

## Arithmetic Sequences

Focus: To understand and apply the concept of arithmetic sequences

Warmup - The starting salary of an employee is \$21 250. If a raise of \$1250 is given each year, in how many years will the employee's salary be \$50 000?

vocabulary

The problem above can be solved using a formula, which we will develop:

A **sequence** is simply an ordered list of numbers (called **terms**) that follow a pattern so that the next term can be determined.

Example: 4, 7, 10, 13, 16      $t_1 =$       $n =$       $t_4 =$       $d =$

- The **first term** in the sequence is labeled  $t_1$
- The **number of terms** in the sequence is  $n$
- Any term of the sequence is  $t_n$  (read  $t$  sub  $n$ ), dependent on the value of  $n$ . For example, the **third term** is  $t_3$ , the **eighth term** is  $t_8$  etc.
- A **finite sequence** has a finite number of terms whereas an **infinite sequence** has an infinite number of terms
- An **ARITHMETIC SEQUENCE** is an ordered list of terms in which the difference between consecutive terms is constant (a **common difference** ( $d$ ))

For the salary problem, what is  $t_1$ ? What is  $d$ ?

$t_1 =$

$d =$

How can you find  $t_2$  using  $t_1$  and  $d$ ?  $t_2 =$      in general:  $t_2 =$

How can you find  $t_3$  using  $t_1$  and  $d$ ?  $t_3 =$      in general:  $t_3 =$

How can you find  $t_4$  using  $t_1$  and  $d$ ?  $t_4 =$      in general:  $t_4 =$

Can you develop a formula for the general term of an arithmetic sequence?

formula for  
an arithmetic  
sequence

Use the formula to answer the problem from the top of the page:

Example 1 – Find the 33<sup>rd</sup> term of the arithmetic sequence:  $-10, -4, 2, 8, 14, \dots$

Example 2 – You notice a few carpenter ants in your basement and by the end of the first month there are 40 ants. The growth produces an arithmetic sequence in which the number of ants increases by approximately 80 ants each month. How many months in total until the colony reaches a population of 3000 ants?

Example 3 – A furnace technician charges \$35 for making a house call, plus \$46 per hour.

- a) Generate the possible charges for the first 4 hours of time.
- b) What is the charge for 10 hours of time?

Example 4 – If an arithmetic sequence starts with  $-21$ , and the 15<sup>th</sup> term is  $105$ , find the common difference,  $d$ .

## Arithmetic Series

*Focus: To understand and apply the concept of Arithmetic Series.*

**Gauss'  
method**

Warmup – When the famous mathematician Gauss was young, his teacher tried to keep him busy and asked him to sum the numbers from 1 to 100. It only took Gauss a few minutes! How did he do this so fast?

Use Gauss' Method to calculate the sum of the numbers from 1 to 8 (show your work).

Building off of the example above, we'll derive a general formula for the sum of an arithmetic series using  $S_n$  as the sum of an arithmetic series,  $t_1$  as the first term,  $n$  as the number of terms, and  $d$  as the common difference.

What does the 100 represent in Gauss' formula?

What does the 101 represent?

Write a general formula for Gauss' method:

**formula for  
the sum of an  
arithmetic  
series**

The sum of an arithmetic series can be determined using the formula

where  $t_1$  = first term,  $t_n$  is the last term,  $n$  is the number of terms, and  $S_n$  is the sum of the first  $n$  terms

From last class,  $t_n = t_1 + (n - 1)d$ , so the formula for  $S_n$  can be written another way.

The sum of an arithmetic series can also be determined using the formula:

Example 1 – Find the sum of:  $-12 + -5 + 2 + 9 + 16 + 23 + 30 + 37 + 44 + 51$

Example 2 – Fireflies flash in patterns to signal location or ward off predators. Suppose a firefly flashes twice in the first minute, four times in the second minute, and six times in the third minute.

- If this pattern continues, what is the number of flashes **in the** 42<sup>nd</sup> minute?
- What is the **total number** of flashes for the male firefly after 42 minutes?

using the correct formula

Example 3 – For the arithmetic series, determine the value of  $n$ :  $t_1 = -6$ ,  $t_n = 21$ ,  $S_n = 75$ . Then, for the following series, determine  $t_1$ :  $d = 0.5$ ,  $S_n = 218.5$ ,  $n = 23$ .

**Sigma Notation**

$\Sigma$

**Example:**

Last value of  $n$

$\sum_{n=1}^6 2n$

Formula for each term

First value of  $n$

$2(1) + 2(2) + 2(3) + 2(4) + 2(5) + 2(6)$

Example 4 – Find (a)  $\sum_{n=3}^7 2n + 1$

(b)  $\sum_{n=1}^6 3 + n$

## How do People Get Paid for Their Work?

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*Focus: Identify, investigate, & calculate using the different payment styles for employment.*

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As a class, let's discuss, name, & record the 4 most popular payment structures for people at their job/career:

Which three will we focus on today?

How often are people usually paid?

Name any benefits you may receive at jobs/careers:

What is a **pension**?

What is **CPP**?

What is **EI**?

What is **gross pay**?  
**net pay**?

Ex1 – An employee has a semi-monthly salary of \$2 687.50. The annual bonus for employees was 6% of the annual salary. How much was the **annual gross pay**?

Ex2 -

Jenna worked 40 hours last week at \$12.55/hr. She had deductions of 14% income tax, 3% EI, 4% CPP, & 5% medical. What is her net pay?

<b>Gross Pay</b>		
<b>Deductions:</b>		
Income Tax		
EI		
CPP		
Medical		
<b>Net Pay</b>		

Ex3 – A graduate student has a salary of \$2000 per month as a teaching assistant, and worked 450 hours at the library at \$16/hr. How much did she gross for the year?

