1.1 - Factoring Review

Greatest Common Factors of a Polynomial

Example 1 – Factor

a) $4s^2 + 14s$

$\frac{4s^2}{2s} \quad \frac{14s}{2s}$

$2s(2s+7)$

b) $-a^2 - a^3$

$\frac{-a^3}{-a^2} - \frac{a^2}{-a^2}$

$-a^2(a+1)$

c) $27n + 36n^2 - 18n^3$

$\frac{-18n^3}{-9n} + \frac{36n^2}{-9n} + \frac{27n}{-9n}$

$-9n(2n^2-4n-3)$

d) $6x^4 - 7x - 8x^3$

$6x^4 - 8x^3 - 7x$

$x(6x^3 - 8x^2 - 7)$

Factoring Polynomials of the Form $ax^2 \pm bx \pm c$ where *a* = *1*

1) Put in order. Then see if there is a GCF to factor out.
2) If *a* = *1*, find two numbers that multiply to ***c*** and add to ***b***.
3) Using the two numbers, write two binomials in brackets.
4) Check by FOIL.

Example 2 – Factor

a) $x^2 + 12x - 28$ $\times -28$, $+ 12$: $14, -2$

$(x+14)(x-2)$

b) $n^2 - 5n - 24$ $\times -24$, $+ -5$: $-8, 3$

$(n-8)(n+3)$

c) $y^2 + 8y + 12$ $\times 12$, $+ 8$: $6, 2$

$(y+6)(y+2)$

d) $-x^2 - 2x + 35$ $\times -35$, $+ 2$: $7, -5$

$-(x^2+2x-35)$

$-(x+7)(x-5)$

e) $2x^2 - 10x - 28$

$2(x^2-5x-14)$

$2(x-7)(x+2)$

f) $4x^2 - 20x - 56$

$4(x^2-5x-14)$

$4(x-7)(x+2)$

Factoring Difference of Squares of the Form $a^2 - b^2$

Characteristics: there is no 'middle term' (x term) & the middle operation is subtract

1) Factor out a GCF if possible. Is middle operation subtract?
2) Make sure the first term AND second term are perfect squares (and first term is positive).
3) Set up a binomial product (two brackets right beside each other).
4) Put a + in the middle of one bracket and a – in the middle of the other.
5) Put the square root of the first term in the first position of each bracket.
6) Put the square root of the second term in the second position of each bracket.
7) FOIL to check.

Example 3 – Factor

a) $x^2 - 16$

$(x+4)(x-4)$

b) $9x^2 - 49$

$(3x+7)(3x-7)$

c) $-100a^2 + 81b^2$

$-(100a^2 - 81b^2)$

$-(10a+9b)(10a-9b)$

OR

$81b^2 - 100a^2$

$(9b+10a)(9b-10a)$

d) $8m^2 - 98n^2$

$2(4m^2 - 49n^2)$

$2(2m+7n)(2m-7n)$

e) $2x^3 + 50x$

$2x(x^2+25)$

↑ sum of squares

f) $-2x^2 + 50x$

$-2x(x-25)$

1.2 – Factoring $ax^2 + bx + c$ when $a \neq 1$

Factoring Polynomials of the Form $ax^2 \pm bx \pm c$ when $a \neq 1$ by decomposition

1) Put in order. If possible, factor out GCF.
2) Identify the *a*, *b*, and *c* values. Make sure ***a*** is positive.
3) Find two numbers that multiply to ***ac*** and add to ***b***.
4) Rewrite the trinomial but split the *bx* term into the two numbers you found.
5) Now there are four terms, so factor by grouping. Factor the first two terms, and then the second two. The resulting brackets must match.
6) Factor the identical brackets out, leaving the components of the second bracket.
7) FOIL to check.

