

2.0 – Naming Triangles and Pythagoras

Name:

Date:

Goal: To learn how to correctly name triangles, their sides and their angles, and to use Pythagoras.

Toolkit:

- Labeling angles and sides of triangles
- All angles in a triangle add to _____
- Pythagoras: $a^2 + b^2 = c^2$ (c is hyp!)
- Labelling triangles from a target angle

Main Ideas:

Definitions

Right triangle –

Equilateral triangle –

Isosceles triangle –

Scalene triangle –

Labelling angles and sides of triangles

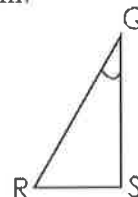
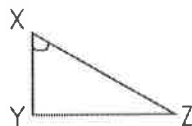
Ex 1) Draw a triangle, $\triangle ABC$, and label all angles and sides.

Label sides using both:

- One lower case variable
- Two endpoints

Three point system of naming angles – An angle is named using the two origins of the angle, and the vertex, with the vertex ALWAYS in the middle!

Ex2) Name each indicated angle using the three point system.

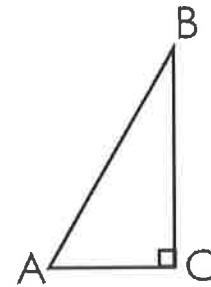


Labelling angles from a target angle

(Only for right triangles!)

In this chapter, we will also want to label the sides of a RIGHT triangle based their position in relation to a target angle which we use as a reference point.

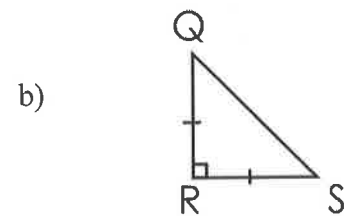
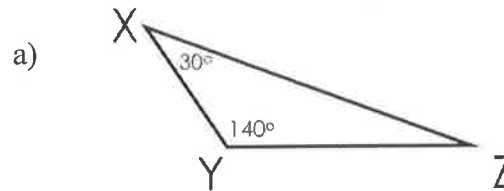
Ex 3) In reference to angle A, label
- the hypotenuse (HYP)
- the side opposite to A (OPP)
- the side adjacent to A (ADJ)



Angles in a triangle

The sum of the angles in a triangle is _____

Ex4) Find the missing angle(s).



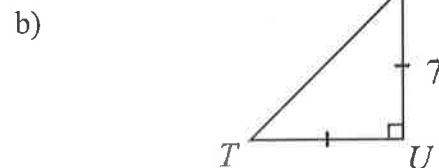
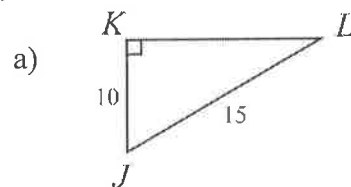
Pythagoras

(Only for right triangles!)

Pythagoras – Remember, “c” MUST be the hypotenuse, or the side across from the right angle!

$$a^2 + b^2 = c^2$$

Ex 5) Name and find the missing side(s) (nearest tenth)



Reflection: Is it possible to have an equilateral triangle that is also a right triangle? Explain.

2.1 – Angles from the Tangent Ratio

Name:

Date:

Goal: to develop the tangent ratio and relate it to the angle of inclination of a line

Toolkit:

- Similar Triangles
- Labeling sides and angles of a triangle

Main Ideas:

Terminology:

Hypotenuse: The longest side of a right triangle (and always opposite the right angle) (HYP)

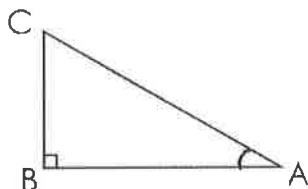
Opposite: The side that does NOT touch the angle (OPP)

Adjacent: The side that DOES touch the angle (and is not the hypotenuse) (ADJ)

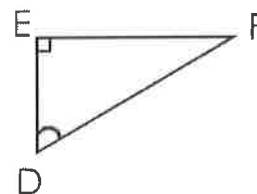
Naming Sides:

We name the sides of a right triangle (a triangle with a 90° angle) in relation to one of its acute angles (one of the angles that is NOT 90°)

Ex1)



TRY:



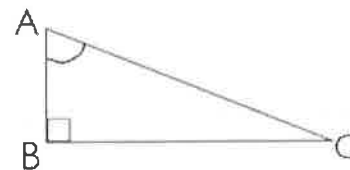
What is trigonometry?

What is the **TANGENT RATIO**?