Ex4 – Joe makes \$13.60 for the first 40 hours per week, then ‘time-and-a-half’ for any hours after. Last week, he worked 49 hours. As well, he does taxes for his friends at \$35 each, and did six of those. How much did he make last week?  
b) What payment styles does Joe have for each of his jobs?

## Commission Pay

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*Focus: To investigate the different types of commission pay structures for jobs/careers.*

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### Warmup:

What are the three payment styles from last day's notes? Define each.

What is **commission**?

What kinds of commission styles exist in jobs/careers?

Ex1 – Evan earns straight commission of 12% at his chemical sales job. Last year, he does \$750 000 in sales. How much did he earn?

Ex2 – Charlotte makes 9% commission and earns an **extra** 5% commission when sales exceed \$500 000. Last year, she did sales of \$810 000. How much did she earn?

Ex3 – Ernie earns an annual salary of \$55 000 with 5.5% commission

on sales. Last year, he did \$475 000 in sales. How much did he earn?

Ex4 – Jenny takes a 'draw against commission'. She was given \$3000 biweekly. She did \$800 000 in sales at 10% commission. Will Jenny get more money at the end of the year, or will she have to pay back some money? How much?

Ex5 – Jeff makes a base salary of \$48 000/yr and earns 1% residual commission on his 3 highest existing accounts. Last year, his three highest accounts had sales of 1.2 million, 1 million, and \$850 000. How much did Jeff earn last year?

## Simple Interest

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*Focus: To understand and apply the simple interest formula to situations.*

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### Warmup:

- 1) Change 62% to dec
- 2) Change 8% to dec
- 3) Change 3.5% to dec
- 4) Change 0.09 to %

If you get a loan, you usually have to pay it back with interest.

- a) What does this mean?
  
- b) Why do you think this is the case?

Can you define **Interest?**

Sometimes, interest works against you. Describe a scenario where this is the case.

Sometimes, interest works for you. Describe a scenario where this is the case.

Why does a bank store money for you and pay you interest, yet they make huge profits?

Ex1 – Suppose you invested \$100 at 4% interest for a year. How much would you now have?

Ex2 – (a) Suppose you invested \$42 000 at 6% for a year. How much would you now have?

b) What if you invested it at simple interest for 12 years? How much would you have?

Here is the formula for **Simple Interest:**

Ex3 – Use the formula to find out how much interest you would pay if you borrowed \$12 000 at 7.5% interest over 8 years.

Sometimes, the situation is under 1 year. In that case, make the applicable **proper fraction** for  $t$ .

Ex4 – You borrow \$5000 for 7 months at 12% simple interest. How much will you pay back in total?

Ex5 – Suppose you won the lottery and invested \$3 000 000 at simple interest at 5%. How much interest would you earn every week?

Ex6 – You make \$540 interest on \$4000 over 3 years. What is the interest percentage?

## Compound Interest

*Focus: To understand and apply the compound interest formula to situations.*

### Warmup

Evaluate:

- a)  $2^7$
- b)  $4^8$
- c)  $(1 + 0.075)^{30}$
- d)  $(1 + 0.12)^4$

From yesterday, simple interest is when you \$500 interest pay/earn interest on the principal (starting) value:

Can you describe what **compound** interest is?

This table shows the clear difference between simple and compound

Ex1 – Eva invests \$30 000 for 3 years at 5% compd int. How much will she have?

Use the exponent

button on your

calculator:

$x^y$  or  $y^x$  or  $^$  or  $x$



$P = \$5000, r = 0.10$  (10%)  
 Ex: Year 1: 10% of \$5000 is \$500 so

you make

Year 2: 10% of \$5000 is \$500 so you make \$500 interest etc...etc...so you make \$500 interest each year

Ex: Year 1: 10% of \$5000 is \$500 so you make \$500 interest  
 Year 2:

Year	Simple Interest			Compound Interest		
	P	I	Total	P	I	Total
1	5000	500	5500	5000	500	5500
2	5000	500	6000	5500		
3	5000	500	6500			
4	5000	500	7000			
5	5000	500	7500			
6	5000	500	8000			
7	5000	500	8500			

interest:

*\*Remember: for compound interest, the interest added each year becomes part of next year's principal, so you make interest on the interest!!*

Year 1:  $(30\ 000)(0.05)(1) =$   
 Year 2:

Ex1 –

Or, you can find the answer using the compound interest formula.

Ex2 – Watch ‘Act 1’ of **Fry’s Bank**.

What do we know so far?

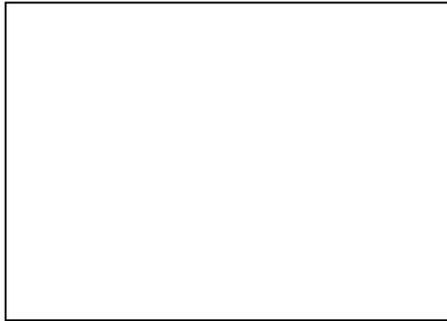
Ex3 – Jesse invests \$3500 at 4% compd int over 9 years. How much will he have?

Ex4 – Fran invests \$40 000 at 4.75% over 14 years. How much more will he have if he uses compound interest rather than simple?

Ex5 – Maddy has \$6303.07 after investing her principal for 6 years at 7% compound int. How much did she start with?

Ex6 – Let’s work out **Fry’s Bank** & then watch the video solution.

Let’s explore the **Compound Interest Formula** a bit further to see why it works:



*continued*

**Using the formula:**

**How much would Fry have if it was simple interest instead?**

