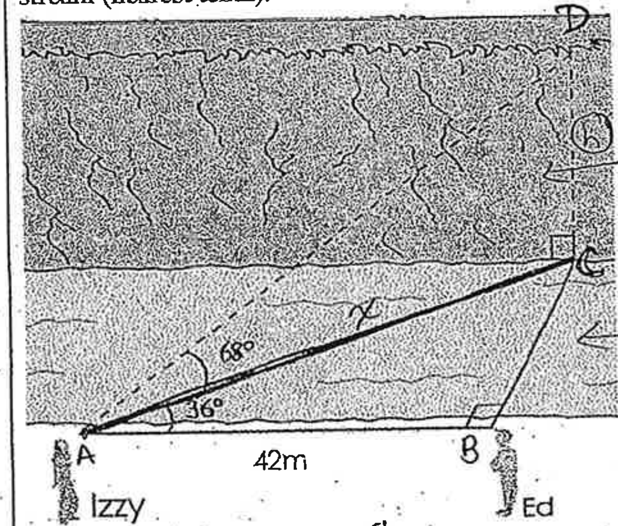
③ BD − BC = CD ☺

Check later? $BD = 5.77$
$BC = 3.25$

$CD = 2.52$

3-D
Hard to picture: try looking for right angles!

Ex3) Izzy and Ed positioned themselves 42 m apart on one side of a stream. Izzy recorded angles, as shown below: Find the height of the cliff on the other side of the stream (nearest tenth).



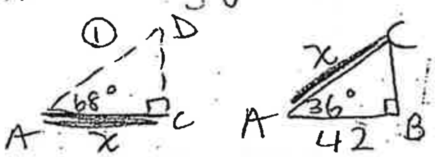
rt Δ "in the air"

right Δ "lying" on stream

Need (h) in  Not enough info. What does it share? AC!

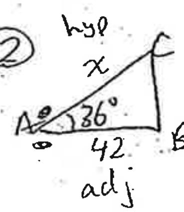
What's the PLAN?

① Label Δs, redraw



② Use ΔABC to get x.

③ use x to get height in ΔACD.

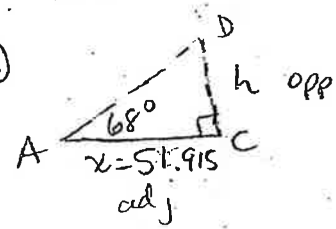
② 

(Cos!)

$\cos 36° = \frac{42}{x}$

$x = \frac{42}{\cos 36°}$

$x = 51.915$

③ 

(tan!)

$\tan 68° = \frac{h}{51.915}$

$h = 51.915 \tan 68$

$h = 128.5$

The height of the cliff is 128.5 m.

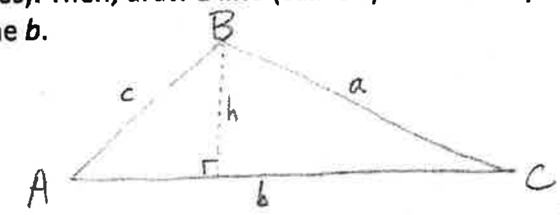
Reflection: What do you have to think about when you draw a diagram with triangles in three dimensions? Right angles don't look right!

The Sine Law

So far, you have learned how to use trigonometry when working with right triangles. Now, you will learn how to use trigonometry for **oblique triangles** (non-right triangles).

developing the sine law

Draw an oblique triangle *ABC* and label the sides *a*, *b*, & *c* (opposite the respective corresponding angles). Then, draw a line (call it *h*) from *B* to *b*, so that it is perpendicular to line *b*.



Write a ratio for sin*A*, and then for sin*C*. Then, solve each for *h*.

$\sin A = \frac{h}{c}$ $\sin C = \frac{h}{a}$

$h = c \sin A$ $h = a \sin C$

Since each ratio is equal to *h*, they must also equal one another.