Example 1 – Factor

a) $2y^2 + 5y + 2$ $\times\ 4$, $+\ 5$, $4, 1$

$2y^2 + 4y + y + 2$

$2y(y+2) + 1(y+2)$

$(y+2)(2y+1)$

b) $8x^2 + 18x - 5$ $\times\ -40$, $+\ 18$, $20, -2$

$8x^2 - 2x + 20x - 5$

$2x(4x-1) + 5(4x-1)$

$(4x-1)(2x+5)$

c) $-5a^2 + 7a + 6$ $\times\ -30$, $+\ -7$, $-10, 3$

$-(5a^2 - 7a - 6)$

$-(5a^2 - 10a + 3a - 6)$

$-(5a(a-2) + 3(a-2))$

$-(a-2)(5a+3)$

d) $18y^2 + 15y - 18$ $\times\ -36$, $+\ 5$, $9, -4$

$3(6y^2 + 5y - 6)$

$3(6y^2 + 9y - 4y - 6)$

$3(3y(2y+3) - 2(2y+3))$

$3(2y+3)(3y-2)$

e) $4x^2 + 14x - 8$ $\times\ -8$, $+\ 7$, $8, -1$

$2(2x^2 + 7x - 4)$

$2(2x^2 + 8x - x - 4)$

$2(2x(x+4) - 1(x+4))$

$2(x+4)(2x-1)$

f) $-3x^2 - 6x + 45$

$-3(x^2 + 2x - 15)$

$-3(x+5)(x-3)$

Example 2 - Factor

a) $x^2 + 8x + 16$

$\times 16$
$+ 8$
$4, 4$

$(x+4)(x+4)$

$(x+4)^2$

b) $4x^2 - 12x + 9$

$\times 36$
$+ -12$
$-6, -6$

$\underline{4x^2 - 6x}\ \underline{-6x + 9}$

$2x(2x-3) - 3(2x-3)$

$(2x-3)(2x-3)$

$(2x-3)^2$

Perfect Square Trinomials:

When the two binomial factors are identical, the original trinomial is deemed a 'perfect square trinomial'.

2.1 – Properties of Rational Expressions

A rational expression is ... a fraction that includes one or more variables.

how zero affects division

Evaluate $\frac{0}{3}$ $=0$

When zero is divided by any non-zero real number, ... the result is 0.

Evaluate $\frac{7}{0}$ = $\emptyset$

Division by zero is undefined because..., for example, you can't put 7 items into zero groups.

undefined values

For the expression $\frac{3}{x-2}$, what value for x is undefined? 2

What is an **undefined value**? Any value for a variable that causes the denominator to be zero, which will cause the expression to be undefined.

Write a rule that explains how to determine undefined values:

If you cannot determine undefined values by inspection, then:

1. Set the denominator equal to 0.
2. Solve the equation

Example 1 - Determine the undefined values for each rational expression

a) $\frac{4a}{3b}$

$3b=0$

$\frac{\cancel{3}b}{\cancel{3}}=\frac{0}{3}$

$b=0$

$\boxed{b\neq 0}$

b) $\frac{x-1}{(x+2)(x-3)}$

$(x+2)(x-3)=0$

$x=-2, 3$

$\boxed{x\neq -2, 3}$

c) $\frac{2y^2}{y^2-4}$

$\frac{2y^2}{(y+2)(y-2)}$

$(y+2)(y-2)=0$

$\boxed{y\neq \pm 2}$

simplifying rational expressions

When simplifying rational expressions:

1) Do any factoring. Determine undefined values

2) Reduce/simplify as much as possible.