THE TANGENT RATIO

If $\angle A$ is an acute angle in a right triangle, then:

$$\text{Tan } A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$



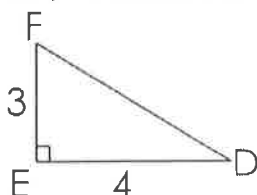
You can use a scientific calculator to find an angle when you know its tangent. The \tan^{-1} operation does this.

- Shift Tan
- or
- 2ndF Tan

Determining the Tangent Ratios for Angles:

*** MAKE SURE CALCULATOR IS IN DEGREE MODE***

Ex2) Determine Tan D and Tan F. Then, determine $\angle D$ and $\angle F$.



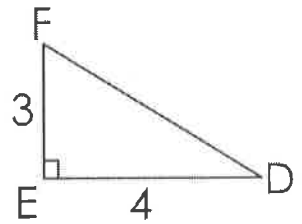
Write Tan D again. At the moment, the Tan D ratio is written as a fraction. Ratios can also be written as decimals. Write Tan D as a decimal:

Tan D =

Trigonometric **ratios** such as tangent can be written as a fraction or a decimal.

Why is the measure of $\angle D = 36.9^\circ$ if the ratio for $\text{Tan } D = \frac{3}{4}$?

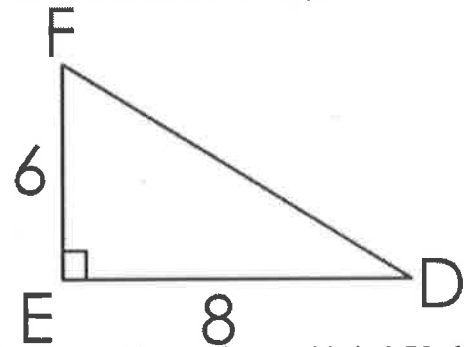
Look at the triangle. If the opposite is less than the adjacent side, $\angle D$ will be less than 45° . Thus, it is 36.9° .
So, why is $\angle F = 53.1^\circ$?



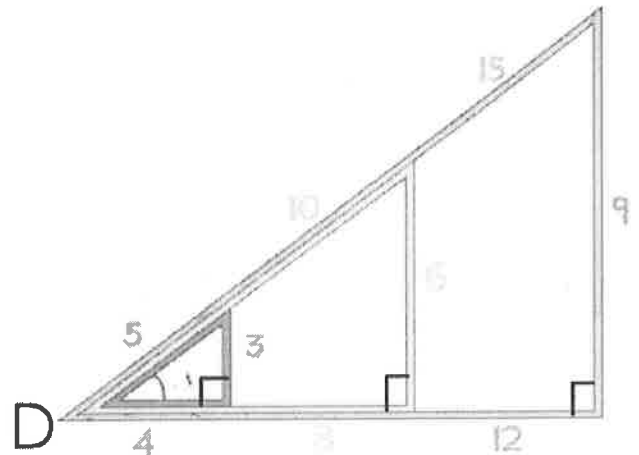
What would the opposite and adjacent sides have to be to have a 45° angle?

What would the other acute angle measure be in this situation?

Back to Triangle DEF:
What would the measure of $\angle D$ be for this triangle?

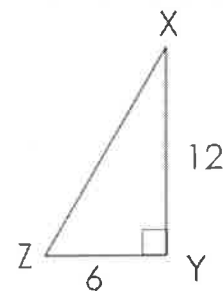


No matter how big the triangle, if the ratio of opposite side to adjacent side is 0.75, then the angle will measure 36.9° .



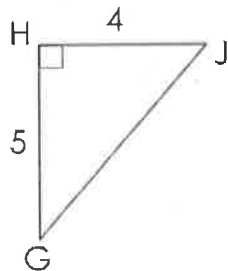
Ex3) Determine Tan X and Tan Z

Now, determine $\angle X$ and $\angle Z$:



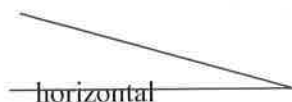
Sketch another right triangle with the same angle measures:

Ex4) Determine the measures of $\angle G$ and $\angle J$ to the nearest tenth of a degree. Start by writing the tangent ratio properly.



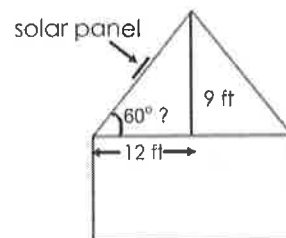
Definition:

Angle of Inclination – This is the ACUTE angle that a line makes with the horizontal



Using the Tan Ratio to Determine the Angle of Inclination:

Ex6) The latitude of Fort Smith, NWT, is approximately 60° . Determine whether this design for a solar panel is best for Fort Smith.



