Arithmetic Sequences

Focus: To understand and apply the concept of arithmetic sequences

Warmup - The starting salary of an employee is $21 250. If a raise of $1250 is given each year, in how many years will the employee's salary be $50 000?

The problem above can be solved using a formula, which we will develop:

vocabulary

A **sequence** is simply an ordered list of numbers (called **terms**) that follow a pattern so that the next term can be determined.

Example: *4, 7, 10, 13, 16* $\boldsymbol{t_1 =}$ $\boldsymbol{n =}$ $\boldsymbol{t_4 =}$ $\boldsymbol{d =}$

- The **first term** in the sequence is labeled $\boldsymbol{t_1}$
- The **number of terms** in the sequence is $\boldsymbol{n}$
- Any term of the sequence is $\boldsymbol{t_n}$ (read t sub n), dependent on the value of $\boldsymbol{n}$. For example, the **third term** is $\boldsymbol{t_3}$, the **eighth term** is $\boldsymbol{t_8}$ etc.
- A **finite sequence** has a finite number of terms whereas an **infinite sequence** has an infinite number of terms
- An **ARITHMETIC SEQUENCE** is an ordered list of terms in which the difference between consecutive terms is constant (a **common difference (*d*)**)

For the salary problem, what is $\boldsymbol{t_1}$? What is $\boldsymbol{d}$?

$\boldsymbol{t_1} =$ $\boldsymbol{d} =$

How can you find $\boldsymbol{t_2}$ using $\boldsymbol{t_1}$ and $\boldsymbol{d}$? $\boldsymbol{t_2} =$ in general: $\boldsymbol{t_2} =$

How can you find $\boldsymbol{t_3}$ using $\boldsymbol{t_1}$ and $\boldsymbol{d}$? $\boldsymbol{t_3} =$ in general: $\boldsymbol{t_3} =$

How can you find $\boldsymbol{t_4}$ using $\boldsymbol{t_1}$ and $\boldsymbol{d}$? $\boldsymbol{t_4} =$ in general: $\boldsymbol{t_4} =$

Can you develop a formula for the general term of an arithmetic sequence?

formula for an arithmetic sequence

Use the formula to answer the problem from the top of the page:

Example 1 – Find the 33^{rd} term of the arithmetic sequence: *-10, -4, 2, 8, 14, …*

Example 2 – You notice a few carpenter ants in your basement and by the end of the first month there are 40 ants. The growth produces an arithmetic sequence in which the number of ants increases by approximately 80 ants each month. How many months in total until the colony reaches a population of 3000 ants?

Example 3 – A furnace technician charges $35 for making a house call, plus $46 per hour.

a) Generate the possible charges for the first 4 hours of time.
b) What is the charge for 10 hours of time?

Example 4 – If an arithmetic sequence starts with -21, and the 15^{th} term is 105, find the common difference, *d*.

Arithmetic Series

Focus: To understand and apply the concept of Arithmetic Series.

Warmup – When the famous mathematician Gauss was young, his teacher tried to keep him busy and asked him to sum the numbers from 1 to 100. It only took Gauss a few minutes! How did he doe this so fast?

Gauss' method

Use Gauss' Method to calculate the sum of the numbers from 1 to 8 (show your work).

Building off of the example above, we'll derive a general formula for the sum of an arithmetic series using S_n as the sum of an arithmetic series, t_1 as the first term, n as the number of terms, and d as the common difference.

What does the 100 represent in Gauss' formula?

What does the 101 represent?

Write a general formula for Gauss' method:

formula for the sum of an arithmetic series

The sum of an arithmetic series can be determined using the formula

where t_1 = first term, t_n is the last term, n is the number of terms, and S_n is the sum of the first n terms

From last class, $t_n = t_1 + (n - 1)d$, so the formula for S_n can be written another way.

The sum of an arithmetic series can also be determined using the formula:

Example 1 – Find the sum of: -12 + -5 + 2 + 9 + 16 + 23 + 30 + 37 + 44 + 51

Example 2 – Fireflies flash in patterns to signal location or ward off predators. Suppose a firely flashes twice in the first minute, four times in the second minute, and six times in the third minute.

a) If this pattern continues, what is the number of flashes **in the** 42nd minute?
b) What is the **total number** of flashes for the male firefly after 42 minutes?

using the correct formula

Example 3 – For the arithmetic series, determine the value of n: $t_1 = -6$, $t_n = 21$, $S_n = 75$. Then, for the following series, determine t_1: $d = 0.5$, $S_n = 218.5$, $n = 23$.

Sigma Notation

$$\Sigma$$

Example:

Last value of *n*

$$\sum_{n=1}^{6} 2n$$

Formula for each term

First value of *n*

$$2(1)+2(2)+2(3)+2(4)+2(5)+2(6)$$

Example 4 – Find (a) $\sum_{n=3}^{7} 2n+1$ (b) $\sum_{n=1}^{6} 3+n$

Geometric Sequences

Focus: To understand and apply the concept of geometric sequences

definition

A **geometric sequence** is a sequence in which the ***ratio*** of consecutive terms is constant.