$c \sin A = a \sin C$

$\frac{\cancel{c} \sin A}{\cancel{c} a} = \frac{\cancel{a} \sin C}{\cancel{a} c}$

$\frac{\sin A}{a} = \frac{\sin C}{c}$

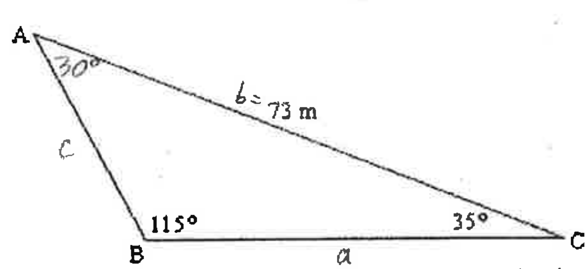
By using similar steps, you can also show the same for *b* and sin*B*.

sine law

For any triangle, the sine law states that the sides of a triangle are proportional to the sines of the opposite angles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad OR \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1 – Solve for side AB and side BC to the nearest tenth.



$\angle A = 180° - 115° - 35° = 30°$

to find side AB: (side c)

$\frac{\sin B}{b} = \frac{\sin C}{c}$; $\frac{\sin 115°}{73} = \frac{\sin 35°}{c}$

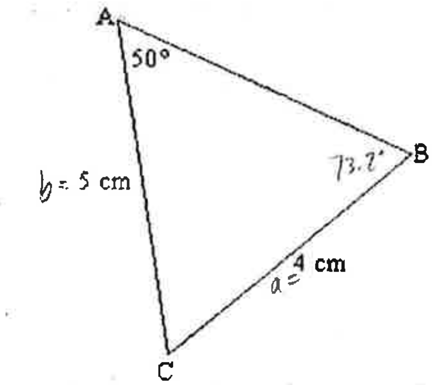
$c \sin 115° = 73 \sin 35°$

$c = \frac{73 \sin 35°}{\sin 115°} = 46.2\text{m}$

To find side BC (side a): $\frac{\sin B}{b} = \frac{\sin A}{a}$; $\frac{\sin 115°}{73} = \frac{\sin 30°}{a}$

$a = \frac{73 \sin 30°}{\sin 115°} = 40.3\text{m}$

Example 2 – Solve for angle B to the nearest degree. Then find angle C to the nearest degree and side AB to the nearest tenth.



$\angle A = 50°$ $\quad a = 4cm$

$\angle B =$ ___ $\quad b = 5cm$

$\angle C =$ ___ $\quad c =$ ___

to find $\angle B$:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 50°}{4} = \frac{\sin B}{5}$$

$$\sin B = \frac{5 \sin 50°}{4}$$

$$\sin B = 0.95756$$

$$\angle B = \sin^{-1} 0.95756$$

$$\angle B = 73.2°$$

$$\angle C = 180 - 50° - 73.2$$

$$\angle C = 56.8° = 57°$$

side AB = side c

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 50°}{4} = \frac{\sin 56.8°}{c}$$

$$c = \frac{4 \sin 56.8°}{\sin 50°}$$

$$c = 4.4 cm$$

information necessary to use the sine law

For oblique triangles, what is the minimum information needed in order to use the sine law to find new information?

Suppose we think of $\angle A$ and side a as 'partners'. Same for the other sets.

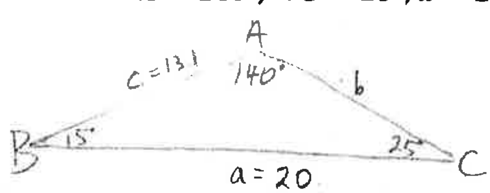
To use sine law, you must know info on 1 full set of 'partners' and half of another 'partnership'.

solving a triangle

When **solving a triangle**, you must find all of the unknown angles and sides.

Example – Sketch and solve the triangle (each answer to the nearest tenth).