Ex7) A 10ft ladder leans against the side of a building with its base 4ft from the wall. What is the angle of inclination of the ladder?

DRAW A DIAGRAM!

Reflection: You have just studied the Tan ratio, which is the ratio of the opposite side to the adjacent side of a right triangle. What are the other two pairs of sides you could have in a right triangle? (think opp, adj, and hyp!)

2.2 – Sides from the Tangent Ratio

Name:

Date:

Goal: Apply the tangent ratio to calculate lengths of sides of triangles

Toolkit:

- Similar Triangles
- Labeling sides and angles of a triangle
- Tan Ratio (opposite and adjacent sides)

Main Ideas:

Terminology:

Direct Measurement: When we use a measuring instrument (eg. Ruler, protractor) to determine a length or an angle.

Indirect Measurement: When we use math concepts (eg. Trig, Pythagoras) to calculate a length or an angle

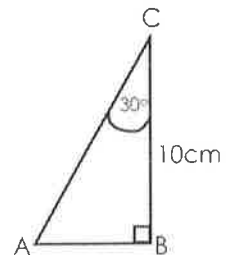
We can use the Tan ratio as a tool to calculate the length of a side of a right triangle *indirectly*.

Steps:

- 1) Use the Tan ratio ($\frac{\text{opposite}}{\text{adjacent}}$) to write an equation
- 2) When we know the measure of an angle (that is NOT the 90° angle!) and the length of one of the legs (not the hypotenuse), solve the equation to determine the length of the other leg.

Determining the Length of a Side **Opposite** a Given Angle:

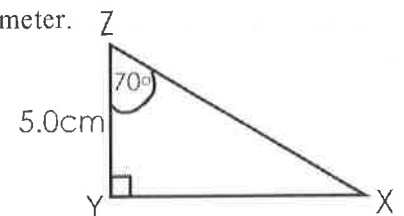
Ex1) Determine the length of AB to the nearest tenth of a centimeter.



REMEMBER:

Calculator **MUST** be in DEGREE mode!

Ex2) Determine the length of XY to the nearest tenth of a centimeter.



Note: when solving a question where you have two equal fractions.....
"multiply the pair, divide by the spare!"

Ex 1. $\frac{3}{1} = \frac{8}{x}$

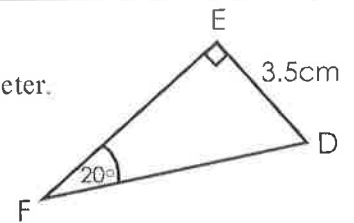
$x = (8 \times 1) \div 3$
 $x = 2.67$

Ex 2. $\frac{2.4}{1} = \frac{y}{11}$

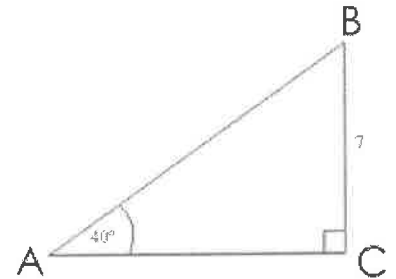
$x = (2.4 \times 11) \div 1$
 $x = 26.4$

Determining the Length of a Side **Adjacent** a Given Angle:

Ex3) Determine the length of EF to the nearest tenth of a centimeter.

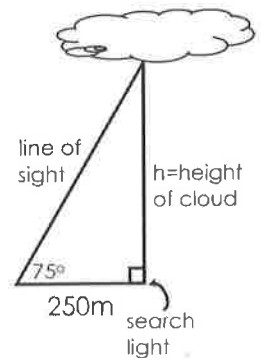


Ex 4) Find side AC to the nearest tenth.



Using the Tan Ratio to Solve an Indirect Measurement Problem:

Ex5) A searchlight beam shines vertically on a cloud. At a horizontal distance of 250m from the searchlight, the angle between the ground and the line of sight to the cloud is 75°. Determine the height of the cloud to the nearest metre.



Ex6) At a horizontal distance of 200m from the base of an observation tower, the angle between the ground and the line of sight to the top of the tower is 8°. How high is the observation tower, to the nearest metre?

Start by sketching and labeling a diagram to represent the information in the problem.....

Reflection: Write, in your own words, how you can find the length of a side by using a known angle and a known side, and using the Tan Ratio.