Example 2 - Simplify the rational expressions. Keep a running list of undefined values.

a) $\frac{3x-3}{6x-6}$ $\boxed{x \neq 1}$

$\frac{3\cancel{(x-1)}}{6\cancel{(x-1)}}$

$\frac{1}{2}$

b) $\frac{x-2}{x^2-4}$ $x \neq \pm 2$

$\frac{\cancel{x-2}}{(x+2)\cancel{(x-2)}}$

$\frac{1}{x+2}$

c) $\frac{3x-6}{2x^2+x-10}$ $\boxed{x \neq 2, \frac{-5}{2}}$

$\frac{3\cancel{(x-2)}}{\cancel{(x-2)}(2x+5)}$

$\frac{3}{2x+5}$

$2x^2+x-10$

$2x^2-4x+5x-10$

$2x(x-2)+5(x-2)$

$(x-2)(2x+5)$

$2x+5=0$

$2x=-5$

$x=\frac{-5}{2}$

d) $\frac{2y^2+y-10}{y^2+3y-10}$ $\boxed{y \neq -5, 2}$

$\frac{\cancel{(y-2)}(2y+5)}{(y+5)\cancel{(y-2)}}$

$\frac{2y+5}{y+5}$

e) $\frac{6-2m}{m^2-9}$ $\boxed{m \neq \pm 3}$

$\frac{2(3-m)}{(m+3)(m-3)}$

$\frac{-2\cancel{(m-3)}}{(m+3)\cancel{(m-3)}}$

$\frac{-2}{m+3}$

f) $\frac{x^2y+xy^2}{xy+y^2}$ $\boxed{y \neq 0, -x}$

$\frac{x\cancel{y}\cancel{(x+y)}}{\cancel{y}\cancel{(x+y)}}$

$=x$

See the bottom of page 71 for *Common Errors

2.2 – Multiplying & Dividing Rational Expressions

multiplication & division review

Warmup – Simplify

a) $\left(\frac{3}{-4}\right)\left(\frac{1}{2}\right)$

$\frac{-3}{8}$

b) $\left(\frac{\cancel{5}^{1}}{\cancel{8}_{2}}\right)\left(\frac{\cancel{-4}^{-1}}{\cancel{15}_{3}}\right)$

$\frac{-1}{6}$

c) $\frac{2}{3} \div \frac{3}{4}$

$\frac{2}{3}\left(\frac{4}{3}\right)$

$\frac{8}{9}$

d) $\frac{\frac{2}{5}}{\frac{-1}{10}}$ $\frac{2}{5} \div \frac{-1}{10}$

$\frac{2}{\cancel{5}_{1}}\left(\frac{\cancel{-10}^{-2}}{1}\right)$

-4

Explain how to multiply fractions:

① Look to 'cross reduce'

② Multiply numerators. Multiply denominators.

③ Simplify? (lowest terms)

Explain how to divide fractions:

① Flip the second fraction and change divide to multiply.

② Follow the 'multiply' steps above.

Example 1 - Simplify and keep a running list of undefined values

a) $\left(\frac{x+3}{2}\right)\left(\frac{x+1}{4}\right)$

$\frac{(x+3)(x+1)}{8}$

b) $\left(\frac{4x^2}{3xy}\right)\left(\frac{y^2}{8x}\right)$ $\boxed{x, y \neq 0}$

$\frac{4x^2y^2}{24x^2y}$

$\frac{y}{6}$

c) $\left(\frac{d}{2\pi r}\right)\left(\frac{2\pi rh}{d-2}\right)$ $\boxed{r \neq 0 \quad d \neq 2}$

$\frac{\cancel{2\pi r}hd}{\cancel{2\pi r}(d-2)}$

$\frac{hd}{d-2}$

Example 2 - Simplify and keep a running list of undefined values

a) $\frac{y^2-9}{r^3-r} \times \frac{r^2-r}{y+3}$ $\boxed{r \neq 0, \pm 1 \quad y \neq -3}$

$\frac{(y+3)(y-3)r(r-1)}{r(r^2-1)(y+3)}$

$\frac{\cancel{r}\cancel{(r-1)}\cancel{(y+3)}(y-3)}{\cancel{r}(r+1)\cancel{(r-1)}\cancel{(y+3)}} = \frac{y-3}{r+1}$

b) $\left(\frac{x^2-x-12}{x^2-9}\right)\left(\frac{x^2-4x+3}{x^2-4x}\right)$ $\boxed{x \neq 0, \pm 3, 4}$