$$\angle A = 140°, \angle C = 25°, a = 20$$



$\angle A = 140°$ $\quad a = 20$

$\angle B =$ ___ $\quad b =$ ___

$\angle C = 25°$ $\quad c =$ ___

$$\angle B = 180 - 140 - 25 = 15°$$

$$\angle B = 15°$$

To find side c: $\frac{\sin A}{a} = \frac{\sin C}{c}$

$$\frac{\sin 140°}{20} = \frac{\sin 25°}{c}; \quad c = \frac{20 \sin 25°}{\sin 140°}$$

$$c = 13.1$$

To find side b: $\frac{\sin A}{a} = \frac{\sin B}{b}$

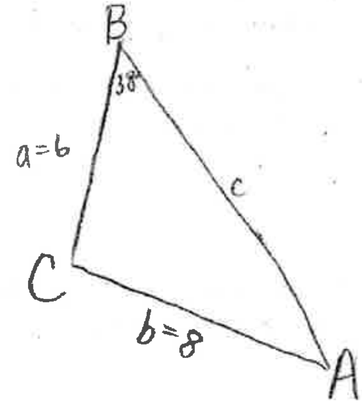
*can't use Pythag as not a right tri

$$\frac{\sin 140°}{20} = \frac{\sin 15°}{b}$$

$$b = \frac{20 \sin 15°}{\sin 140°}; \quad b = 8.1$$

Example – Solve the triangle (round to the nearest whole number).

$< B = 38°, b = 8, a = 6$



$\angle A =$ ___ $\quad a = 6$

$\angle B = 38°$ $\quad b = 8$

$\angle C =$ ___ $\quad c =$ ___

To find $\angle A$: $\dfrac{\sin B}{b} = \dfrac{\sin A}{a}$

$\dfrac{\sin 38°}{8} = \dfrac{\sin A}{6}$

$\sin A = \dfrac{6 \sin 38°}{8}$

$\sin A = 0.4617461$

$\angle A = \sin^{-1} 0.4617461$

$\angle A = 27.4998 = 27°$

$\angle A = 27°$

To find $\angle C$:

$180 - 38 - 27.4998 = 114.5002$

$\angle C = 115°$

To find side c:

*cannot use Pythag!

$\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\dfrac{\sin 38°}{8} = \dfrac{\sin 114.5}{c}$

$c = \dfrac{8 \sin 114.5°}{\sin 38°}$

$c = 11.8 = 12$

$c = 12$

The Cosine Law

For right triangles, the trigonometric ratios sine, cosine, and tangent can be used to find unknown sides and angles. For oblique triangles, **sine law** and **cosine law** must be used.

An effective way to work with oblique triangles is to imagine the angle and its opposite side as 'partners'. Thus, angle *A* and side *a* are partners, <*B* and *b* are partners, and <*C* and *c* are partners.

In order to use the sine law, you must know one full set of partners and half of another set. If you know only half of each set of the three partners, at least two of which are sides, you must use **cosine law**.

Example – For each oblique triangle, state which law you would use.

a) x =30cm, y =28cm, z =32cm (b) <C =27°, a =17m, c =13m (c) <J =41°, k =16cm, p =14cm

- 3 half partners
- all are sides

COSINE LAW

full set of partners & half of another

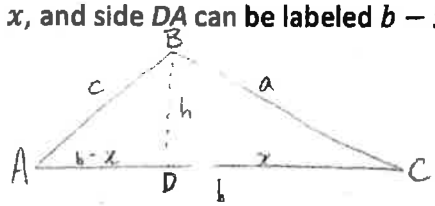
SINE LAW

3 half partners, 2 of which are sides

COSINE LAW

deriving cosine law

1. The **cosine law** can be developed by starting with oblique ΔABC and drawing vertical line h from <B to side b. Where h meets side b, call that vertex D. Side CD can then be labeled x, and side DA can be labeled $b - x$.



2. For ΔBCD, find $\cos C$ and rearrange the equation to isolate x. Then write a Pythagorean equation for ΔBCD.

$$\cos C = \frac{x}{a} \; ; \; x = a\cos C$$

$$x^2 + h^2 = a^2$$

3. Next, for ΔABD, write a Pythagorean equation. Then FOIL $(b - x)^2$. Can you see where a^2 can now replace a part of the equation? What can you replace for x?

$$h^2 + (b-x)^2 = c^2$$

$$h^2 + (b-x)(b-x) = c^2$$

$$h^2 + b^2 - 2bx + x^2 = c^2$$

$$\underbrace{x^2 + h^2} + b^2 - 2bx = c^2$$

$$a^2 + b^2 - 2b\underbrace{x}_{a\cos C} = c^2$$

$$c^2 = a^2 + b^2 - 2ba\cos C$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$ Cosine law!

cosine law

The **cosine law** describes the relationship between the cosine of an angle and the lengths of the three sides of any triangle.