2.4 – The Sine and Cosine Ratios

Name:

Date:

Goal: to develop and apply the sine and cosine ratios to determine angle measures

Toolkit:

- Labeling sides and angles of a triangle
- What you have learned about the Tan ratio
- Angle of elevation vs depression

(HYP) – **Hypotenuse:** The longest side of a right triangle (and always opposite the right angle)

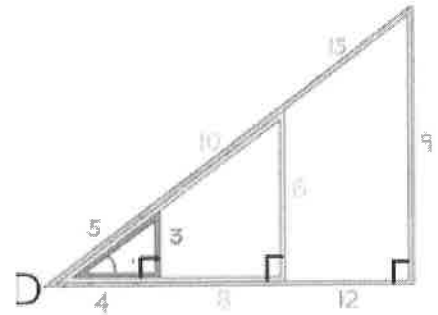
(OPP) – **Opposite:** The side that does NOT touch the angle

(ADJ) – **Adjacent:** The side that DOES touch the angle (and is not the hypotenuse)

Main Ideas:

Remember from yesterday that:
 $\tan D = \frac{\text{opposite side}}{\text{adjacent side}}$

If the ratio of opposite to adjacent doesn't change, then angle D doesn't change. Look at angle D compared to the length of the opposite vs. the length of the adjacent sides.



You can also compare the ratio of other pairs of sides compared to the target angle. Tangent ratio is opposite side divided by adjacent side, but you can make a ratio with each of these sides and the hypotenuse, and these ratios can be related to the size of the target angle:

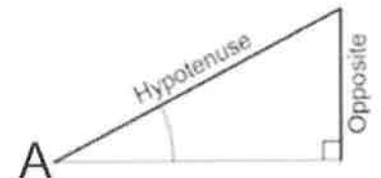
THE SINE RATIO

If $\angle A$ is an acute angle in a right triangle, then

$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

What values will the sine ratio always be between?

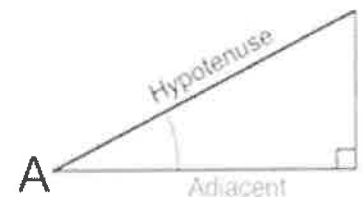
If the opposite side is quite small compared to the hypotenuse, will the target angle be closer to 0° or 90° ?



THE COSINE RATIO

If $\angle A$ is an acute angle in a right triangle, then

$$\cos A = \frac{\text{length of side adjacent } \angle A}{\text{length of hypotenuse}}$$



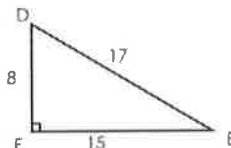
What values will the cosine ratio always be between?

If the adjacent side is quite small compared to the hypotenuse, will the target angle be closer to 0° or 90° ?

S O H C A H T O A

Determining the Sine and Cosine of an Angle, and Determining the Measure of the Angle

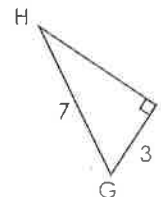
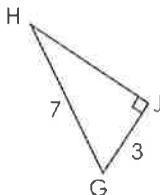
Ex1) a) In triangle DEF, identify the side opposite $\angle D$, the side adjacent to $\angle D$, and the hypotenuse



b) Determine the ratios $\sin D$ and $\cos D$, and give the values as decimals (nearest hundredth)

c) Determine angles D and E to the nearest tenth

Ex2) Determine the measures of $\angle G$ and $\angle H$ to the nearest tenth of a degree.



Using Sine or Cosine to Solve a Problem

Ex3) A water bomber is flying at an altitude of 5500 ft. The plane's radar shows that it is 9000 ft from the target site in a forest fire. What is the angle of elevation of the plane measured from the target site, to the nearest degree?

Reflection: What will you do to remember the calculator steps when finding an ANGLE (whether it's a sin, cos, or tan problem)?

2.5 – Missing Sides from Sine and Cosine

Name:

Date:

Goal: to use the sine and cosine ratios to determine lengths indirectly.

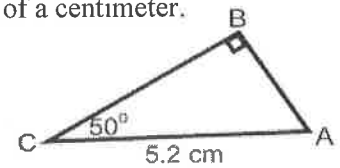
Toolkit:

- What you have learned about the Tan ratio
- Angle of elevation vs depression
- SOHCAHTOA

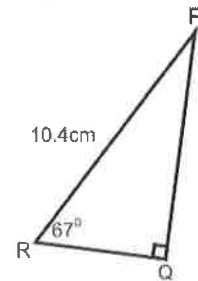
Main Ideas:

Using the sine or cosine ratio to determine the length of a leg

Ex1) Determine the length of side a to the nearest tenth of a centimeter.