$\frac{\cancel{(x-4)}\cancel{(x+3)}\cancel{(x-3)}(x-1)}{\cancel{(x+3)}\cancel{(x-3)}x\cancel{(x-4)}}$

$\frac{x-1}{x}$

Example 3 - Simplify and keep a running list of undefined values

a) $\frac{m^2-6m-7}{m^2-49} \div \frac{m^2+8m+7}{m^2+7m}$ $\boxed{m \neq \pm 7, 0, -1}$

$\frac{(m-7)(m+1)}{(m+7)(m-7)} \div \frac{(m+7)(m+1)}{m(m+7)}$

$\frac{m\cancel{(m+7)}\cancel{(m-7)}\cancel{(m+1)}}{(m+7)\cancel{(m-7)}\cancel{(m+7)}\cancel{(m+1)}}$

$\frac{m}{m+7}$

b) $\frac{3x+12}{3x^2-5x-12} \div \frac{12}{3x+4} \times \frac{2x-6}{x+4}$ $x \neq 3, -4, \frac{-4}{3}$

$\frac{3(x+4)}{(x-3)(3x+4)} \div \frac{12}{3x+4} \cdot \frac{2(x-3)}{x+4}$

$\frac{{}^{1}\cancel{3}(x+4)\cancel{(3x+4)}}{{}_{4}\cancel{12}(x-3)\cancel{(3x+4)}} \cdot \frac{2(x-3)}{x+4}$

$\frac{2\cancel{(x-3)}\cancel{(x+4)}}{4\cancel{(x-3)}\cancel{(x+4)}} = \frac{1}{2}$

$3x^2-5x-12$ $\quad$ $\times -36$, $+ -5$, $-9, 4$

$3x^2-9x+4x-12$

$3x(x-3)+4(x-3)$

$(x-3)(3x+4)$

undefined value:

$3x+4=0$

$3x=-4$

$x=\frac{-4}{3}$

2.3 – Adding & Subtracting Rational Expressions

adding & subtracting review

Warmup – Simplify each expression

a) $\frac{4\cdot 5}{4\cdot 6}-\frac{3\cdot 3}{8\cdot 3}$ b) $-\frac{5\cdot 2}{5\cdot 3}+\frac{4\cdot 3}{5\cdot 3}$ c) $\frac{7x+1}{x}-\frac{5x-2}{x}$ $\boxed{x\neq 0}$ d) $\frac{7\cdot 4x}{6x^2\cdot 4x}-\frac{3\cdot 3}{8x^3\cdot 3}$ $\boxed{x\neq 0}$

a) $\frac{20}{24}-\frac{9}{24}$

$\frac{11}{24}$

b) $\frac{-10}{15}+\frac{12}{15}$

$\frac{2}{15}$

c) $\frac{7x+1-(5x-2)}{x}$

$\frac{7x+1-5x+2}{x}$

$\frac{2x+3}{x}$

d) $\frac{28x-9}{24x^3}$

Write the steps to adding/subtracting fractions:

1. Get common denominators.
2. Add or subtract numerators.
3. Look to reduce.

Example 1 - Simplify and identify all undefined values

a) $\frac{10y-1}{4y-3}-\frac{8-2y}{4y-3}$ $\boxed{y\neq \frac{3}{4}}$ b) $\frac{2x\cdot x}{xy\cdot x}+\frac{4\cdot y}{x^2\cdot y}-\frac{3\cdot x^2y}{1\cdot x^2y}$ c) $\frac{3}{3x+6}+\frac{1}{x+2}$ $\boxed{x\neq -2}$

$4y-3=0$
$4y=3$
$y=\frac{3}{4}$

a) $\frac{10y-1-(8-2y)}{4y-3}$

$\frac{10y-1-8+2y}{4y-3}$

$\frac{12y-9}{4y-3}$

$\frac{3\cancel{(4y-3)}}{\cancel{4y-3}}$

$=3$

b) $\boxed{x, y\neq 0}$

$\frac{2x^2+4y-3x^2y}{x^2y}$

c) $\frac{\cancel{3}}{\cancel{3}(x+2)}+\frac{1}{x+2}$

$\frac{1}{x+2}+\frac{1}{x+2}$

$\frac{2}{x+2}$

Steps: 1) Factor as much as possible. Do any relevant reducing.
2) List undefined values
3) Get common denominators.
4) Add or subtract numerators
5) Do any further factoring and/or reducing.

