

Key

Name: _____

Date: _____

Arithmetic / Interest Unit Review

1) What are the following in terms of arithmetic sequences / series:

n = number of terms

t_n = last term

t_1 = first term

d = common difference

S_n = sum of arithmetic series

2) For the following arithmetic sequence, find the 58th term: 120, 114, 108, ...

$$t_1 = 120$$

$$t_n = t_1 + (n-1)d$$

$$d = -6$$

$$t_{58} = 120 + (58-1)(-6)$$

$$n = 58$$

$$t_{58} = 120 + (-342)$$

$$t_{58} = -222$$

3) Find the first term of an arithmetic sequence with $t_{11} = 100$ and $d = 8$.

$$n = 11$$

$$t_n = t_1 + (n-1)d$$

$$d = 8$$

$$100 = t_1 + (11-1)(8)$$

$$t_{11} = 100$$

$$100 = t_1 + 80$$

$$t_1 = 20$$

4) If the first term of an arithmetic sequence is -25, and the last term is 20, with a common difference of 3, find the number of terms.

$$t_1 = -25$$

$$t_n = t_1 + (n-1)d$$

$$t_n = 20$$

$$20 = -25 + (n-1)(3)$$

$$d = 3$$

$$45 = 3(n-1)$$

$$15 = n-1$$

$$n = 16$$

5) Molly starts with 21 cards and adds 18 baseball cards each month to her collection. How many cards will she have in 3 years? *Think carefully about what your 'n' value will be.

21, 39, 57, 75, ...

↑
after
1 month

$$n = 36 + 1 = 37$$

$$t_1 = 21$$

$$d = 18$$

$$t_n = t_1 + (n-1)d$$

$$t_{37} = 21 + (37-1)(18)$$

$$t_{37} = 21 + 648$$

$$t_{37} = 669$$

6) Jon is 14 and has a birthday party. Every 3 years after that, up to and including his 50th birthday, he has another party. How many parties has Jon had in total?

$$14, 17, 20, \dots, 50 \quad t_n = t_1 + (n-1)d \quad \left| \begin{array}{l} 12 = n-1 \\ n = 13 \end{array} \right.$$

$$t_1 = 14 \quad 50 = 14 + (n-1)(3)$$

$$d = 30 \quad 36 = 3(n-1)$$

$$t_n = 50 \quad \left. \vphantom{t_n = 50} \right| \text{13 parties}$$

7) Find the sum of the arithmetic series: $-10 + -1 + 8 + \dots + 80$

$$t_1 = -10 \quad \left| \begin{array}{l} t_n = t_1 + (n-1)d \\ n = 11 \end{array} \right. \quad \left. \vphantom{t_n = t_1 + (n-1)d} \right| \begin{array}{l} S_{11} = 5.5(70) \\ S_{11} = 385 \end{array}$$

$$d = 9 \quad 80 = -10 + (n-1)(9) \quad S_n = \frac{n}{2}(t_1 + t_n)$$

$$t_n = 80 \quad 90 = 9(n-1) \quad S_{11} = \frac{11}{2}(-10 + 80)$$

$$\text{need } n \quad 10 = n-1$$

8) Find the sum of an arithmetic series that starts at 37, has a common difference of -6, and has 27 terms.

$$t_1 = 37 \quad S_n = \frac{n}{2}[2t_1 + (n-1)d] \quad \left. \vphantom{S_n = \frac{n}{2}[2t_1 + (n-1)d]} \right| \begin{array}{l} S_{27} = 13.5(74 + (-156)) \\ S_{27} = -1107 \end{array}$$

$$d = -6$$

$$n = 27 \quad S_{27} = \frac{27}{2}[2(37) + (27-1)(-6)]$$

9) If an arithmetic series has $S_7 = 175$, and $t_7 = 40$, find t_1 .

$$n = 7 \quad S_n = \frac{n}{2}(t_1 + t_n) \quad \left. \vphantom{S_n = \frac{n}{2}(t_1 + t_n)} \right| \begin{array}{l} 50 = t_1 + 40 \\ t_1 = 10 \end{array}$$

$$S_7 = 175 \quad 175 = \frac{7}{2}(t_1 + 40)$$

$$t_7 = 40 \quad 175 = 3.5(t_1 + 40)$$

10) If an arithmetic series has a first term of 3, and $S_{14} = 406$, find d .