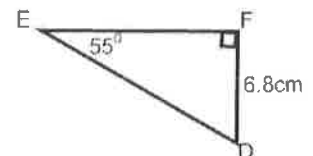


Ex2) Determine the length of side r to the nearest tenth of a centimeter.



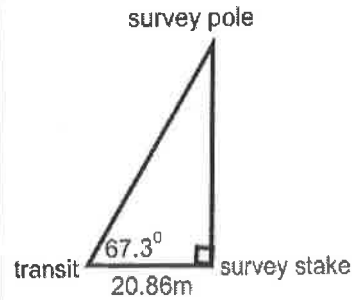
Using the sine or cosine ratio to determine the length of the hypotenuse

Ex3) Determine the length of side f to the nearest tenth of a centimeter.



Solving an Indirect Measurement Problem

Ex4) A surveyor made the measurements shown in the diagram. How could the surveyor determine the distance from the transit to the survey pole to the nearest hundredth of a metre?



Ex5) From a radar station, the angle of elevation of an approaching airplane is 32.5° . The horizontal distance between the plane and the radar station is 35.6km. How far is the plane from the radar station to the nearest tenth of a kilometer? (Draw a picture!)

Reflection: Explain when you might use the sine or cosine ratio instead of the tangent ratio to determine the length of a side in a right triangle.

2.6 – Solving Triangles

Name:

Date:

Goal: Use a trigonometric ratio to solve a problem involving a right triangle

Toolkit:

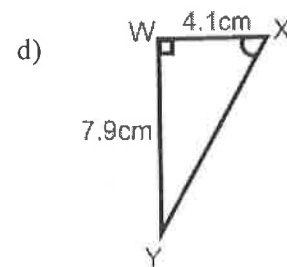
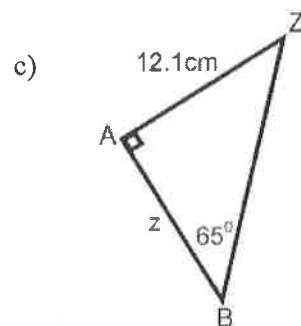
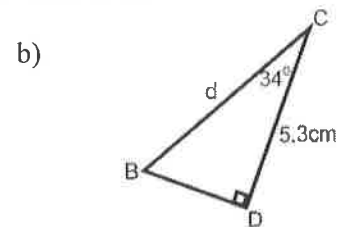
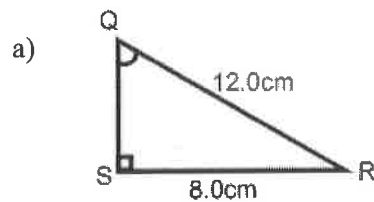
- $S \frac{O}{H} C \frac{A}{H} T \frac{O}{A}$ (SOHCAHTOA!)
- The sum of the angles in any triangle is 180°
- Pythagoras $\rightarrow a^2 + b^2 = c^2$

Main Ideas:

Which Trig Ratio should be used?

Find the missing angle or side using trig...

Ex1) To determine the measure of the indicated angle or side, which trig ratio would you use? Why? Then find the indicated angle or side, to the nearest tenth of a degree.

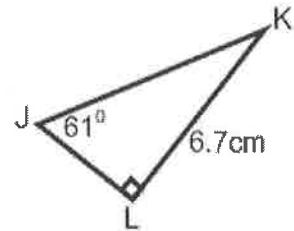


How do you *SOLVE* a triangle?

Solving a triangle means to determine the measures of all the angles and the lengths of all the sides in a triangle. We will need to use:

- $S^o_H C^o_H T^o_A$
- The sum of the angles in any triangle is 180°
- Pythagoras $\rightarrow a^2 + b^2 = c^2$

Ex2) Solve $\triangle JKL$. Give measures to the nearest tenth.



How do you solve a triangle without the picture of the triangle?

Ex3) In right triangle $\triangle KMN$, $\angle M = 90^\circ$, $KM = 8\text{cm}$, and $MN = 9\text{cm}$. Solve this triangle. Give measures to the nearest tenth.

(Draw and label the triangle, then solve!)

Reflection: What is the advantage of determining the unknown angle before the unknown sides?