Example 2 - Simplify and identify all undefined values

a) $\frac{4}{p^2-1}+\frac{3}{1-p}$ $\boxed{p \neq \pm 1}$

$$\frac{4}{(p+1)(p-1)} - \frac{3}{p-1}$$

$$\frac{4}{(p+1)(p-1)} - \frac{3(p+1)}{(p+1)(p-1)}$$

$$\frac{4-3(p+1)}{(p+1)(p-1)}$$

$$\frac{4-3p-3}{(p+1)(p-1)} = \frac{1-3p}{(p+1)(p-1)}$$

b) $\frac{x-2}{x^2+x-6}-\frac{x^2+6x+5}{x^2+4x+3}$ $\boxed{x \neq -3, -1, 2}$

$$\frac{\cancel{x-2}}{(x+3)\cancel{(x-2)}} - \frac{(x+5)\cancel{(x+1)}}{(x+3)\cancel{(x+1)}}$$

$$\frac{1}{x+3} - \frac{(x+5)}{x+3}$$

$$\frac{1-(x+5)}{x+3}$$

$$\frac{1-x-5}{x+3} = \frac{-x-4}{x+3}$$

c) $\frac{1}{x^2-1}-\frac{2}{x^2+x}$ $\boxed{x \neq 0, \pm 1}$

$$\frac{1}{(x+1)(x-1)} - \frac{2}{x(x+1)}$$

$$\frac{1x}{x(x+1)(x-1)} - \frac{2(x-1)}{x(x+1)(x-1)}$$

$$\frac{x-2(x-1)}{x(x+1)(x-1)}$$

$$\frac{x-2x+2}{x(x+1)(x-1)}$$

$$\frac{-x+2}{x(x+1)(x-1)}$$

d) $\frac{3x+9}{x^2+7x+10}+\frac{14}{x^2+3x-10}$ $\boxed{x \neq \pm 2, -5}$

$$\frac{3(x+3)^{(x-2)}}{(x+5)(x+2)_{(x-2)}} + \frac{14^{(x+2)}}{(x+5)(x-2)_{(x+2)}}$$

$$\frac{3(x+3)(x-2)+14(x+2)}{(x+5)(x+2)(x-2)}$$

$$\frac{3(x^2-2x+3x-6)+14x+28}{(x+5)(x+2)(x-2)}$$

$$\frac{3(x^2+x-6)+14x+28}{(x+5)(x+2)(x-2)}$$

$$\frac{3x^2+3x-18+14x+28}{(x+5)(x+2)(x-2)}$$

$3x^2+15x+2x+10 \longrightarrow$

$3x(x+5)+2(x+5)$

$(x+5)(3x+2)$

$$\frac{3x^2+17x+10}{(x+5)(x+2)(x-2)}$$

$$\frac{\cancel{(x+5)}(3x+2)}{\cancel{(x+5)}(x+2)(x-2)} = \frac{3x+2}{(x+2)(x-2)}$$

2.4 – Mixed Operations

When simplifying rational expressions with mixed operations, ORDER OF OPERATIONS is to be followed (**BEDMAS**).