Warmup – Suppose you have the geometric sequence 4, 12, 36, 108, …

a) What is t_1?
b) What do you multiply by to get the next term (this is the ***r*** value)?
c) Is the sequence geometric (see the definition above)? In other words, is the ***r*** value consistent throughout the sequence?
d) What is t_5? Explain how you got t_5. Write a general formula for this.
e) Show how to get t_5 using only t_1 and r.
f) Show how to get t_8 using only t_1 and r.
g) What do you notice about the exponent on r compared to n?
h) Write a general formula for t_n for any geometric sequence:

Geometric Sequence formula

The general term of a geometric sequence where n is a positive integer is:

OR

where t_1 is the first term, n is the number of terms, r is the common ratio, and t_n is a general term

common ratio

For a geometric sequence, the **common ratio (*r*)**, can be found by taking any term (except the first) and dividing that term by the preceding term. So $r = \frac{t_n}{t_{n-1}}$

Example – Are the following sequences geometric (ie. Is the ***r*** value consistent)?

a) 2, 4, 6, 8

b) 4, 10, 25, 62.5

Example – Find t_{18} for the following: *3, -6, 12, -24, …*

Example – Find t_1 if $t_5 = 567$ and $t_6 = 1701$.

Example – Bacteria reproduce by splitting into two. Suppose there were three bacteria originally present in a sample. How many bacteria will there be after 8 generations?

Example – Suppose a photocopier can reduce a picture to 60% of its original size. If the picture is originally 42cm long, what length will it be after five successive reductions (to the nearest hundredth)?

Example – In 1990 the population of Canada was approximately 26.6 million. The population projection for 2025 is approximately 38.4 million. If this projection were based on a geometric sequence, what would be the annual growth rate?

Percentages

If a question involves percent **growth**, ***r*** must be greater than 1.

Ex. If there is 30% growth each year, what is the ***r*** value for the problem?

If a question involves a percent reduction, ***r*** must be less than 1 and must represent the percent remaining (not the percent lost).

Ex. If you reduce the size of your savings by 25% per year, what is ***r***?

Simple Interest

Focus: To understand and apply the simple interest formula to situations.

Warmup:

1) Change 62% to dec
2) Change 8% to dec
3) Change 3.5% to dec
4) Change 0.09 to %

If you get a loan, you usually have to pay it back with interest.

a) What does this mean?

b) Why do you think this is the case?

Can you define **Interest**?

Sometimes, interest works against you. Describe a scenario where this is the case.

Sometimes, interest works for you. Describe a scenario where this is the case.

Why does a bank store money for you and pay you interest, yet they make huge profits?

Ex1 – Suppose you invested $100 at 4% interest for a year. How much would you now have?

Ex2 – (a) Suppose you invested $42 000 at 6% for a year. How much would you now have?

b) What if you invested it at simple interest for 12 years? How much would you have?

Here is the formula for **Simple Interest:**

Ex3 – Use the formula to find out how much interest you would pay if you borrowed $12 000 at 7.5% interest over 8 years.

Sometimes, the situation is under 1 year. In that case, make the applicable **proper fraction** for ***t***.

Ex4 – You borrow $5000 for 7 months at 12% simple interest. How much will you pay back in total?

Ex5 – Suppose you won the lottery and invested $3 000 000 at simple interest at 5%. How much interest would you earn every week?

Ex6 – You make $540 interest on $4000 over 3 years. What is the interest percentage?

Compound Interest

Focus: To understand and apply the compound interest formula to situations.

Warmup

Evaluate:

a) 2^7

b) 4^8

c) $(1 + 0.075)^{30}$

d) $(1 + 0.12)^4$

Use the exponent button on your calculator: ☐

x^y or ***y***^{x} or **^** or ***x***

From yesterday, simple interest is when you pay/earn interest on the principal (starting) value:

P = $5000, r = 0.10 (10%)

Ex: Year 1: 10% of $5000 is $500 so you make $500 interest

Year 2: 10% of $5000 is $500 so you make $500 interest

etc…etc…so you make $500 interest each year

Can you describe what **compound** interest is?

Ex: Year 1: 10% of $5000 is $500 so you make $500 interest

Year 2:

This table shows the clear difference between simple and compound interest:

Year	Simple Interest			Compound Interest		
	P	I	Total	P	I	Total
1	5000	500	5500	5000	500	5500
2	5000	500	6000	5500		
3	5000	500	6500			
4	5000	500	7000			
5	5000	500	7500			
6	5000	500	8000			
7	5000	500	8500			

**Remember: for compound interest, the interest added each year becomes part of next year's principal, so you make interest on the interest!!*

Ex1 – Eva invests $30 000 for 3 years at 5% cmpd int. How much will she have?

Year 1: (30 000)(0.05)(1) =

Year 2:

Ex1 –

Or, you can find the answer using the compound interest formula.

Ex2 – Watch '**Act 1**' of ***Fry's Bank***.
What do we know so far?

Ex3 – Jesse invests \$3500 at 4% cmpd int over 9 years. How much will he have?

Ex4 – Fran invests \$40 000 at 4.75% over 14 years. How much more will he have if he uses compound interest rather than simple?

Ex5 – Maddy has \$6303.07 after investing her principal for 6 years at 7% compound int. How much did she start with?

Ex6 – Let's work out ***Fry's Bank*** & then watch the video solution.

Let's explore the **Compound Interest Formula** a bit further to see why it works:

continued

Using the formula:

How much would Fry have if it was simple interest instead?