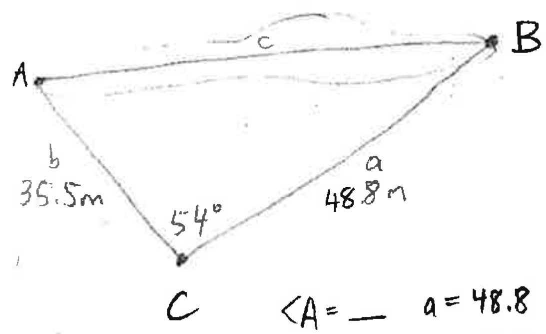
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Cosine law can also be written as $a^2 = b^2 + c^2 - 2bc\cos A$ **OR**

$$b^2 = a^2 + c^2 - 2ac\cos B$$

Example 1 – Kohl wants to find the distance between two points, *A* and *B*, on opposite sides of a pond. She locates a point *C* that is 35.5m from *A* and 48.8m from *B*. If the angle at *C* is 54°, determine the distance *AB*, to the nearest tenth of a metre.

3 half partners, 2 of which are sides
COSINE LAW



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (48.8)^2 + (35.5)^2 - 2(48.8)(35.5)\cos 54°$

$c^2 = 2381.44 + 1260.25 - 2036.56$

$c^2 = 1605.13$

$c = \sqrt{1605.13}$

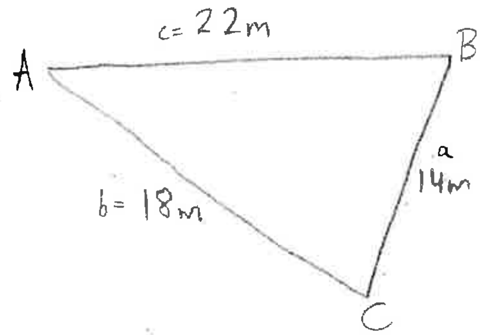
$c = 40.1\text{m}$

The pond is 40.1m long

$\angle A =$ ___ $a = 48.8$
$\angle B =$ ___ $b = 35.5$
$\angle C = 54°$ $c =$ ___

Example 2 – A triangular brace has side lengths 14m, 18m, and 22m. Determine the measure of the angle opposite the 18m side, to the nearest degree.

Must find $\angle B$



$b^2 = a^2 + c^2 - 2ac\cos B$

$18^2 = 14^2 + 22^2 - 2(14)(22)\cos B$

$324 = 196 + 484 - 616\cos B$

$324 = 680 - 616\cos B$

$-680 \quad -680$

$\frac{-356}{-616} = \frac{-616\cos B}{-616}$

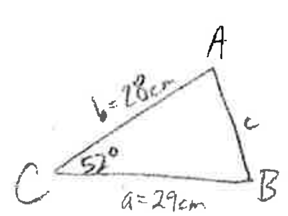
$0.57792 = \cos B$

$\angle B = \cos^{-1} 0.57792$

$\angle B = 54.7° \approx 55°$

using cosine law & sine law

Example 3 – In Δ*ABC*, *a* = 29cm, *b* = 28cm, and <*C* = 52°. Sketch a diagram and determine the length of the unknown side and the measures of the unknown angles, to the nearest tenth.



$\angle A =$ ___ $a = 29$
$\angle B =$ ___ $b = 28$
$\angle C = 52°$ $c =$ ___

must start with cos law
(no full set of partners)

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (29)^2 + (28)^2 - 2(29)(28)\cos 52°$

$c^2 = 841 + 784 - 999.834$

$c^2 = 625.166$

$c = \sqrt{625.166}$

$c = 25\text{ cm}$

To find $\angle A$:

$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin 52°}{25} = \frac{\sin A}{29}$

$\sin A = \frac{29 \sin 52°}{25}$

$\sin A = 0.914$

$\angle A = \sin^{-1} 0.914$

$\angle A = 66.1°$

$\angle C = 180 - 52 - 66.1$

$\angle C = 61.9°$