Example 1 – Simplify & identify all undefined values

a) $\frac{x+5}{x+6}+\frac{1}{x+4}\div\frac{x+6}{x^2-x-20}$ $\boxed{x\neq -6,-4,5}$

$\frac{x+5}{x+6}+\frac{1}{x+4}\div\frac{x+6}{(x-5)(x+4)}$

$\frac{x+5}{x+6}+\frac{(x-5)\cancel{(x+4)}}{\cancel{(x+4)}(x+6)}$

$\frac{x+5}{x+6}+\frac{x-5}{x+6}$

$\frac{x+5+x-5}{x+6}$

$\frac{2x}{x+6}$

b) $\left(\frac{x-3}{x^2-9}+\frac{x+3}{x^2+6x+9}\right)\left(\frac{x+3}{x+1}\right)$ $\boxed{x\neq \pm 3,-1}$

$\left(\frac{\cancel{x-3}}{(x+3)\cancel{(x-3)}}+\frac{\cancel{x+3}}{\cancel{(x+3)}(x+3)}\right)\left(\frac{x+3}{x+1}\right)$

$\left(\frac{1}{x+3}+\frac{1}{x+3}\right)\left(\frac{x+3}{x+1}\right)$

$\frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x+1)}$

$\frac{2}{x+1}$

Complex Fractions – Rational Expressions that contain fractions in the numerators and/or denominators.

Example 2 - Simplify and identify all undefined values

$\frac{2-\frac{4}{y}}{y-\frac{4}{y}}$ $\boxed{y\neq 0,\pm 2}$

$\left(\frac{2\cdot y}{1\cdot y}-\frac{4}{y}\right)\div\left(\frac{y\cdot y}{1\cdot y}-\frac{4}{y}\right)$

$\left(\frac{2y-4}{y}\right)\div\left(\frac{y^2-4}{y}\right)$

$\frac{2(y-2)}{y}\div\frac{(y+2)(y-2)}{y}$

$\frac{2\cancel{y}\cancel{(y-2)}}{\cancel{y}(y+2)\cancel{(y-2)}}$

$=\frac{2}{y+2}$

Steps:

1) Get a common denominator for the numerator and then the denominator of the complex fraction.

2) Write each as one fraction.

3) Rewrite the division in a side-by-side manner and simplify.

Example 3 – Simplify and identify all undefined values

a) $\dfrac{\frac{2}{5x}-\frac{3}{x^2}}{\frac{7}{2x}+\frac{3}{4x^2}}$ $\boxed{x \neq 0, \frac{-3}{14}}$

$$\left(\frac{2\cdot x}{5x\cdot x}-\frac{3\cdot 5}{x^2\cdot 5}\right)\div\left(\frac{7\cdot 2x}{2x\cdot 2x}+\frac{3}{4x^2}\right)$$

$$\frac{2x-15}{5x^2}\div\frac{14x+3}{4x^2}$$

$$\frac{4\cancel{x^2}(2x-15)}{5\cancel{x^2}(14x+3)}$$

$$=\frac{4(2x-15)}{5(14x+3)}$$

b) $\dfrac{\frac{1}{x-1}+\frac{2}{x+2}}{\frac{2}{x+2}-\frac{1}{x-3}}$ $\boxed{x \neq -2, 1, 3, 8}$

$$\left(\frac{1(x+2)}{(x-1)(x+2)}+\frac{2(x-1)}{(x+2)(x-1)}\right)\div\left(\frac{2(x-3)}{(x+2)(x-3)}-\frac{1(x+2)}{(x-3)(x+2)}\right)$$

$$\left(\frac{x+2+2(x-1)}{(x-1)(x+2)}\right)\div\left(\frac{2(x-3)-1(x+2)}{(x+2)(x-3)}\right)$$

$$\left(\frac{x+2+2x-2}{(x-1)(x+2)}\right)\div\left(\frac{2x-6-x-2}{(x+2)(x-3)}\right)$$

$$\frac{3x}{(x-1)(x+2)}\div\frac{x-8}{(x+2)(x-3)}$$

$$\frac{3x\cancel{(x+2)}(x-3)}{(x-1)\cancel{(x+2)}(x-8)}$$

$$=\frac{3x(x-3)}{(x-1)(x-8)}$$

2.5 – Rational Equations (Non-Quadratic)

A rational equation is an equation containing at least one rational expression. Remember, when working with an equation, whatever you do to one side, you do to the other side.