$$t_1 = 3 \quad S_n = \frac{n}{2}[2t_1 + (n-1)d] \quad \left. \vphantom{S_n = \frac{n}{2}[2t_1 + (n-1)d]} \right| \begin{array}{l} 58 = 6 + 13d \\ 52 = 13d \\ d = 4 \end{array}$$

$$n = 14 \quad 406 = \frac{14}{2}[2(3) + (14-1)d]$$

$$S_{14} = 406 \quad 406 = 7[6 + 13d]$$

11) Find $\sum_{n=2}^5 4n - 5$

$$[4(2) - 5] + [4(3) - 5] + [4(4) - 5] + [4(5) - 5]$$

$$3 + 7 + 11 + 15$$

$$\text{36}$$

12) If you started with \$17 and then were given \$19 after the first minute, then \$21 after the next minute, and this pattern continued, how much would you have after 30 minutes?

$$S_n = \frac{n}{2} [2t_1 + (n-1)d]$$

$$17 + 19 + 21 + \dots$$

$$t_1 = 17$$

$$n = 31$$

$$d = 2$$

$$S_{31} = \frac{31}{2} [2(17) + (31-1)(2)]$$

$$S_{31} = 15.5 [34 + 60]$$

$$S_{31} = \underline{\$1457}$$

13) Calculate the simple interest when \$8500 is invested at 4.6% for 7 years.

$$I = Prt$$

$$I = (8500)(0.046)(7)$$

$$I = \$2737$$

$$\underline{\$2737}$$

14) Calculate the number of years that \$2500 is invested at 6% in order to make \$750 in simple interest.

$$I = Prt$$

$$t = \frac{750}{150} = 5$$

$$750 = (2500)(0.06)t$$

$$750 = 150t$$

$$\underline{5 \text{ years}}$$

15) \$5000 is invested at 3.5% for 4 years simple interest. After 4 years, all of that money is taken and invested at 5% for 6 years simple interest. How much in total will you have after this?

$$I = Prt$$

$$I = (5000)(0.035)(4) = 700$$

$$\text{Total} = 5000 + 700 = 5700$$

$$I = Prt = (5700)(0.05)(6) = 1710$$

$$\text{Total} = 5700 + 1710 = 7410$$

$$\underline{\$7410}$$

16) \$3000 is invested for 4 years at compound interest of 7%. How much total money will you have after 4 years?

$$A = P(1+r)^t$$

$$A = 3000(1+0.07)^4$$

$$A = 3000(1.07)^4$$

$$A = 3932.39$$

$$\underline{\$3932.39}$$

17) Micah ends up with \$21 522.79 after 7 years of compound interest at 3%. How much did he originally invest?

$$A = P(1+r)^t$$

$$P = \$17 500$$

$$21522.79 = P(1+.03)^7$$

$$21522.79 = P(1.03)^7$$

$$21522.79 = P(1.229873865)$$

$$\underline{\$17 500}$$

18a) Joanie wins \$2 000 000 in a lottery. She decides to invest it at 5.5% compound interest for 5 years. How much will she have in total at that point?

$$A = P(1+r)^t$$

$$A = 2\,000\,000(1+0.055)^5$$

$$A = 2\,000\,000(1.055)^5$$

$$\underline{\$2\,613\,920.01}$$

b) How much more interest will she make with compound interest compared to if she invested with the same terms at simple interest?

$$I = Prt$$

$$I = (2\,000\,000)(0.055)(5)$$

$$I = 550\,000$$

$$\text{Total} = 2\,000\,000 + 550\,000 = 2\,550\,000$$

$$\text{difference: } \begin{array}{r} 2\,613\,920.01 \\ - 2\,550\,000.00 \\ \hline 63\,920.01 \end{array}$$

$$\underline{\underline{63\,920.01}}$$

$$\underline{\underline{\$63\,920.01}}$$