2.7A – Applications, One Triangle

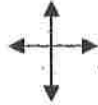
Name:

Date:

Goal: to apply trigonometry to solve problems with one right triangle

Toolkit:

- $S \frac{O}{H} C \frac{A}{H} T \frac{O}{A}$ (SOHCAHTOA!)
- The sum of the angles in any triangle is 180°
- Pythagoras $\rightarrow a^2 + b^2 = c^2$
- Perimeter
- Equilateral, isosceles, scalene
- Compass



Main Ideas:

Ex1) A whale-watching boat is stopped near a rock to look at some sea lions. Then it goes 95m due west to head towards a possible whale sighting. The captain points out a pod of whales, which the radar shows are 140m north of the boat. How far are the whales from the sea lions, and what is the angle at the rock (between the boat's path and the whales' direct line to the sea lions)?
Answer to the nearest tenth.

Ex2) As Sam is driving, she sees a sign telling her that the road has a 7% grade (i.e., a rise of 7 meters for a horizontal change of 100m).

- a) What is the angle of inclination of the road? (nearest degree)
- b) If she travels 500m along the road, how much has she risen vertically? (nearest meter)

Reflection: What are two things YOU can do (there are lots!) to help make sure you can solve an application question like the ones in this section?

2.7B – Applications, Two Triangles

Name:

Date:

Goal: to apply trigonometry to solve problems with two right triangles

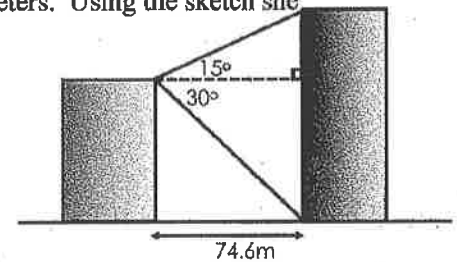
Toolkit:

- $S \frac{O}{H} C \frac{A}{H} T \frac{O}{A}$ (SOHCAHTOA!)
- The sum of the angles in any triangle is 180°
- Pythagoras $\rightarrow a^2 + b^2 = c^2$
- A PLAN: you'll need to come up with a PLAN to use more than one triangle to help you answer the question before you jump in.
- Try re-drawing the pieces.

Main Ideas:

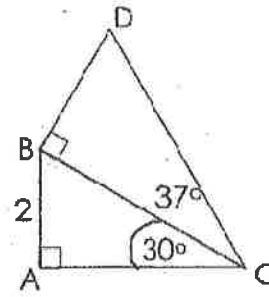
2-D

Ex1) From the top of one building, a surveyor measures the angle of elevation to the top of another (taller!) building, and the angle of depression to the base of the other building. The distance between the buildings is 74.6 meters. Using the sketch she made, find the height of the buildings (nearest tenth).

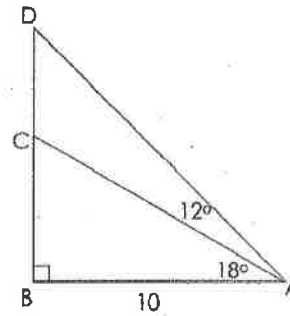


Ex2) For each questions, write out a PLAN to find the missing side CD.

a) Find the length CD. What's the PLAN?



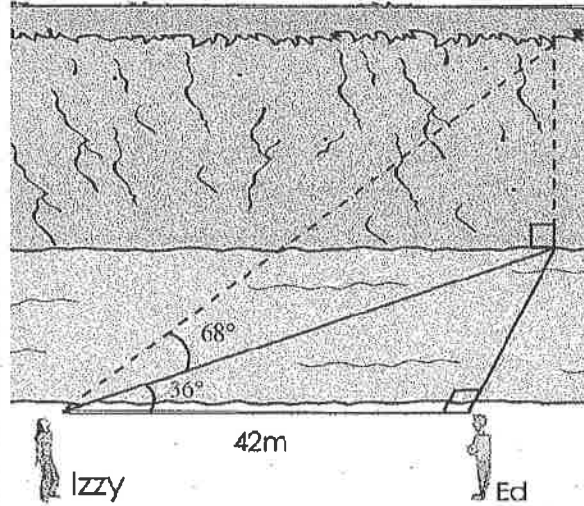
b) Find the length CD. What's the PLAN?



3-D

Hard to picture: try looking for right angles!

Ex3) Izzy and Ed positioned themselves 42 m apart on one side of a stream. Izzy recorded angles, as shown below. Find the height of the cliff on the other side of the stream (nearest tenth).



Reflection: What do you have to think about when you draw a diagram with triangles in three dimensions?