Steps to solving rational equations:

1) Factor each denominator if possible.
2) Identify any undefined values (and do this throughout).
3) Multiply both sides of the equation by what would be the lowest common denominator in order to eliminate the fractions.
4) Solve the equation.
5) Check your solutions.

Example 1 - Solve

a) $\frac{(6)x}{2} + \frac{7(6)}{3} = \frac{5(6)}{\cancel{6}}$

$3x + 14 = 5$

$3x = -9$

$x = -3$

b) $\frac{5}{3x}(9x) - \frac{1}{9}(9x) = \frac{4}{x}(9x)$ $\boxed{x \neq 0}$

$15 - x = 36$

$+x \qquad +x$

$x + 36 = 15$

$x = -21$

Example 2 – Solve

a) $\frac{2x}{x-4}(x-4) = \frac{8}{x-4}(x-4) + 1(x-4)$ $\boxed{x \neq 4}$

$2x = 8 + x - 4$

$2x = x + 4$

$-x \quad -x$

$\cancel{x = 4}$ extraneous

$\emptyset$ no solutions

b) $\frac{9}{y-3} - \frac{4}{y-6} = \frac{18}{y^2-9y+18}$ $\boxed{y \neq 3, 6}$

$\frac{9(y-3)(y-6)}{y-3} - \frac{4(y-3)(y-6)}{y-6} = \frac{18(y-3)(y-6)}{(y-6)(y-3)}$

$9(y-6) - 4(y-3) = 18$

$9y - 54 - 4y + 12 = 18$

$5y - 42 = 18$

$5y = 60$

$y = 12$

When a solution is the same as an undefined value, it is called an EXTRANEOUS solution.

Example 3 – Solve

a) $\frac{1}{x-4} - \frac{1}{x-2} = \frac{2x}{x^2-6x+8}$ $\boxed{x \neq 2, 4}$

$$\frac{1}{x-4}(x-4)(x-2) - \frac{1}{x-2}(x-4)(x-2) = \frac{2x}{(x-4)(x-2)}(x-4)(x-2)$$

$$x-2-(x-4) = 2x$$

$$x-2-x+4 = 2x$$

$$2 = 2x$$

$$x = 1$$

b) $\frac{4}{2x-1} = \frac{2}{x+3}$ $\boxed{x \neq -3, \frac{1}{2}}$

$$\frac{4}{\cancel{2x-1}}(2x-1)(x+3) = \frac{2}{\cancel{x+3}}(2x-1)(x+3)$$

$$4(x+3) = 2(2x-1)$$

$$4x+12 = 4x-2$$

$$-4x \quad -12 \quad -4x \quad -12$$

$$0 = -14$$

$$\emptyset$$

c) $\frac{5}{x-7} - \frac{1}{2x} = \frac{9x+7}{2x^2-14x}$ $\boxed{x \neq 0, 7}$

$$\frac{5}{x-7}2x(x-7) - \frac{1}{2x}2x(x-7) = \frac{(9x+7)}{2x(x-7)}2x(x-7)$$

$$10x-(x-7) = 9x+7$$

$$10x-x+7 = 9x+7$$

$$9x+7 = 9x+7$$

$$0 = 0$$

$$\infty$$

infinite solutions
(except $x \neq 0, 7$)

***Do only the questions that have been specifically assigned for your homework in section 2.5**

2.7 – Applications of Rational Equations (Non-Quadratic)

There is no fool-proof way to solve a word problem. You should try to read the problem carefully, create a *'Let'* statement for your variable, build your equation (sometimes using a table or diagram for assistance), and solve the equation. Then do a check.

Template:

$$\frac{\text{time together}}{\text{time alone 1}} + \frac{\text{time together}}{\text{time alone 2}} = 1$$

Example 1 – Stella takes 4 hours to paint a room. It takes Jose 3 hours to paint the same area. How long will the paint job take if they work together?

Let x = time it will take together (hr)

	Time to Paint (hours)	Fraction of Work Done in 1 hour	Fraction of Work Done in x hours
Stella	4	$\frac{1}{4}$	$\frac{x}{4}$
Jose	3	$\frac{1}{3}$	$\frac{x}{3}$
Together	x	$\frac{1}{x}$	$\frac{x}{x} = 1$

$$\frac{x}{4} + \frac{x}{3} = 1$$

$$(12)\frac{x}{4} + (12)\frac{x}{3} = 1(12)$$

$$3x + 4x = 12$$

$$7x = 12$$

$$x = \frac{12}{7} = 1\frac{5}{7} \text{ hrs.}$$

$$\frac{5}{7}\text{hr} \times \frac{60\text{ min}}{1\text{ hr}} = 43\text{ mins}$$

They can paint the room together in 1 hr 43 mins.

Example 2 – Jenny takes 5 hours to install laminate flooring in the kitchen by herself. Mike can do the job alone in 6 hours. How long would it take them if they did it together? Let x = time together to install flooring (hr)

using the template:

$$\frac{x}{5} + \frac{x}{6} = 1$$

$$\frac{x(30)}{5} + \frac{x(30)}{6} = 1(30)$$

$$6x + 5x = 30$$

$$11x = 30$$

$$x = \frac{30}{11} = 2\frac{8}{11}\text{ hr}$$

$$\frac{8}{11}\text{hr} \times \frac{60\text{ min}}{1\text{ hr}} = 44\text{min}$$

They can floor the kitchen together in 2hr 44min.

Example 3 – Evan works twice as fast as JJ. If it takes them 13 minutes & 20 seconds together to shovel snow from the driveway, how long would it take JJ by himself? 800 sec

Let x = time it takes Evan (sec)
Let $2x$ = time it takes JJ (sec)

using template: $\boxed{x \neq 0}$

$$\frac{800}{x} + \frac{800}{2x} = 1$$

$$\frac{800}{x}(2x) + \frac{800}{2x}(2x) = 1(2x)$$

$$1600 + 800 = 2x$$

$$2x = 2400$$

$$x = 1200\text{ sec}$$

in mins: $\frac{1200}{60}$

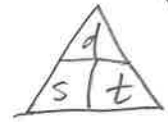
$$= 20\text{ mins.}$$

It takes Evan 20 mins by himself.

It will take JJ 40 mins by himself

Example 4 – A speedboat can travel 108km downstream in the same time it can travel 78km upstream. If the current of the river is 10km/h, what is the speed of the boat in still water?

$$s = \frac{d}{t} \qquad d = st \qquad t = \frac{d}{s}$$



	d (km)	s (km/h)	t (h)	Equation
downstream	108	$x+10$	$\frac{108}{x+10}$	$\frac{108}{x+10} = \frac{78}{x-10}$
upstream	78	$x-10$	$\frac{78}{x-10}$	

Let x = speed of boat in still water (km/h)

$$\frac{108}{x+10} = \frac{78}{x-10} \qquad \boxed{x \neq \pm 10}$$

The speed of the boat in still water is 62 km/h.

$$108(x-10) = 78(x+10)$$

$$108x - 1080 = 78x + 780$$

$$30x = 1860$$

$$x = 62 \text{ km/h.}$$

Enrichment Problem: Example 5 – A car travels from home to work at an average speed of 80km/h, and because of traffic, returns from work at an average speed of 50km/h. What is the average speed of the entire trip (to work and back) if the distance to work is 20km (to the nearest tenth)?

$$\text{Average Speed} = \frac{\text{total distance}}{\text{total time}} = \frac{20+20}{0.25+0.4} = \frac{40}{0.65} = 61.5 \text{ km/h.}$$

	d (km)	s (km/h)	time ($\frac{d}{s}$)
home to work	20	80	0.25
work to home	20	50	0.4

The average speed of the trip is 61.5 km/h.